The Dark Matter Problem

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Implications of astronomical data.

- James Jeans (1922): **Local Dark Matter** - vertical motions of stars near the plane of our Galaxy – spatial density of known stars insufficient.

- Fritz Zwicky (1933): **Global Dark Matter** - velocities in the Coma cluster of Galaxies - velocities too large by factor of 10 if taking into account estimated visible mass.

- Bancock (1939): **rotation curves of galaxies** in spiral galaxies – Andromeda Galaxy M31 – unexpectedly high rotational velocity in outer region.

Implications of astronomical data

- Einasto (1974) - **dynamics and morphology of companion galaxies** (pairs of galaxies) – **Dwarf Spheroidal Galaxies (M/L ratio up to 1000 in solar units)** - the mass of galactic coronas or halos *exceeds* the mass of polulations of known stars by one order of magnitude.

- This astronomical evidence of *dark matter* is confirmed by recent observations and analysis, note that *local dark matter* is probably baryonic (low mass and luminosity stars, jupiters,…), *global dark matter*, if it corresponds to an exotic particle species, is probably non-baryonic (*primordial nucleosynthesis constraints*). Another way to explain the observations correpsonds to **Modified Newtonian Gravity or generally modified gravity models.**

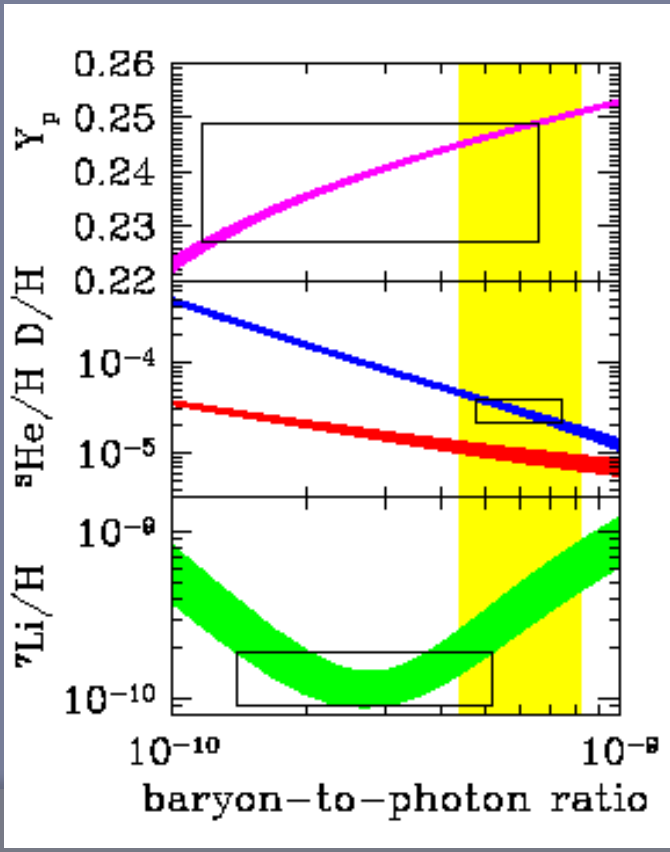
Arguments for Dark Matter from Cosmology

- **Cosmological expansion**: from the expansion speed one can estimate the critical density (flat FLRW universe model) The mean density can be estimated from (luminous) masses of galaxies and of intergalactic gas – the mean (luminous) density of universe is a few percent of the critical density. Observations (CMBR, Cosmic Microwave Background Radiation) imply that on large scales the universe is very close to flatness.

- **Structure formation and the CMBR**: primordial density fluctuations are the seeds for structure formation – this process is rather slow – in the radiation dominated epoch baryonic matter does not reach suitable perturbation amplitude - radiation pressure slows the process - collisionless but gravitating dark matter is not affected significantly.

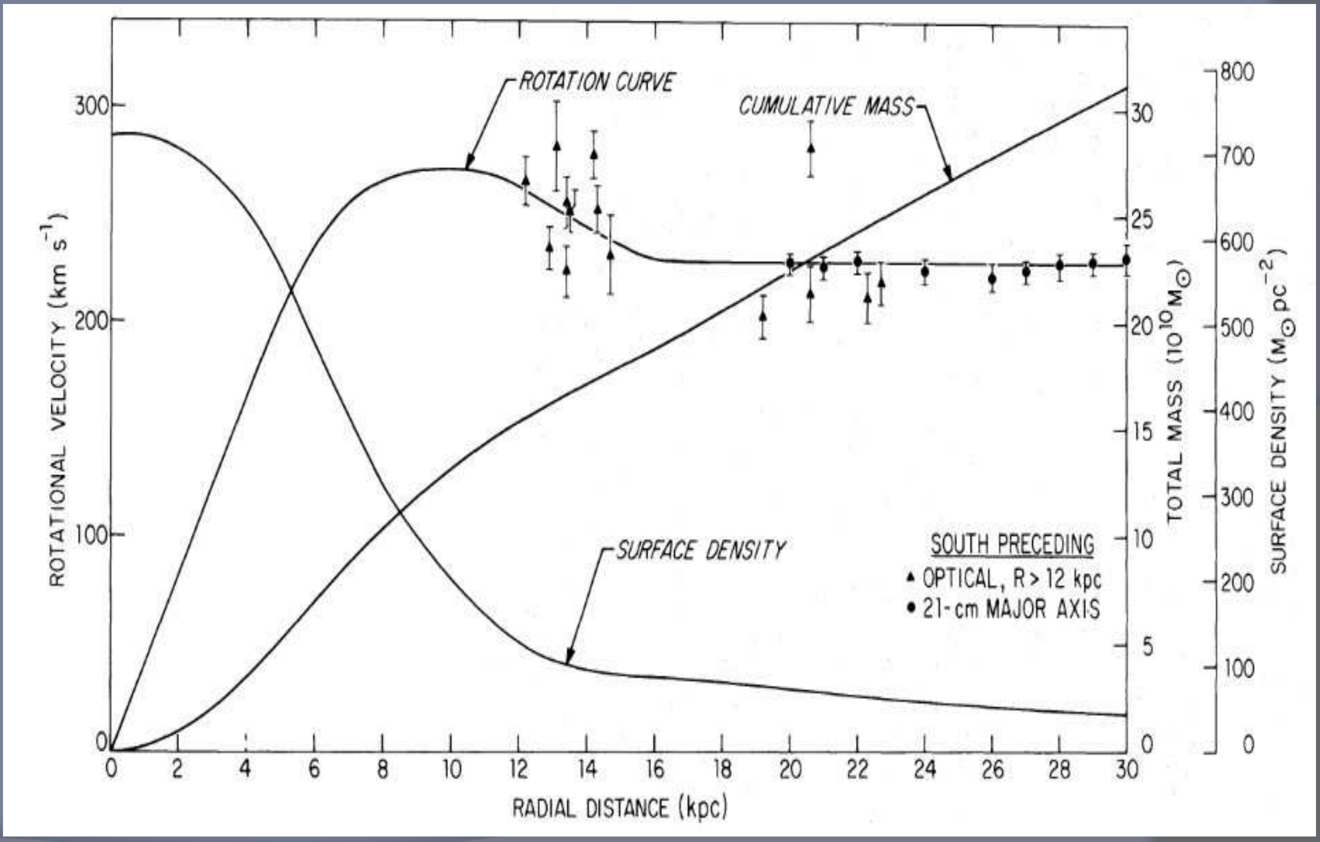
Implications for the nature of Dark Matter from cosmology

- Primordial nucleosynthesis: the total amount of baryonic matter cannot be higher than 0.04 of the critical density – the denser the primordial proton and neutron plasma, the greater the amount of light elements that will be formed.



Methods.

- Velocity dispersions – using virial relations one obtains the total mass M.
- Analysis of rotation curves in spiral galaxies – need for additional mass distributions.

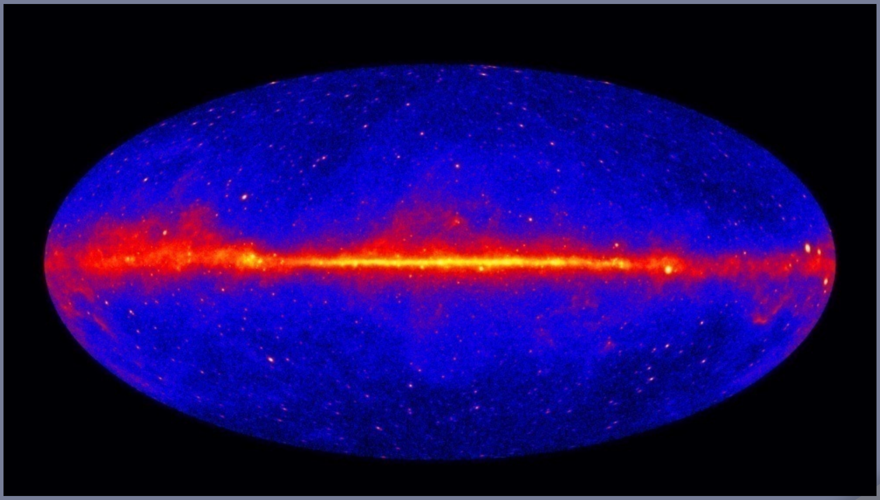


Methods

- Considering tidal effects on satellite or companion galaxies - one obtains a constraint the total mass.

Methods

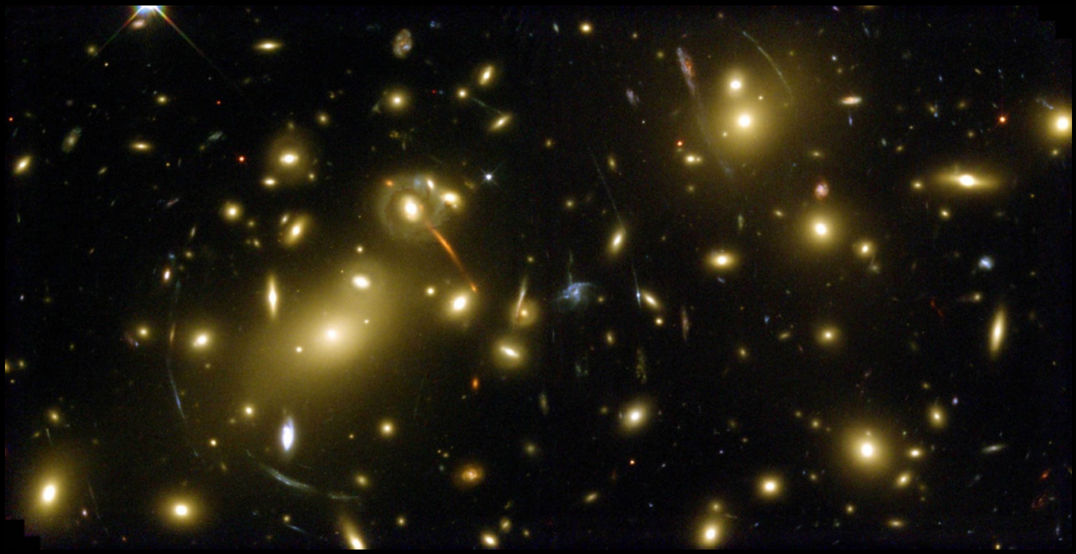
- X-ray data on galaxies and galaxy clusters: hot gas moves under the influence of the gravitational field of the systems. Advantages: collisional fluid is isotropically distributed and the hydrostatic method yields mass as function of radius (not only total mass).



Methods

- **Galactic and extragalactic lensing:**

strong, weak and micro-lensing.



Both previous methods (X-rays and gravitational lensinig)

CONFIRM

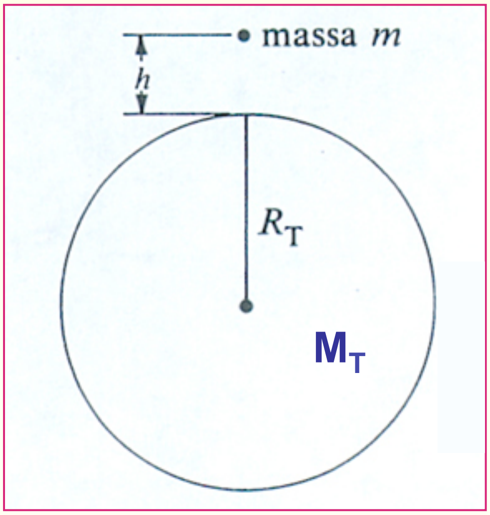
the results for the mass using the virial method, see *Jaan Einasto, "Dark Matter", ArXiv:0901.0632 (2010), asro-ph.CO*

Prerequisites for the Virial Method: newtonian gravity.

$$\vec{F} = -G\,\frac{m_1 m_2}{r^2}\,\hat{r}$$

$$G = 6,67 \cdot 10^{-11} N\,\frac{m^2}{kg}$$

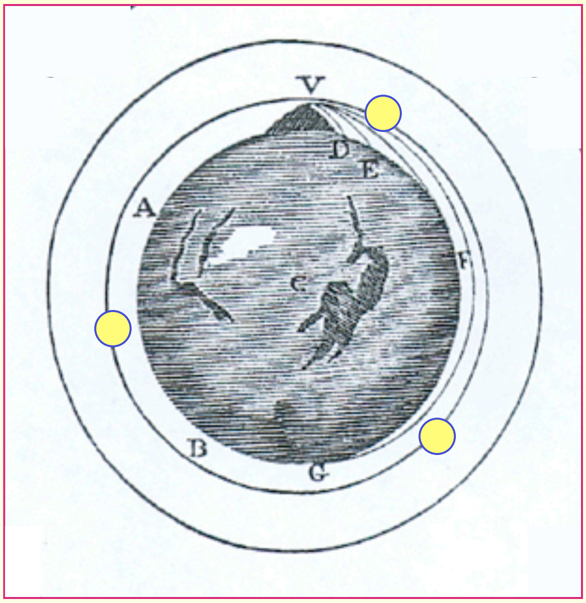
The value of G is known only empirically.



$$F = G\,\frac{mM_T}{(r)^2} = G\,\frac{mM_T}{(R_T+h)^2} \sim G\,\frac{mM_T}{R_T^2} = ma$$

$$a = g = G\,\frac{M_T}{R_T^2}$$

Newton's insight



Prediction of Newton for the fall of the moon in 1s.

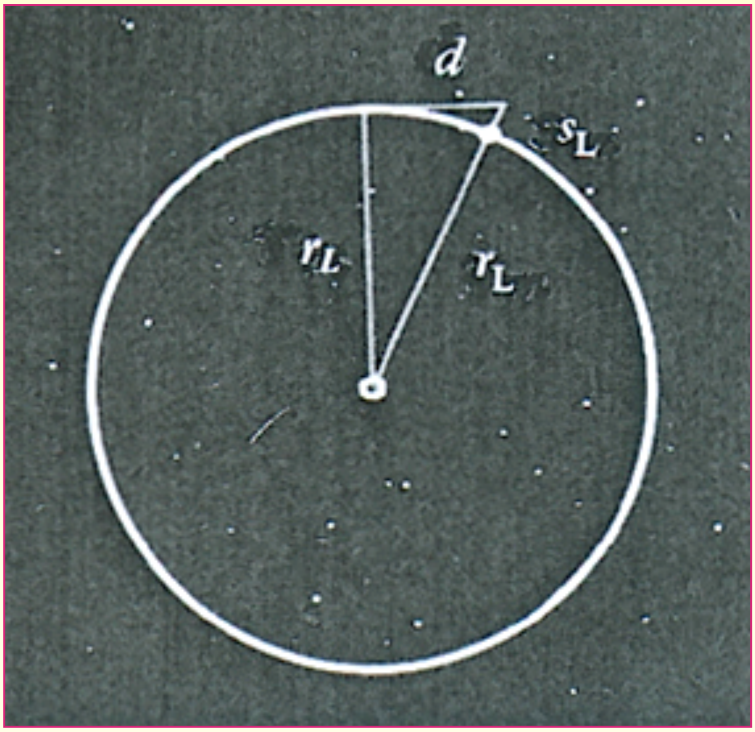
$$s_{mela} = s_m = \frac{1}{2}gt^2 \qquad s_{Luna} = s_L = \frac{1}{2}at^2$$

$$g = G\frac{M_T}{R_T^2} > a = G\frac{M_T}{r_L^2}$$

$$\frac{s_L}{s_m} = \frac{\frac{1}{2}at^2}{\frac{1}{2}gt^2} = \frac{a}{g} = \frac{\frac{GM_T}{r_L^2}}{\frac{GM_T}{R_L^2}} = \frac{R_T^2}{r_L^2} = \frac{1}{60^2} = \frac{1}{3600}$$

$$s_L = \frac{4,9m}{3600} = 0,0014m = 0,14cm$$

An apple close to the earth should fall 4,9 m in 1 s;
In the same time, the moon should fall 0,14 cm.



Pithagoras:

$$(r_L + s_L)^2 = r_L^2 + 2r_L s_L + s_L^2 = r_L^2 + d^2$$

$$2r_L s_L + s_L^2 = d^2;\ s_L^2 \ll r_L \Rightarrow 2r_L s_L = d^2$$

$$s_L = \frac{d^2}{2r_L}$$

$$\frac{d}{2\pi r_L} = \frac{1s}{1mese} = 4,2 \cdot 10^{-7}$$

$$d = 2\pi r_L (4,2 \cdot 10^{-7}) = 2\pi\,(384400\,Km)(4,2 \cdot 10^{-7}) = 1,01\,Km$$

$$s_L = \frac{(1,01Km)^2}{2 \cdot 384400\,Km} = 13.10^{-7} Km = 0,13\,cm$$

Keplerian orbits

$$G \frac{M_S m}{r^2} = m a = m \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM}{r}} \qquad \mathrm{v} = \frac{2\pi r}{T}$$

$$G \frac{m M_S}{r^2} = \frac{m\, 4\pi^2 r^2}{r T^2}$$

$$T^2 = \left(\frac{4\pi^2}{G M_S} \right) r^3$$

$$T^2 = k r^3$$

The Virial Theorem

It is sometimes referred to as a statistical theorem, and states that, if the system is in a steady state then:

$$2\overline{T} + \overline{V} = 0$$

$\overline{T}$ Is the mean (total) kinetic energy averaged over an indetermined interval of time

$\overline{V}$ Is the mean (total) potential energy averaged over an indetermined interval of time

The Virial Theorem

In the case of a two body system, with one the of masses being much greater than the other:

$$2 \cdot \frac{v^2}{2} m - \frac{GMm}{r} = 0$$

and thus one gets keplerian velocities:

$$v = \sqrt{\frac{GM}{r}}$$

The Virial Method

1) Measure the average radial velocity w.r.t. the sun

2) Determine the velocity dispersion

3) Determine the total mass of the system

TABLE 1—(*continued*)

Star	M_{bol} [e]	$(V-K)_0$ [e]	V	B-V	Disk No.	Julian Date	Exposure (min)	R	This Velocity	Other Velocity	Ref [b]	Notes
						NGC 6838 = M71 (Cudworth 1985a)						
A	...	...	10.70	0.53	2654-11	6222.984	2.5	10.7	-27.1 ± 0.7	-17 ± 7	PS83	0,f
					2656-15	6223.986	4	9.7	-27.0 ± 0.8			
B,V2	-3.13	4.66	12.08	1.87	2654-9	6222.980	7.2	5.5	-18.9 ± 1.3			95
					2656-13	6223.982	5.5	4.7	-19.2 ± 1.5			
A4	-2.35	3.55	12.20	1.69	2651-19	6221.902	6	10.3	-25.4 ± 0.7	-25	C80	94
A9	-1.38	3.37	12.94	1.57	2651-24	6221.931	12	11.2	-24.8 ± 0.7			86
I	...	...	12.42	1.57	2656-11	6223.976	8.2	10.4	-15.5 ± 0.7			93
S	-1.34	3.12	12.94	1.51	2651-22	6221.920	12	10.7	-23.7 ± 0.7			81
1-21	-1.08	2.84	13.02	1.49	2651-28	6221.956	15	12.7	-21.5 ± 0.6			95
1-29	-3.39	5.6	12.76	1.84	2651-17	6221.896	8	9.4	-23.6 ± 0.8			90
1-36	...	...	12.79	1.25	2651-30	6221.964	10	8.5	-22.2 ± 0.9	-24 ± 11	GN78	64
1-45	-2.18	3.60	12.36	1.76	2654-2	6222.948	12	9.2	-22.8 ± 0.8	-22	C80	94
					2654-4	6222.958	12	9.8	-23.2 ± 0.8	-29 ± 14	GN78	
										-22 ± 18	GN78	
1-46	-2.31	3.64	12.29	1.75	2656-9	6223.968	12	8.8	-23.2 ± 0.9	-26	C80	93
										-16 ± 10	GN78	
1-53	...	...	12.97	1.61	2652-1	6221.974	10	9.6	-25.1 ± 0.8			95
					2654-7	6222.972	15	9.5	-24.2 ± 0.8			
1-56	...	...	13.14	1.38	2656-7	6223.958	20	11.7	-20.9 ± 0.7			96
1-64	...	...	13.10	1.53	2653-26	6222.908	17	12.5	-17.2 ± 0.6	-24 ± 40	HS78	95
					2656-4	6223.943	6	7.7	-17.4 ± 1.0	-9 ± 40	HS78	
1-66	...	...	13.01	1.40	2653-28	6222.919	15	13.1	-20.0 ± 0.6			81
1-77	-1.89	3.57	12.65	1.73	2651-26	6221.941	12	9.7	-27.1 ± 0.8	-16 ± 19	GN78	73
1-113	-2.25	3.81	12.43	1.80	2653-24	6222.895	10	10.3	-21.5 ± 0.7	-29 ± 11	GN78	95
										-21 ± 18	GN78	

[a] In km s^{-1}.

The Virial Method

In the case of an isotropic velocity dispersion, the mean square velocity can be expressed as:

$$v^2 = 3\sigma^2$$

The effective virial relation implies:

$$2 \cdot \frac{1}{2} M (3\sigma)^2 \approx \left(\frac{G M^2}{R} \right)$$

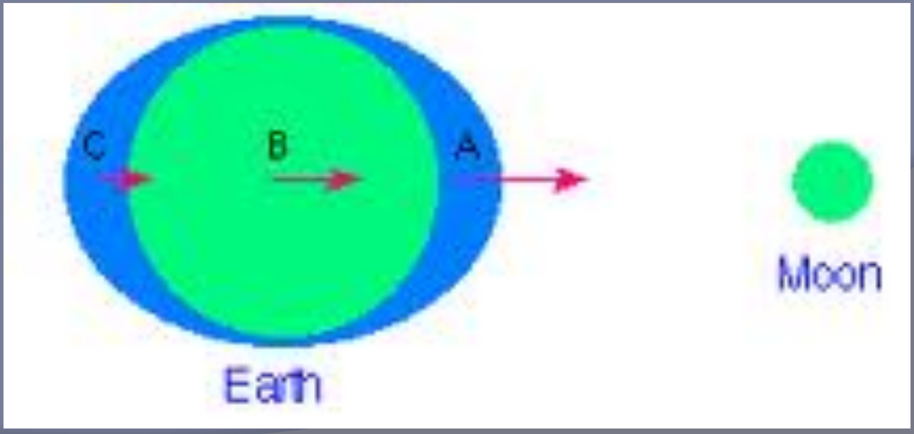
$$M \approx \left(\frac{R (3\sigma)^2}{G} \right)$$

Tidal effects

The effects of gravity can be expressed as a combination of two parts:

Gravity = “Volume contracting part” + “Volume preserving distortion-tidal part”

The second part is due to the non-uniformity of the gravitational field and generates tidal distortions



Tidal effects

Tidal distorsions are a "differential" effects.

$$\Delta F = G\frac{mM}{(R-r)^2} - G\frac{mM}{(R+r)^2}$$

A Taylor-McLaurin expansion yields:

$$\Delta F \approx 4G\frac{mMr}{R^3}$$

Tidal effects

In order for the compianion or satellite galaxy to resist to tidal disruption, the intensity of its own gravitational field must balance the tidal differential:

$$\Delta F \approx 4G \frac{m\, M\mathrm{r}}{R^{3}} \approx \frac{1}{4} \frac{G\, m^{2}}{r^{2}}$$

This equation is satisfied only for a given radius of the companion galaxy:

$$r^{3} \approx \frac{1}{16} R^{3} \frac{m}{M}$$

Modified Gravity, see

"Modified Newtonian Dynamics", Benoit Famaey, Stacy McGaugh – ArXiv: 1112.3960 (2012), astroph.CO

A critical acceleration seems to play an important role in the dynamics of systems ranging from galaxies to clusters of galaxies, of the order:

$$a \approx 10^{-10}\ ms^{-2}$$

Note that the acceleration scale related to the cosmological constant is of the same order:

$$a^2 \approx \Lambda$$

Milgrom's emprirical Law.

Newton's gravity is valid if:

$$\mathrm{g} \rangle a$$

Modified Newtonian Gravity is taken into account if:

$$\mathrm{g} \langle a$$

Formally, this law can be expressed as:

$$\mu\left(\frac{g}{a}\right)\vec{g} = \vec{g}(n)$$

With g(n) being the newtonian acceleration and

$\mu(x) \rightarrow 1$ if $x \rangle 1$

$\mu(x) \rightarrow x$ if $x \langle 1$

Dieletric analogy.

$$\mu(\mathrm{E})E = \frac{Q}{4\pi\varepsilon R^2}$$

$$\mu(\mathrm{E})$$

Is the "gravitational permittivity"

Bekenstein-Milgrom MOND

$$\nabla \bullet \left[\mu\left(\frac{|\nabla\Phi|}{a} \right) \nabla\Phi \right] = 4\pi G\rho$$

This **generalises non-linearly** the Poisson equation, and is analoguous to **Gauss' law** for a free electric charge distribution:

$$\nabla \bullet \left[\mu\varepsilon\vec{E} \right] = \rho$$

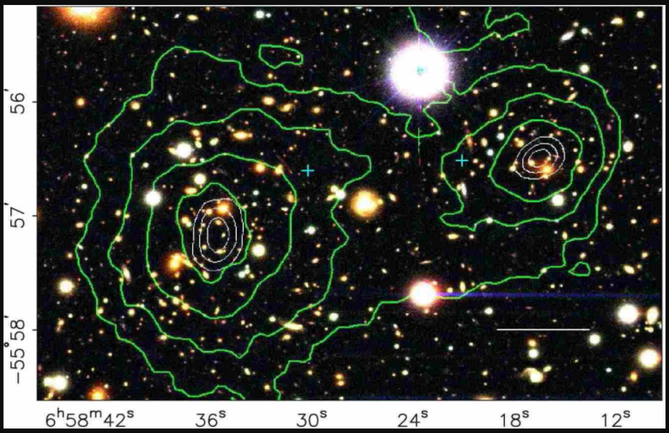
and yields

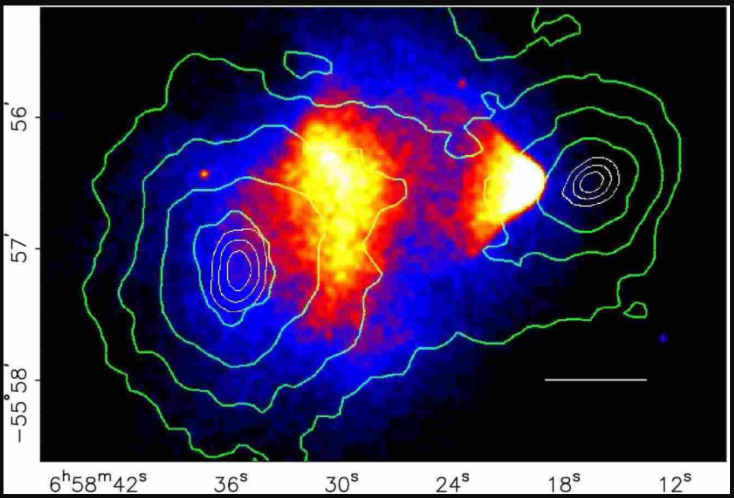
$$\mu\left(\frac{g}{a}\right)\vec{g} = \vec{g}(n) + \vec{S} \qquad \nabla \bullet \vec{S} = 0$$

Challenges to MOND

- Dynamical mass of clusters of galaxies
- Second:third acoustic peak in the CMBR
- The “Bullet cluster” (1E0657-56), and the “rich cluster” (Cl0024+17) observations – dark matter concentrations

The "Bullet" cluster





The non-linearity of the Bekenstein-Milgrom model implies that convergence regions can be different from baryon concentration zones. A residual missing mass is needed, as for clusters in general: hot baryonic matter, like 2eV neutrinos.

The “rich” cluster

- A weak lensing mass recontruction shows the presence of a ring-like structure, offset from both gas and galaxies in the cluster.

- In certain MOND models, this effect can occur. In the simplest models, however, additional collisionless matter required, like 2eV neutrinos.

Challenges to CDM models

- the challenges are mainly unobserved predictions,
- astrophysical complexity to explain data,
- no explanation of the critical acceleration a.

Neither the CDM models or the MOND models ought to be discarded: both have challenges to address and deserve attention and need to be developed.