Coendomorphism bialgebroid and chain complexes.

(Joint works with A. Ardizzoni and C. Menini.)

Abstract.
A commutative Hopf algebroids are exactly presheaves of groupoids in affine schemes (there is in fact a bi-equivalence). Following SGA 3 (Exposé V), this is in fact a particularity to the affine case of the notion of groupoid scheme acting on an other scheme (the base scheme). As a mathematical object, Hopf algebroids appears in different branches. In algebraic topology, any (symmetric) rings spectrum and any formal groups laws, induce a commutative Hopf algebroid. While, the most interesting examples in algebraic geometry are constructed using the theory of Tannakian categories, introduced by N. Saavedra Rivano and completed by P. Deligne.

The main goal that many Mathematicians were persecuting, was always trying to recognizes some given category as a category of comodules over some Hopf algebroid or may be a bialgebroid. In this way, although with elementary techniques, B. Pareigis success to construct a (no commutative) Hopf algebra whose category of comodules is monoidally equivalent to the category of unbounded complexes over a vector spaces (over a base field). This kinds of equivalences suggest that certain categories of comodules can be endowed with a model (might be monoidal) structures. Namely this is one of our main motivations to further investigating the relation between categories of complexes and comodules.

In this talk, i will show the main steps which are needed in order to prove that for every associative and unital ring R, there exists a left R-bialgebroid $\mathcal{L}(R)$ such that the category of chain complexes of left R-modules is equivalent (monoidally if R is commutative) to the category of left comodules over an epimorphic image of $\mathcal{L}(R)$. Our approach relies heavily on the non commutative theory of Tannaka reconstruction, and the generalized faithfully flat descent for small additive categories, or rings with enough orthogonal idempotents.