

## Advances in algebra 1: Categorical and geometric methods in representation theory

PhD course

PhD course at the University of Verona, 5 November 2025 – 28 November 2025 + Lecture Series in February, 2026.

The lecture series (February 2026) will be given by Jan Schröer (University of Bonn) on geometric representation theory.

### Course information for the period 5/11/2025 – 28/11/2025.

- Instructor: Georgios Dalezios
- Topics to be covered: Purity, functor categories and localisations
- Place: Dipartimento di Informatica - Università di Verona
- Assessment: In class presentation (30 min). Precise date and topics will be decided during the course.
- contact: georgios.dalezios@univr.it

**Course content.** The course will focus on certain interactions between purity, functor categorical techniques and localisations of abelian categories, with applications to finite dimensional algebras.

The course will start by introducing pure exact sequences, pure projective and pure injective modules, with examples from ring theory and representation theory. It is natural to consider functor categories for the study of purity, for instance via the tensor embedding of a module category to a certain functor category. We will discuss this in detail and prove the existence of pure injective hulls.

The use of functor categories is not only limited to purity. We will study Auslander algebras via functor categorical techniques, and prove for instance that they have global dimension at most 2. This will lead us to related results such as Auslander's formula and correspondence.

Understanding of the aforementioned results, usually involves the use of localisations of abelian categories. The last part of the course, will be about localisations and their relation to subcategories of module categories. Time permitting, the above topics will be related to concepts such as ring epimorphisms, cotorsion pairs, tilting and approximation theory.

### Date and time of lectures.

- Wednesday 5 November 2025, 13:30 - 15:30, Aula D (Ca' Vignal 1)
- Friday 7 November 2025, 15:30 - 17:30, Aula E (Ca' Vignal 1)
- Wednesday 12 November 2025, 10:30 - 12:30, Sala Riunioni
- Friday 14 November 2025, 10:30 - 12:30, Sala Riunioni
- Wednesday 19 November 2025, 10:30 - 12:30, Sala Riunioni
- Friday 21 November 2025, 10:30 - 12:30, Sala Riunioni
- Wednesday 26 November 2025, 10:30 - 12:30, Sala Riunioni
- Friday 28 November 2025, 10:30 - 12:30, Sala Riunioni

**Where to study from.** Most of the material is quite folklore by now and contained in excellent books. Consulting these sources is usually more practical than going through the originals. For instance, an elementary account to Purity is given in Lam [5, Ch.2, §4J] and Stenström [7, Ch.1, §11]. Functor categories and the tensor embedding are covered in Prest

[6, Ch.12, §1] and the basic techniques of localisations can be found in Stenström [7, Ch.VI]. Further references will be provided during the course.

#### REFERENCES

- [1] W. Crawley-Boevey, *Locally finitely presented additive categories*, Commun. Algebra **22** (1994), no. 5, 1641–1674.
- [2] P. Gabriel, *Des catégories abéliennes*, Bull. Soc. Math. Fr. **90** (1962), 323–448.
- [3] H. Krause, *Homological theory of representations*, Camb. Stud. Adv. Math., vol. 195, Cambridge: Cambridge University Press, 2022.
- [4] ———, *The spectrum of a locally coherent category*, J. Pure Appl. Algebra **114** (1997), no. 3, 259–271.
- [5] T. Y. Lam, *Lectures on modules and rings*, Grad. Texts Math., vol. 189, New York, NY: Springer, 1999.
- [6] Mike Prest, *Purity, spectra and localisation.*, Encycl. Math. Appl., vol. 121, Cambridge: Cambridge University Press, 2009.
- [7] Bo Stenström, *Rings of quotients. An introduction to methods of ring theory*, Grundlehren Math. Wiss., vol. 217, Springer, Cham, 1975.