Deriving Bisimulations by Simplifying Partitions

Isabella Mastroeni

VMCAI 2008

INTRODUCTION

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COMPLETENESS: ... models the precision of an abstract domain wrt an operator.

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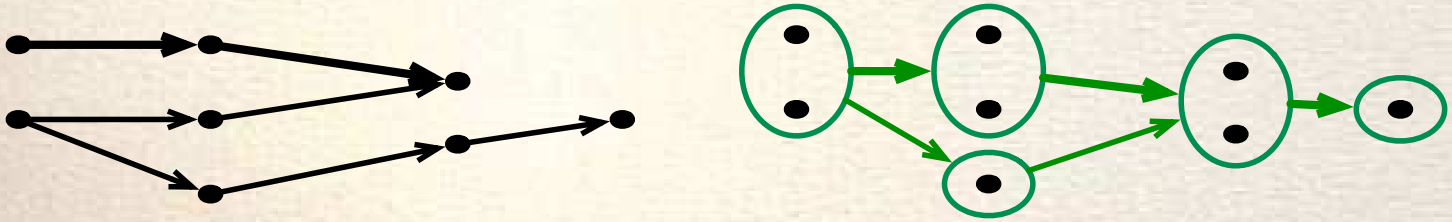

C

Abstraction A

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A not complete [GQ'01,RT'05]

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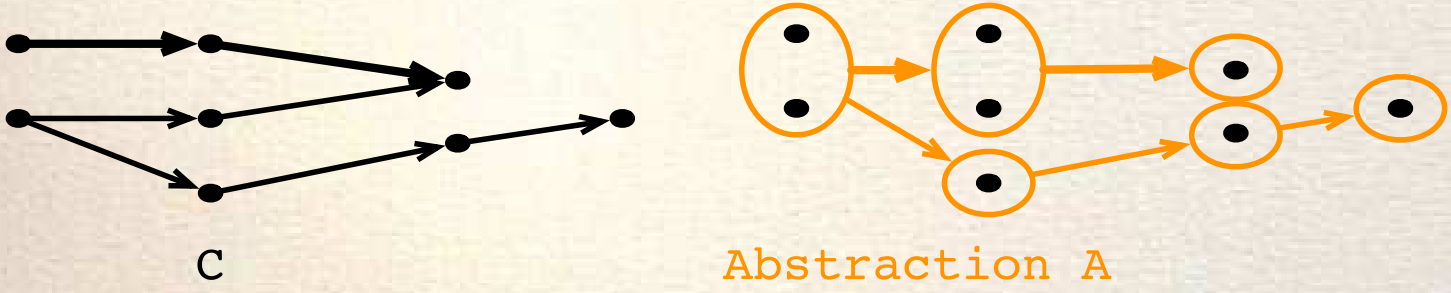

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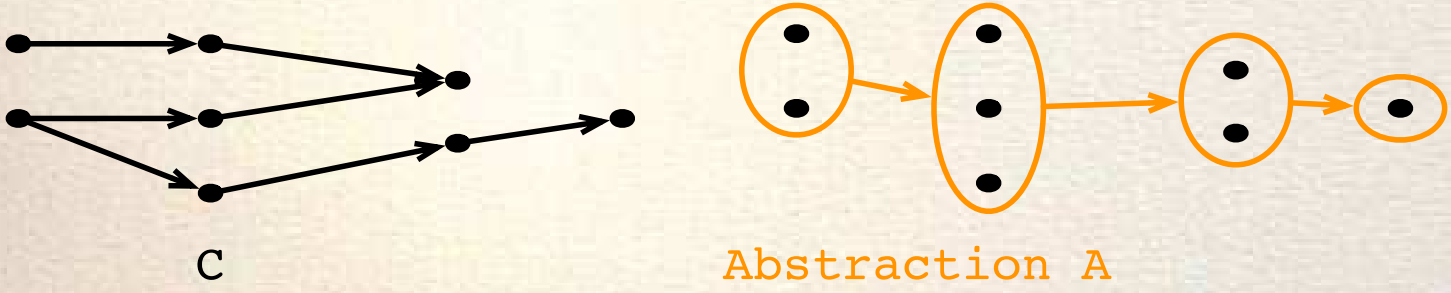

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ABSTRACT INTERPRETATION

Consider the complete lattice $< C, \leq, \wedge, \vee, \bot, \top >$, $A_i \in \mathit{uco}(C)$

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$\mathsf{A} \equiv \rho(C)$

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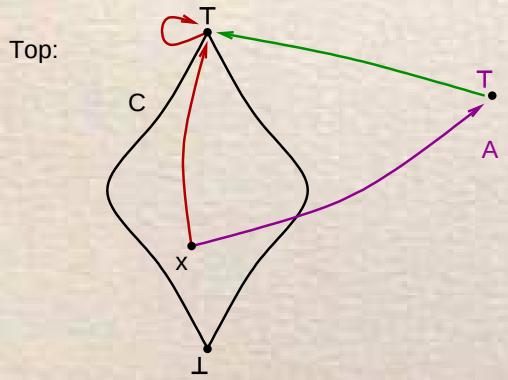

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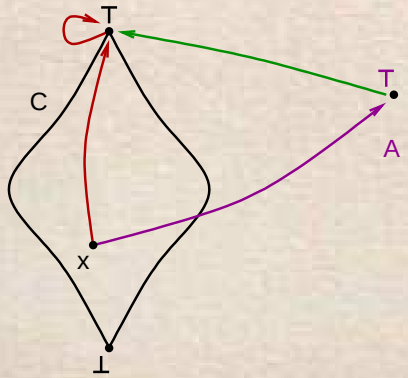

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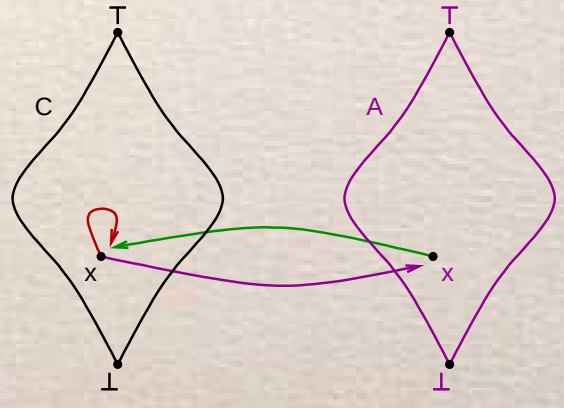

PARTITIONS VS ABSTRACT DOMAINS

Partitions uniquely correspond to particular abstract domains: PARTITIONING [RT'04,HM'05]

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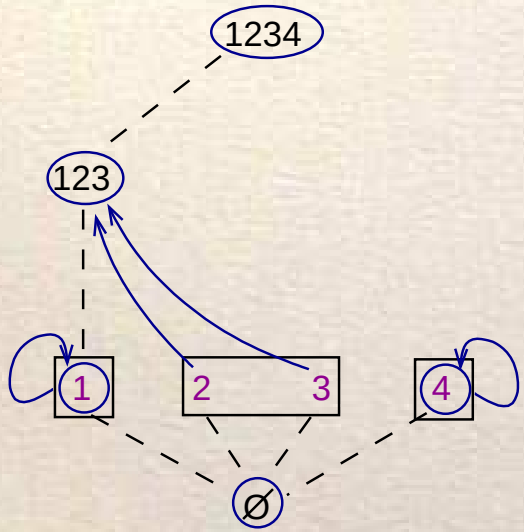

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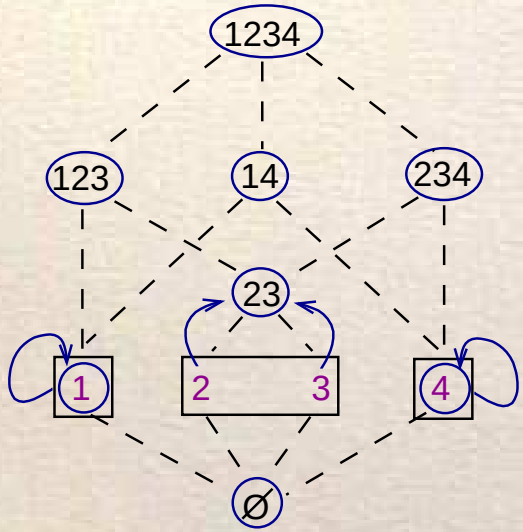

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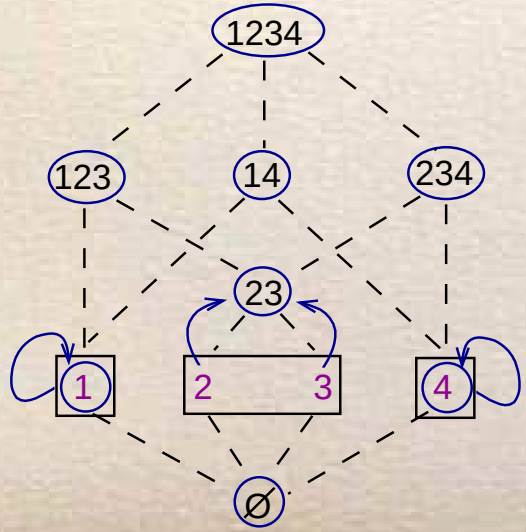

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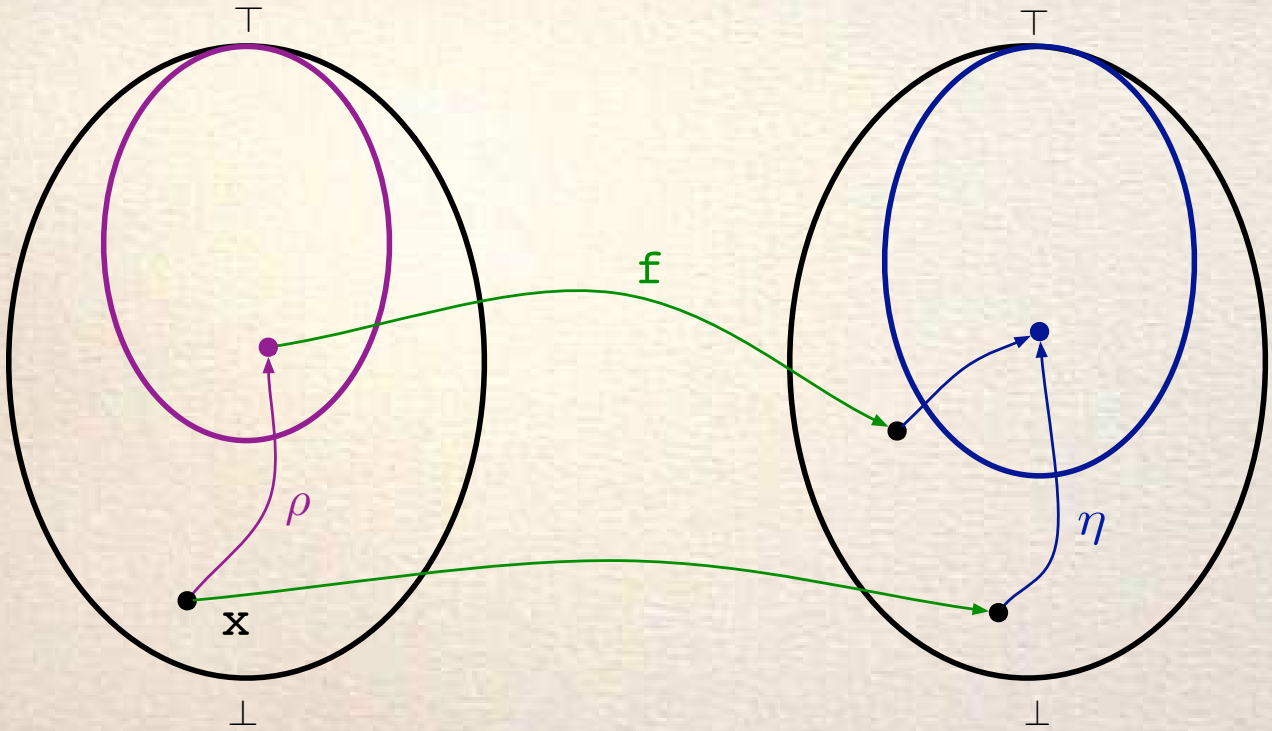
- $\mathsf{R} \in \mathit{Eq}(C) \;\Rightarrow\; \mathit{Clo}^{\mathsf{R}}(X) \stackrel{\mathsf{def}}{=} \bigcup_{x \in X} [x]_{\mathsf{R}}$
- $\Pi(\eta) \stackrel{\mathsf{def}}{=} \mathit{Clo}^{\mathit{Rel}^{\eta}} \sqsubseteq \eta$

COMPLETENESS



BACKWARD COMPLETENESS: $\eta \circ f \circ \rho = \eta \circ f$

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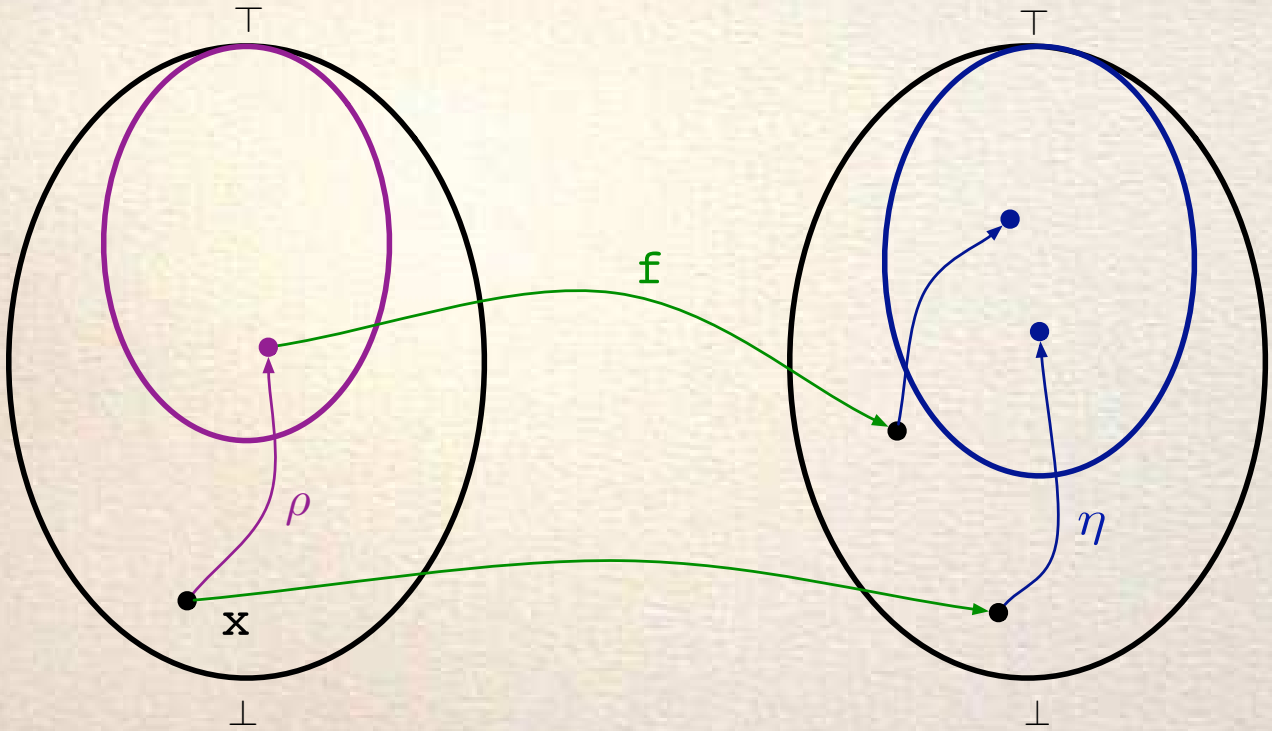

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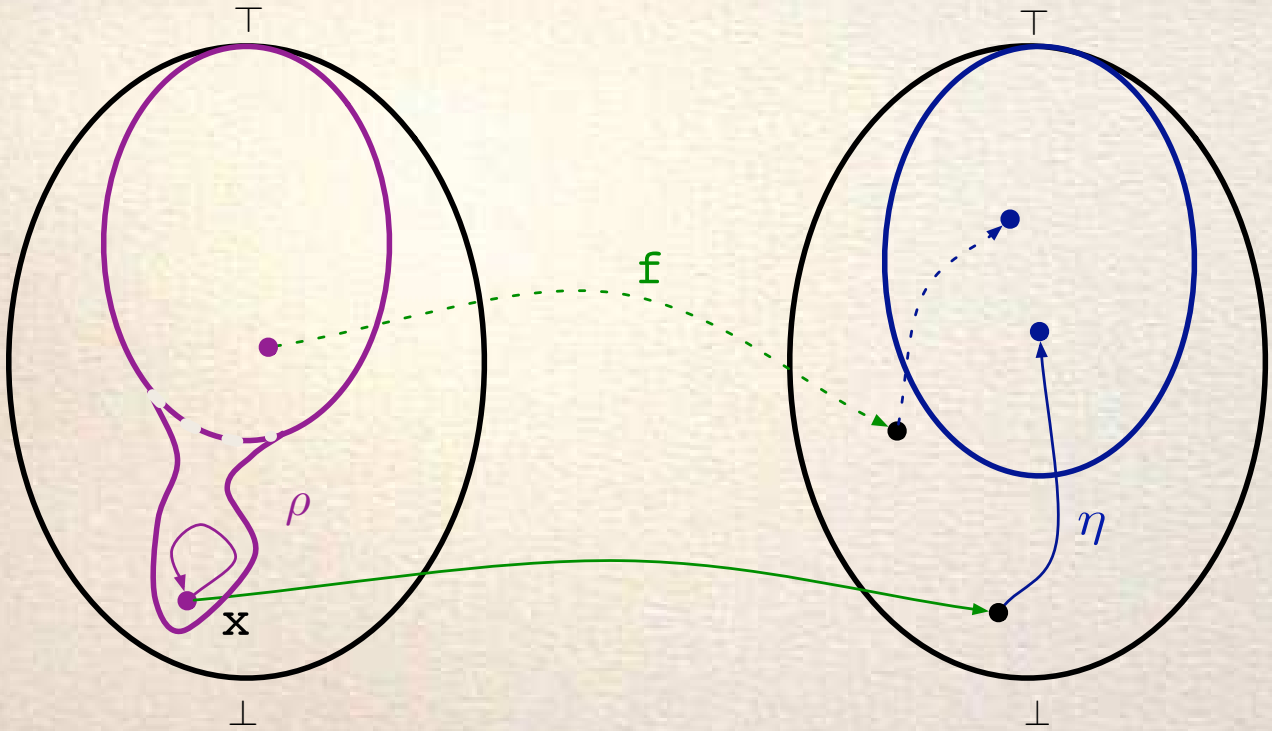

Making BACKWARD COMPLETE: Refining input domains [GRS'00]

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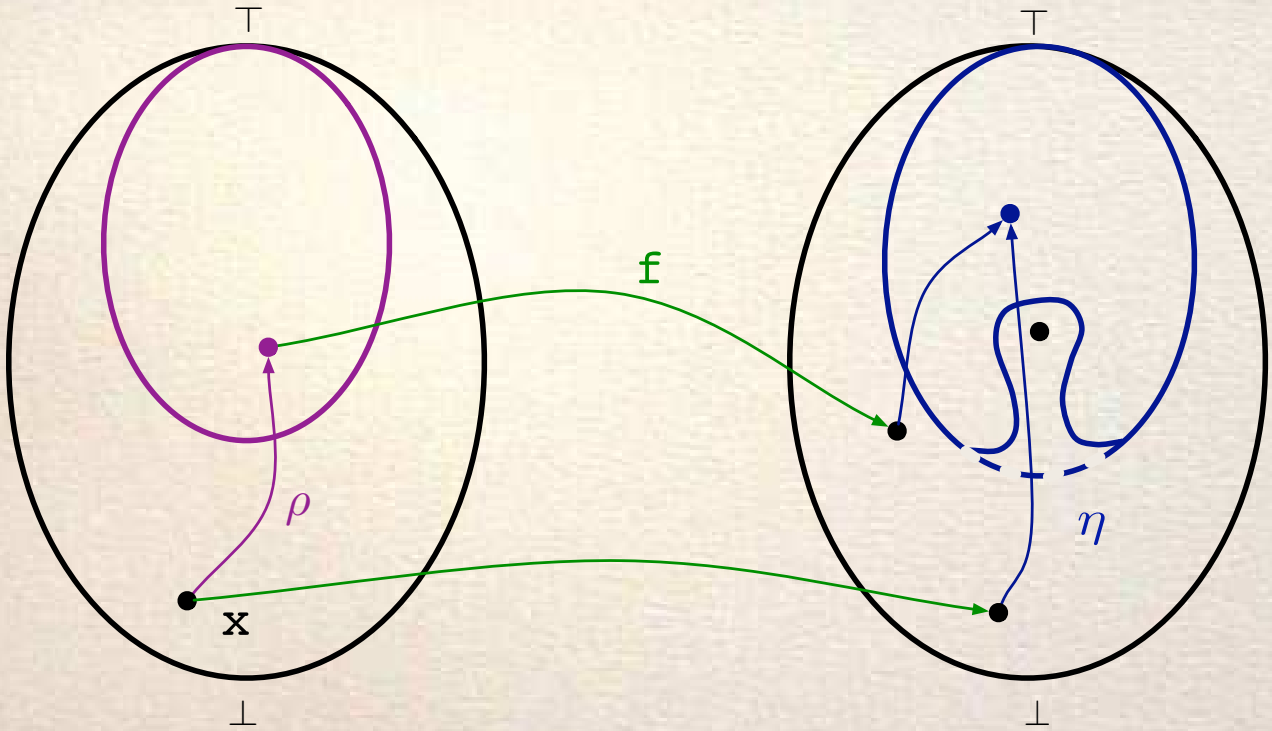

Making BACKWARD COMPLETE: Simplifying output domains [GRS'00]

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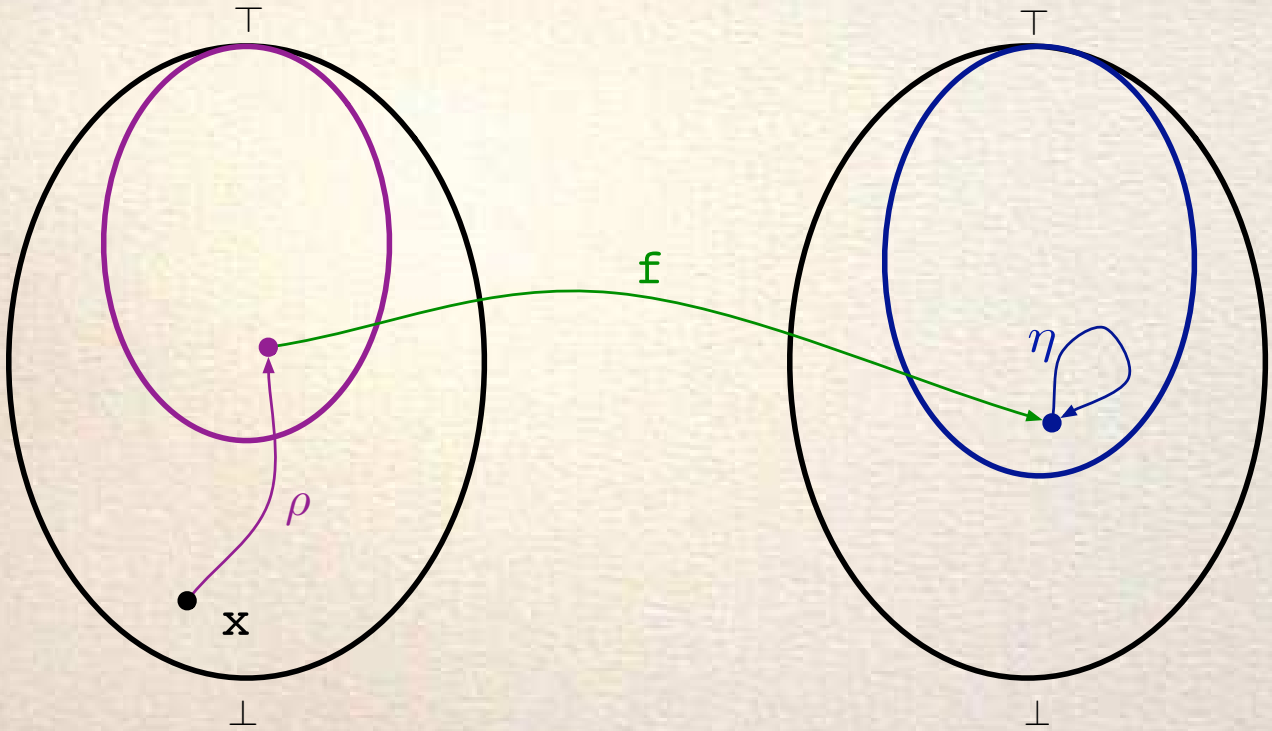

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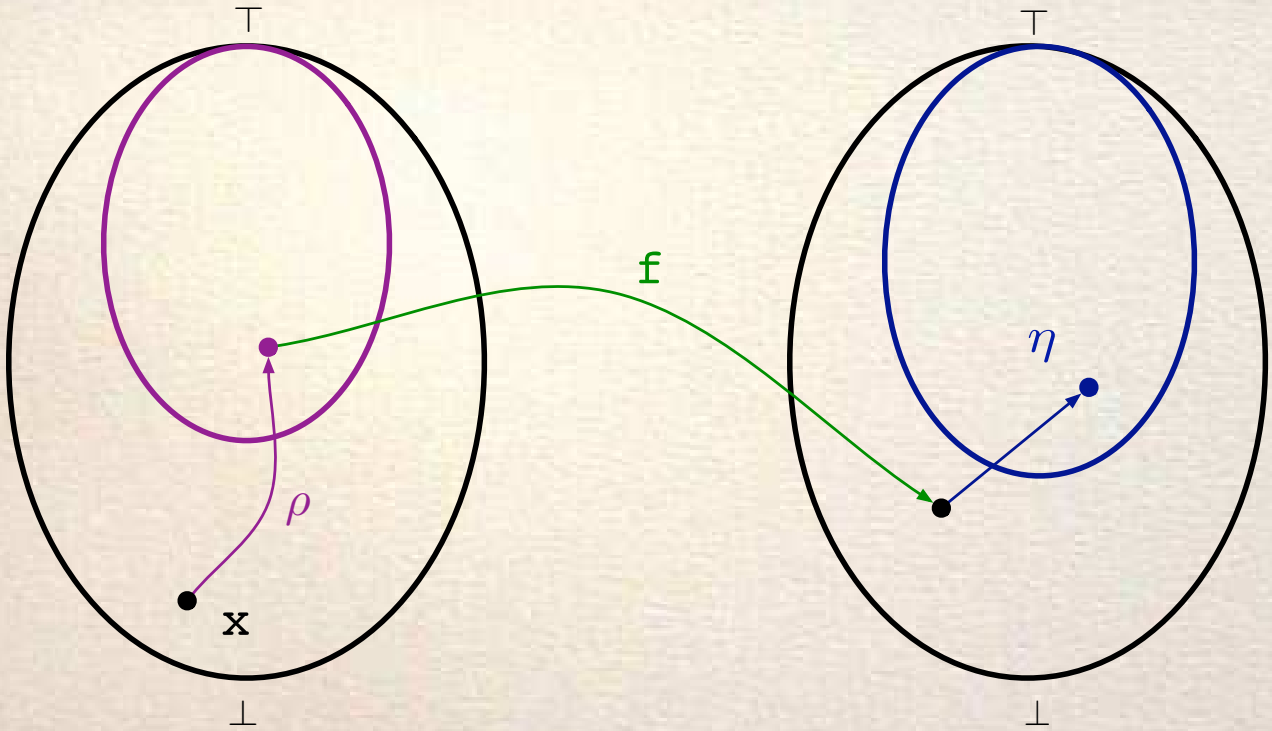

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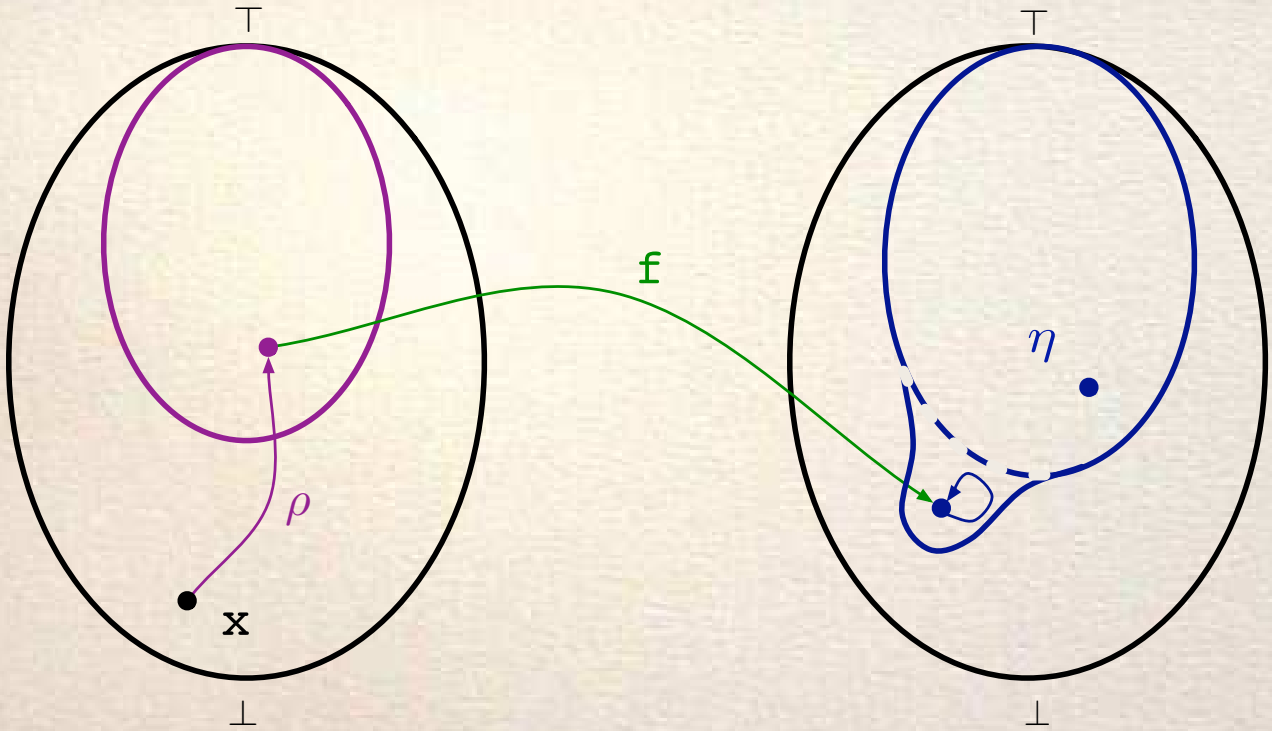

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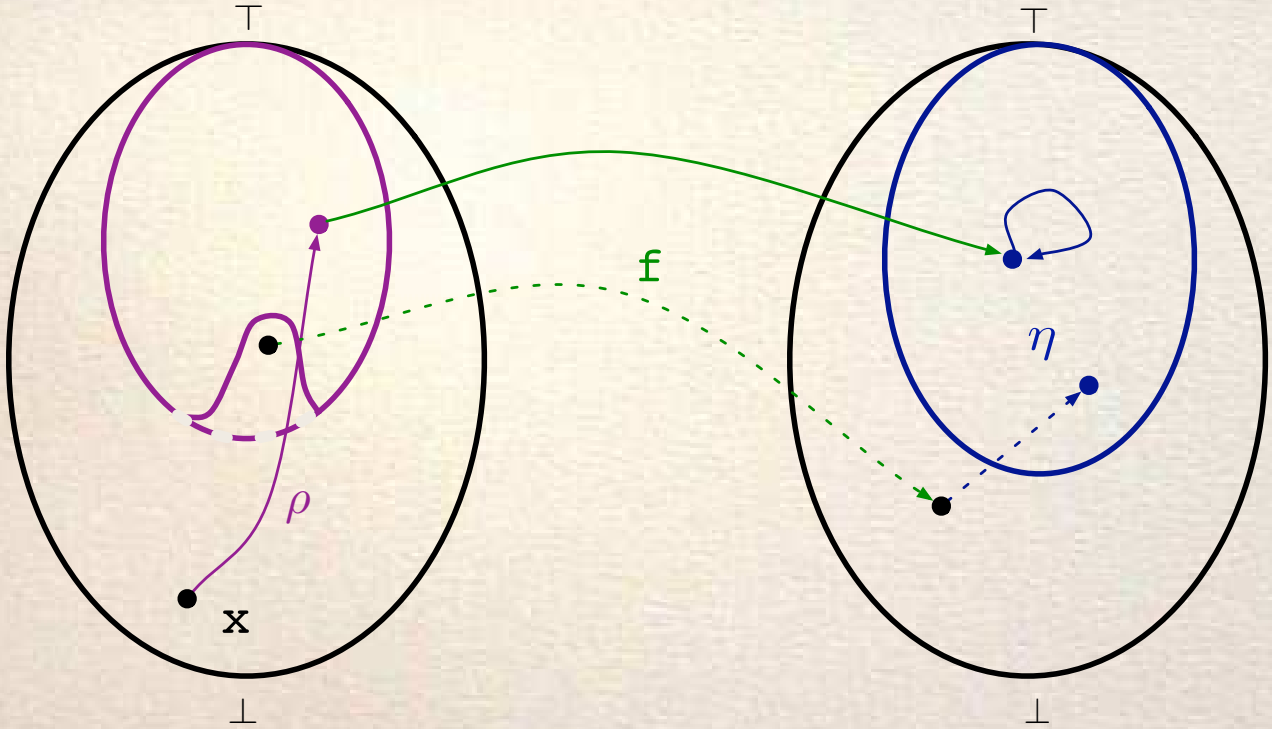

Making FORWARD COMPLETE: Simplifying input domains [GQ'01]

STABILITY

Let S and R, resp., an output and an input partition, let *p* be a binary relation:

STABILITY: S is *stable* wrt R if $\forall X \in \mathsf{S}$, $Y \in \mathsf{R}$ we have

$$X \cap p(Y) \neq \varnothing \;\Rightarrow\; X \subseteq p(Y)$$

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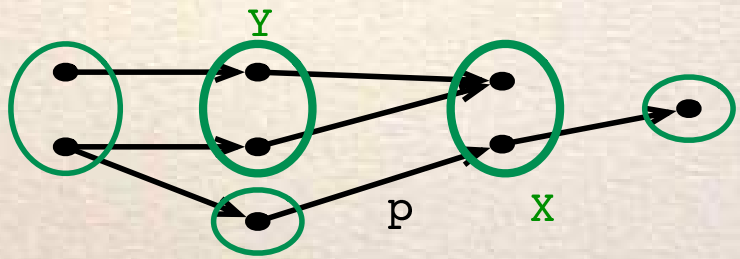

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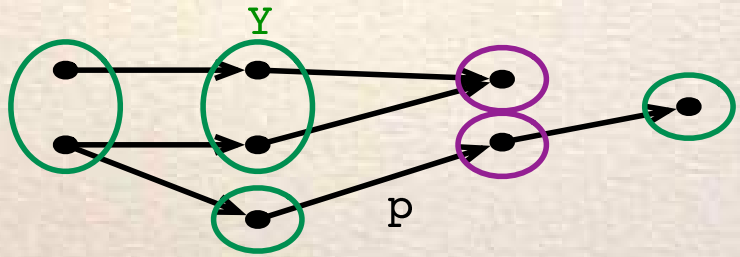

MAIN CONTRIBUTION

WHAT ALREADY EXISTS:

- A correspondence between *stability* and forward completeness [RANZATO & TAPPARO '05];
- A *refinement* algorithm for *partition stability* [PAIGE & TARJAN'87];
- A *refinement* transformer for *abstract domain completeness* [GIACOBAZZI ET AL.'00, GIACOBAZZI & QUINTARELLI'01];
- A *simplification* transformer for *abstract domain completeness* [GIACOBAZZI ET AL.'00, GIACOBAZZI & QUINTARELLI'01];;

MAIN CONTRIBUTION

WHAT DOES NOT EXIST:

- A characterization of completeness for partitions;
- A notion of partition stability/completeness for the *backward* direction;
- A *simplification* algorithm for *partition stability*

STABILITY VS FORWARD COMPLETENESS

F-STABILITY: $X \cap f(Y) \neq \varnothing \;\Rightarrow\; X \subseteq f(Y)$ [PT'87,RT'05]

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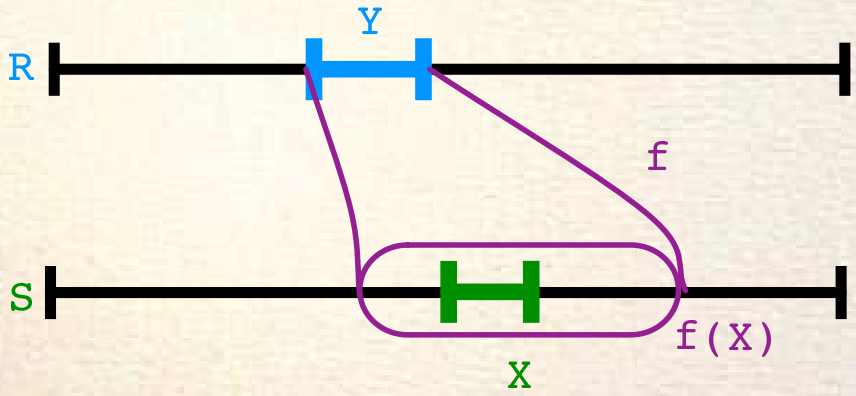

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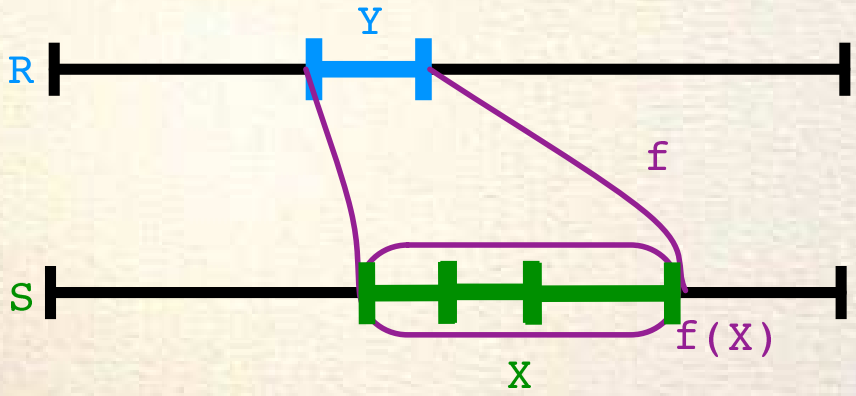

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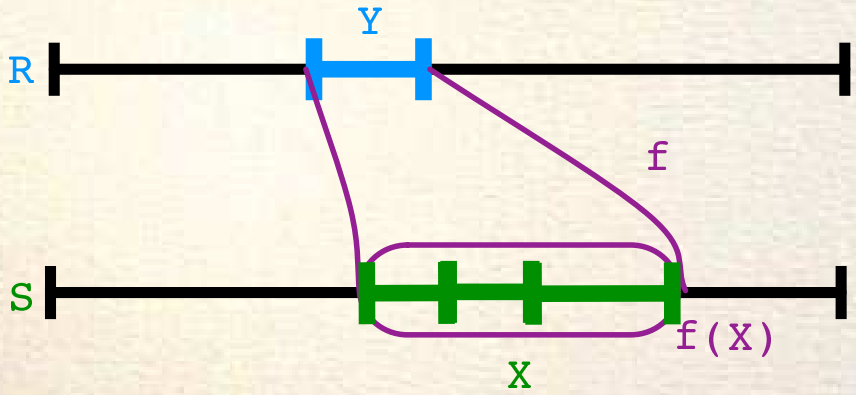

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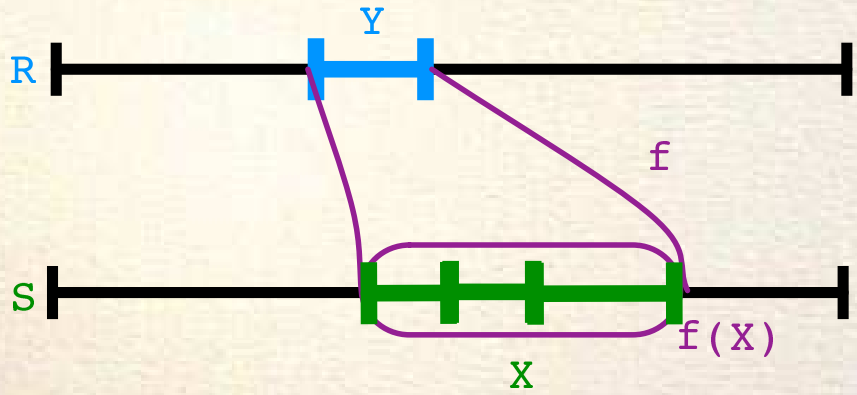
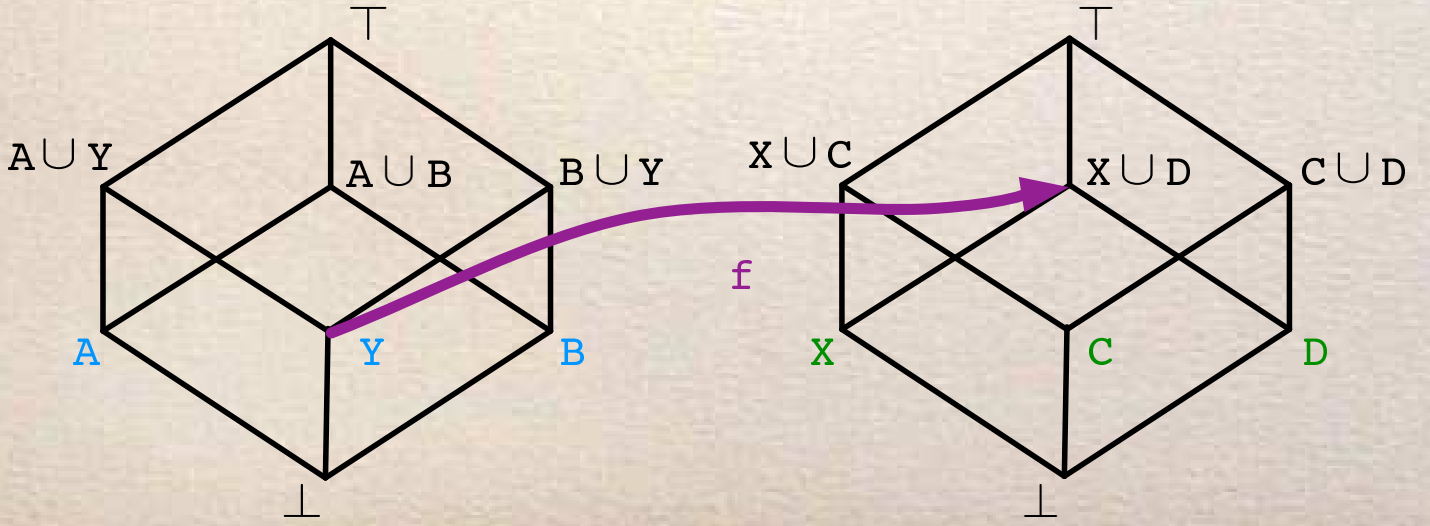


F-COMPLETENESS: $[f([x]_R)]_S = f([x]_R) \;\Leftrightarrow\; (\forall X \in Clo_R \;\Rightarrow\; f(X) \in Clo_S)$



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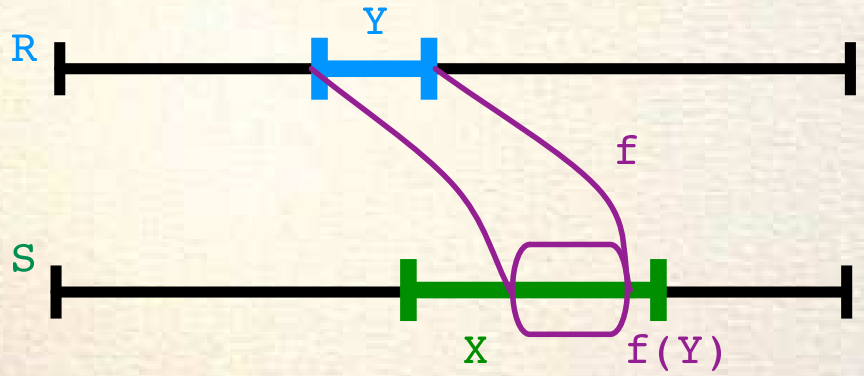

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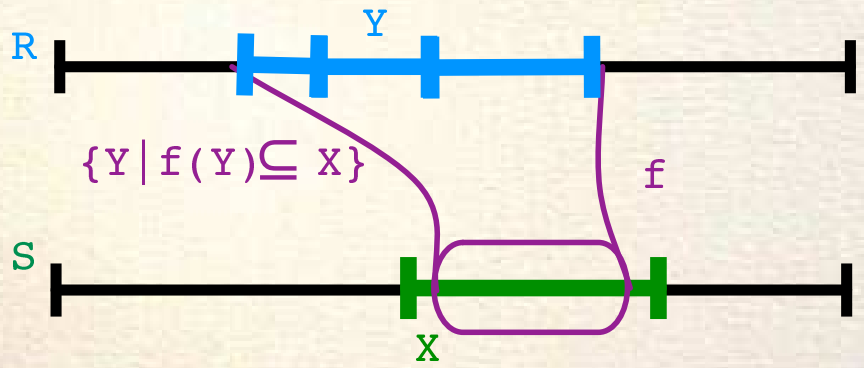

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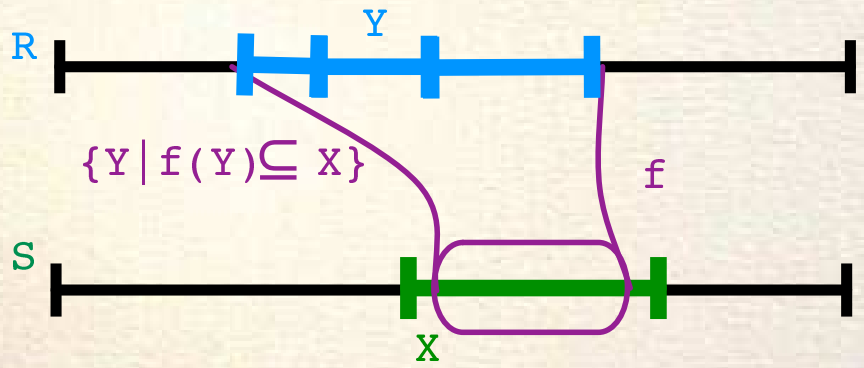

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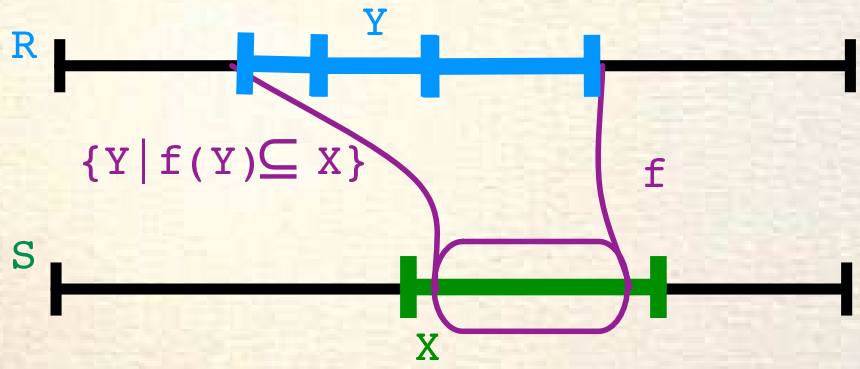
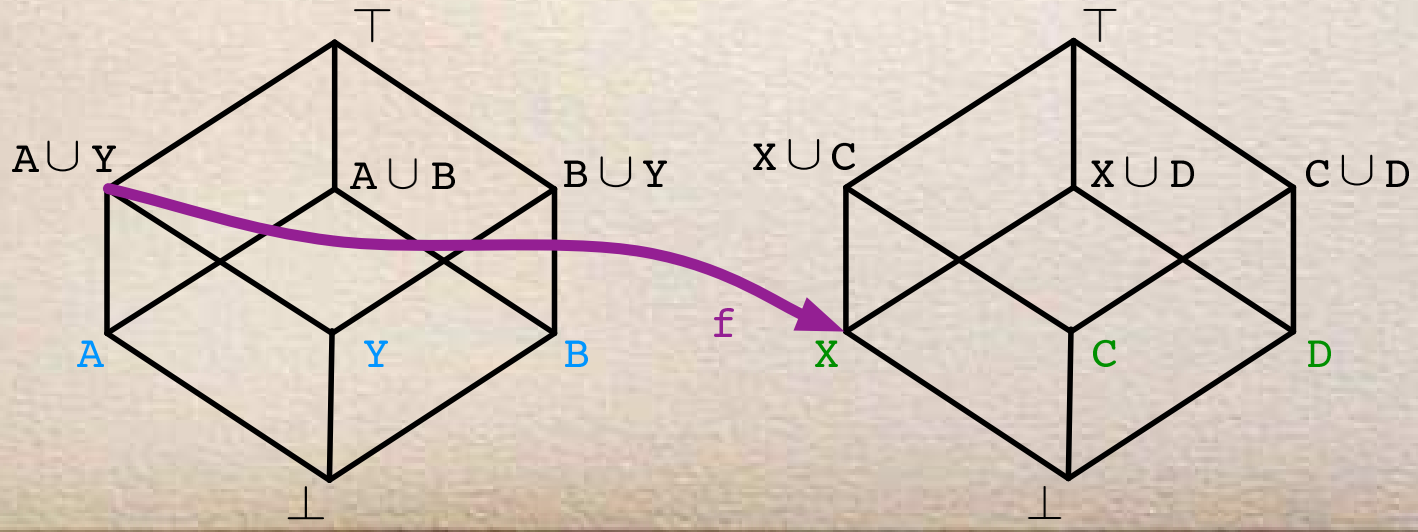


B-COMPLETENESS:

$[f([x]_{\mathsf{R}})]_{\mathsf{S}} = [f(x)]_{\mathsf{S}} \;\Leftrightarrow\; (\forall X \in Clo_{\mathsf{S}} \;\Rightarrow\; \max\left\{\, Y \;\middle|\; f(Y) \subseteq X \,\right\} \in Clo_{\mathsf{R}})$



BACKWARD VS FORWARD

- A domain is *backward complete* wrt f iff it is *forward complete* wrt $f^+ = \lambda X.\ \bigcup \left\{ \ Y \ \middle| \ f(Y) \subseteq X \ \right\}$;
- A (not trivial) partition is *backward stable* wrt f iff it is *forward stable* wrt $f^{-1} = \lambda X.\ \left\{ \ y \ \middle| \ f(y) \in X \ \right\}$;
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A backward problem can always be transformed in a forward one, but the viceversa is not always possible!

REFINING FOR STABILITY: PT GENERALIZED

The best refinement towards forward stability always exists!
[PT'87]

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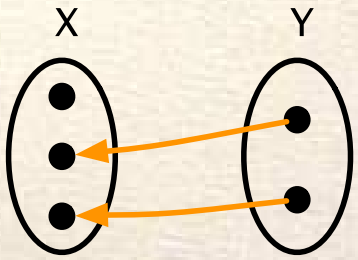

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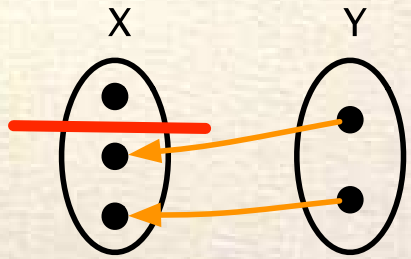

REFINING FOR STABILITY: PT GENERALIZED

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[PT'87]

$$\texttt{PTSplit}_f(S,\mathsf{P}) : \begin{cases} \mathsf{P} : \text{Partition} \\ \text{Partition obtained from } \mathsf{P} \text{ by replacing} \\ \text{each block } B \in \mathsf{P} \text{ with } B \cap f(S) \text{ and } B \setminus f(S) \end{cases}$$

$$\texttt{PTRefiners}_f(\mathsf{P}) \stackrel{\text{def}}{=} \left\{ S \;\middle|\; \mathsf{P} \neq \texttt{PTSplit}_f(S,\mathsf{P}) \;\wedge\; \exists \{B_i\}_i \subseteq \mathsf{P}.\, S = \bigcup_i B_i \right\}$$

$$\texttt{PT-Algorithm}_f : \begin{cases} \textbf{while } (\mathsf{P} \text{ is not stable}) \textbf{ do} \\ \quad \textbf{choose } S \in \texttt{PTRefiners}_f(\mathsf{P}); \\ \quad \mathsf{P} := \texttt{PTSplit}_f(S,\mathsf{P}); \\ \textbf{endwhile} \end{cases}$$

[RT'05]

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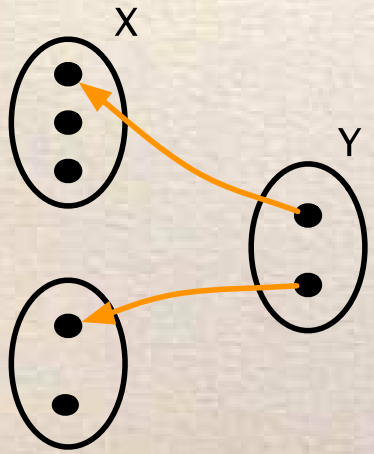

REFINING FOR STABILITY: PT GENERALIZED

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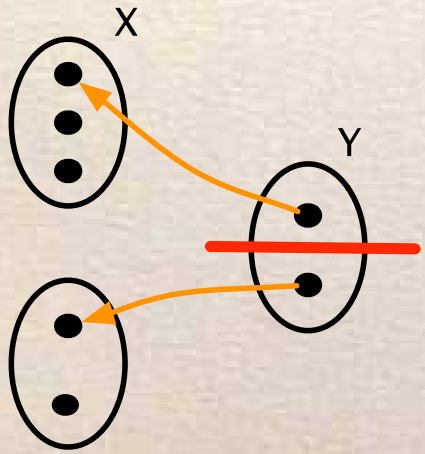

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We can use the PT algorithm since a backward problem wrt f corresponds always to a forward problem wrt f^{-1}.

SIMPLIFYING FOR STABILITY

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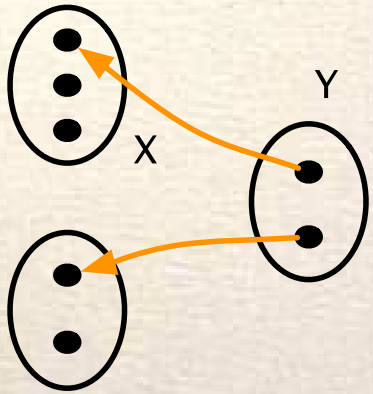

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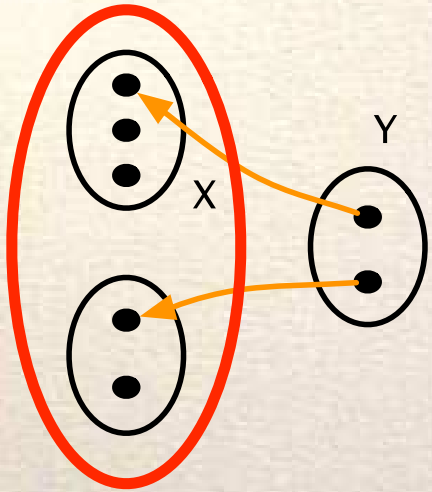

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$$\texttt{PTSimplifiers}_f(S) \stackrel{\text{def}}{=} \left\{ X \;\middle|\; X \cap f(S) \neq \varnothing \right\}$$

$$\texttt{PTMerge}_f(S, \texttt{P}) : \begin{cases} \text{Partition obtained from } \texttt{P} \text{ by replacing} \\ \text{all the blocks } X \in \texttt{PTSimplifiers}_f(S) \text{ with} \\ \bigcup \texttt{PTSimplifiers}_f(S) \end{cases}$$

$$\texttt{DPT-Algorithm}_f : \begin{cases} \textbf{while } (\texttt{P} \text{ is not stable}) \textbf{ do} \\ \quad \textbf{choose } S \in \texttt{PTSimplifiers}_f(\texttt{P}); \\ \quad \texttt{P} := \texttt{PTMerge}_f(S, \texttt{P})); \\ \textbf{endwhile} \end{cases}$$

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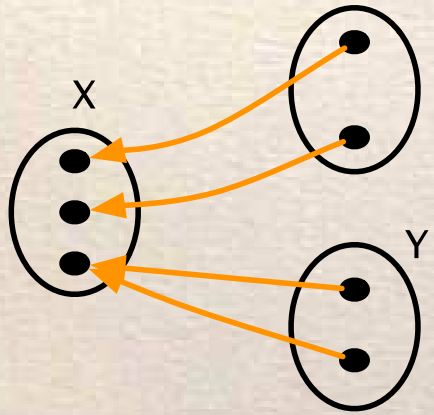

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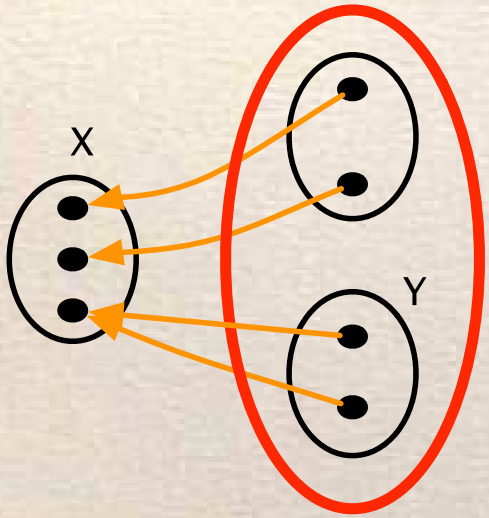

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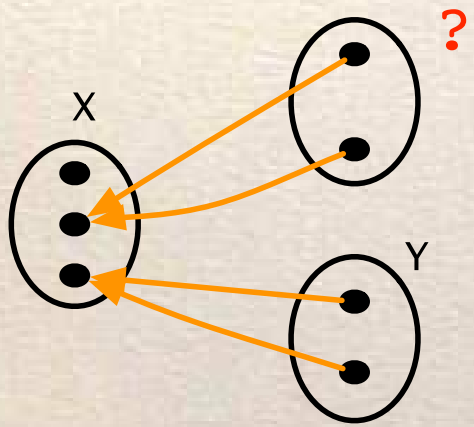

SIMPLIFYING FOR STABILITY

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EXAMPLE:

Consider $f(x) = 2x$. $\top$ is not stable ($f(\top) = even \subset \top$).

Consider R $= \{even, odd\}$ (Parity partition), then

$even \cap f(odd) \neq \varnothing$ since $6 \in f(3)$ and

$even \not\subseteq f(odd)$ since $4 \notin f(odd)$

$\Rightarrow$ A forward stable simplification does not exist!

STABILITY IN ABSTRACT NON INTERFERENCE

ABSTRACT NON INTERFERENCE: [GM'04]

Fixed an observation of public input, the variation of private input has not to *interfere* with the observation of the public output.

$$\Downarrow$$

$$\forall l_1, l_2 \in \mathbb{V}^{\mathsf{L}}, h_1, h_2 \in \mathbb{V}^{\mathsf{H}}. \eta(l_1) = \eta(l_2) \Rightarrow \rho([\![P]\!](h_1, l_1)^{\mathsf{L}}) = \rho([\![P]\!](h_2, l_2)^{\mathsf{L}})$$

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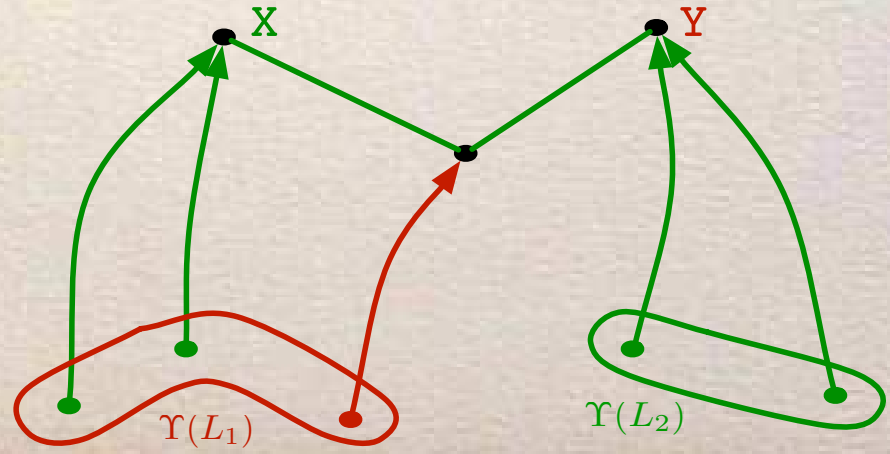

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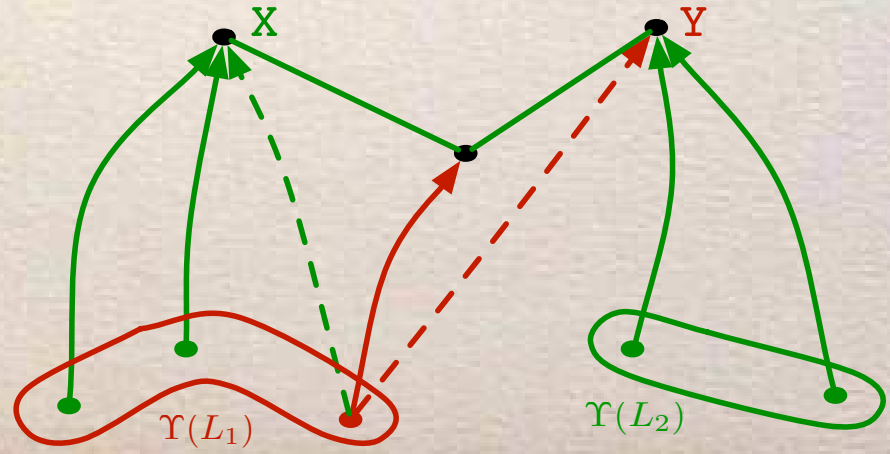
$Y \cap \Upsilon(L_1) \neq \varnothing$ and $\Upsilon(L_1) \not\subseteq Y$



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THEOREM: The domain $\left\{ X \;\middle|\; X \text{ is backward stable wrt } \Upsilon \right\}$ is the strongest harmless attacker for deterministic systems.

STABILITY FOR OPACITY

OPAQUE PREDICATE: A predicate ϕ over the semantics of a system, is opaque wrt the observation function *obs* if, for every execution t_1 satisfying ϕ there is an execution t_2 which does not satisfy ϕ, such that $obs(t_1) = obs(t_2)$. [Bryans et al. '05]

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EXAMPLE:
$\phi = 3|(x^3 - x)$, attacker capability $\alpha = \{\mathbb{Z}, 3\mathbb{Z}, \mathbb{Z} \setminus 3\mathbb{Z}, \varnothing\}$

$\Rightarrow$ If the attacker can observe the predicate as the bca of all the function composing ϕ then $obs(2) \cap \phi = \mathbb{Z} \cap \phi \neq \varnothing$, while $obs(2) \not\subseteq \phi$.

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$\Rightarrow$ *Completeness* can be exploited for certifying the resilience of opaque predicates to reverse engineering;

$\Rightarrow$ *Opacity* provides new expectations in seeking domain transformers increasing incompleteness,

DISCUSSION

- We extend the existing notion of stability (corresponding to forward completeness for partitions) also to the backward direction;
- We dualize the existing refinement algorithm for stability in order to simplify partitions;
- The simplification algorithm can be considered for simplifying abstract models in abstract model checking;
- We show fields of computer science where the new stability notion models existing concepts:
  - The strongest harmless attacker in abstract non-interference [Giacobazzi & Mastroeni ’04, Hunt & Mastroeni ’05]
  - Opacity for abstract observations of programs