THE PER MODEL OF ABSTRACT NON-INTERFERENCE

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and

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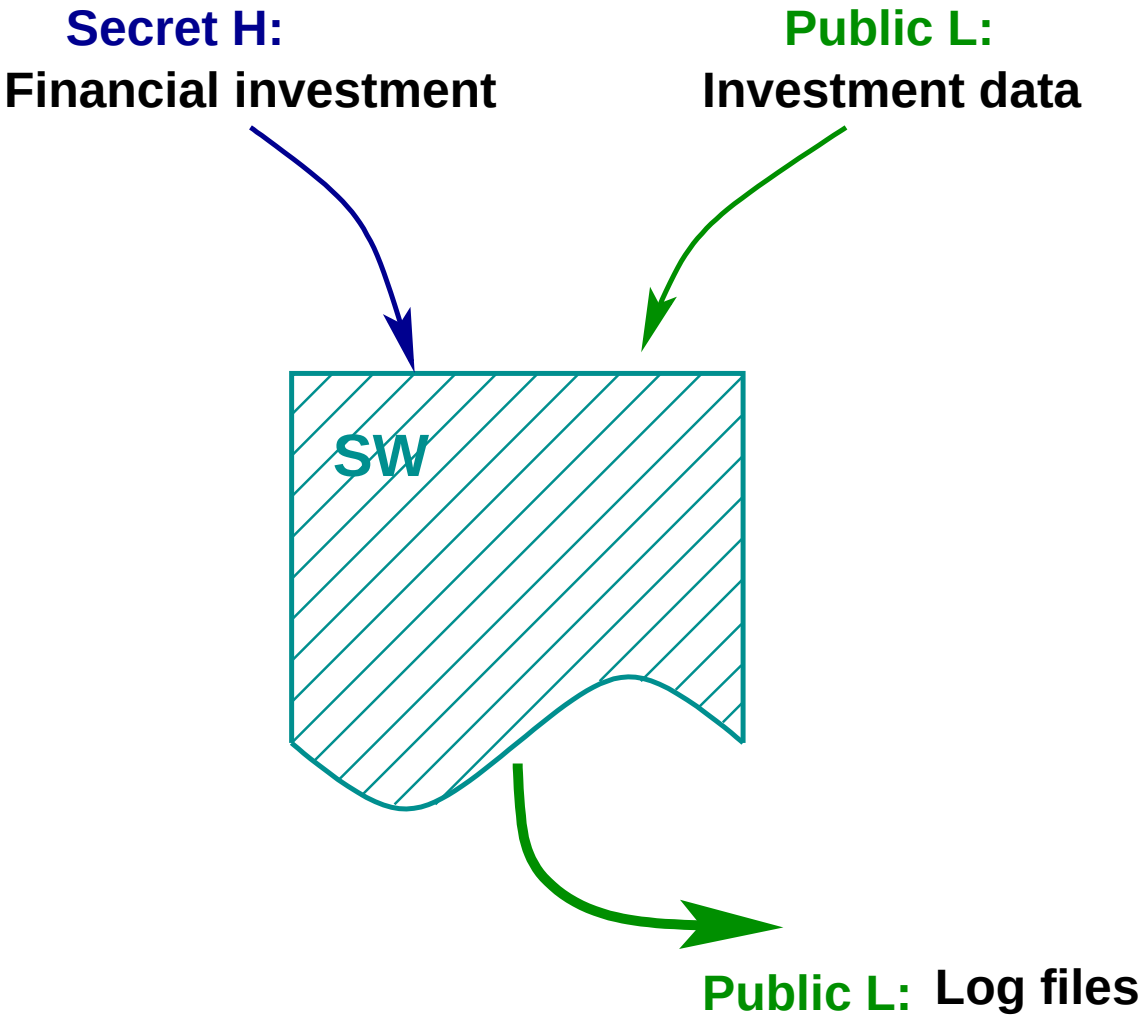
Kansas State University, Manhattan, KS, USA

SAS, September 9th, 2005

The Problem: Non-Interference



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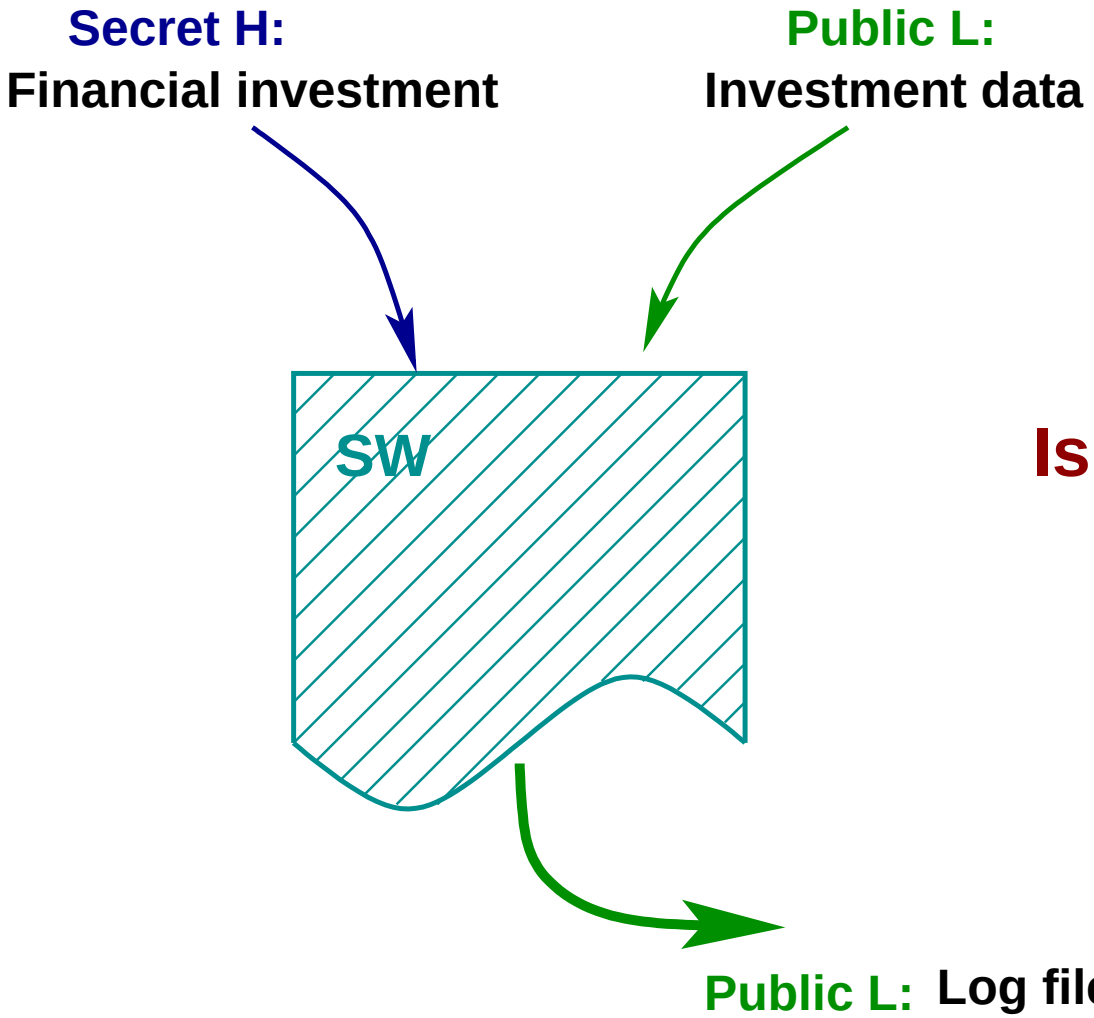
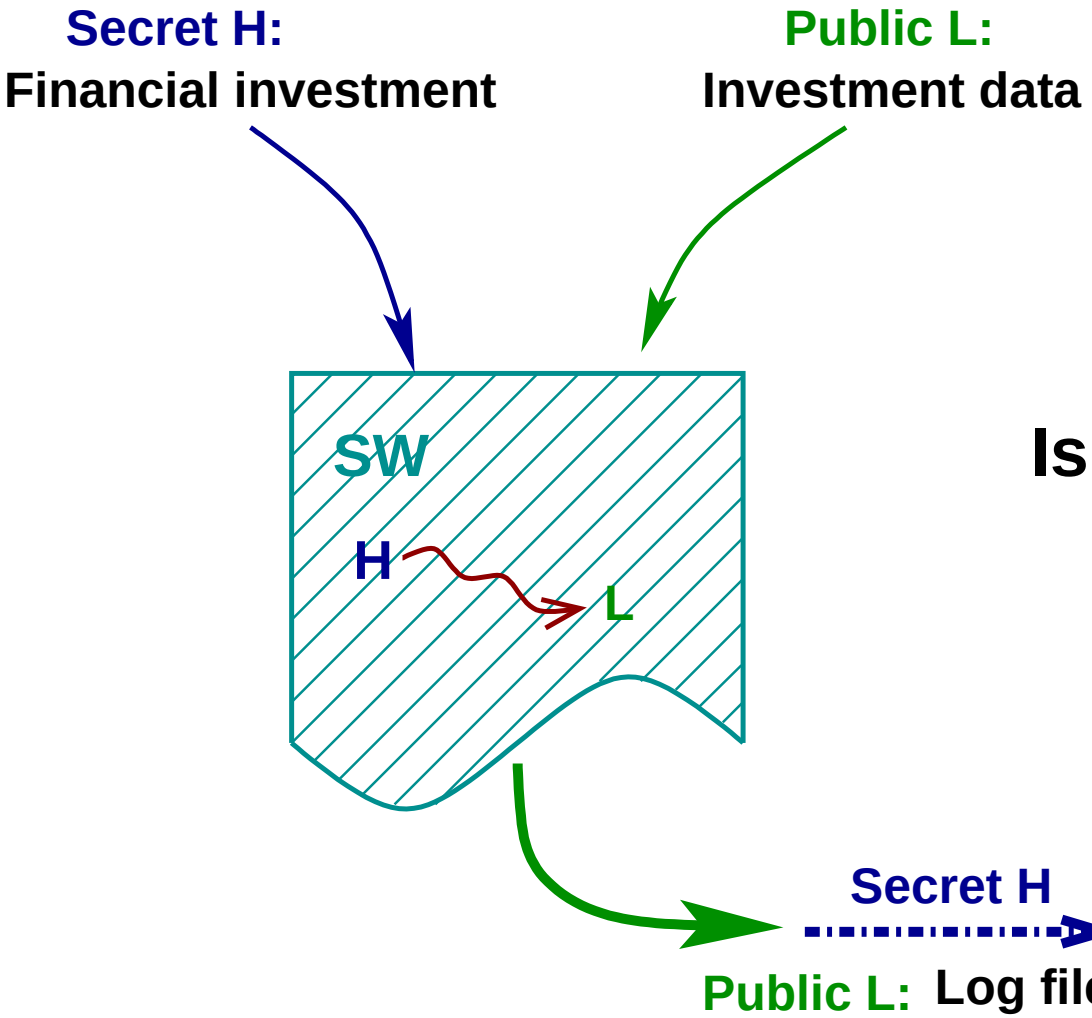


Is it secure?

The Problem: Non-Interference



Is it secure? NO

Background: The PER model

SECURITY PROPERTY: States which classes have not to interfere with other classes of objects.

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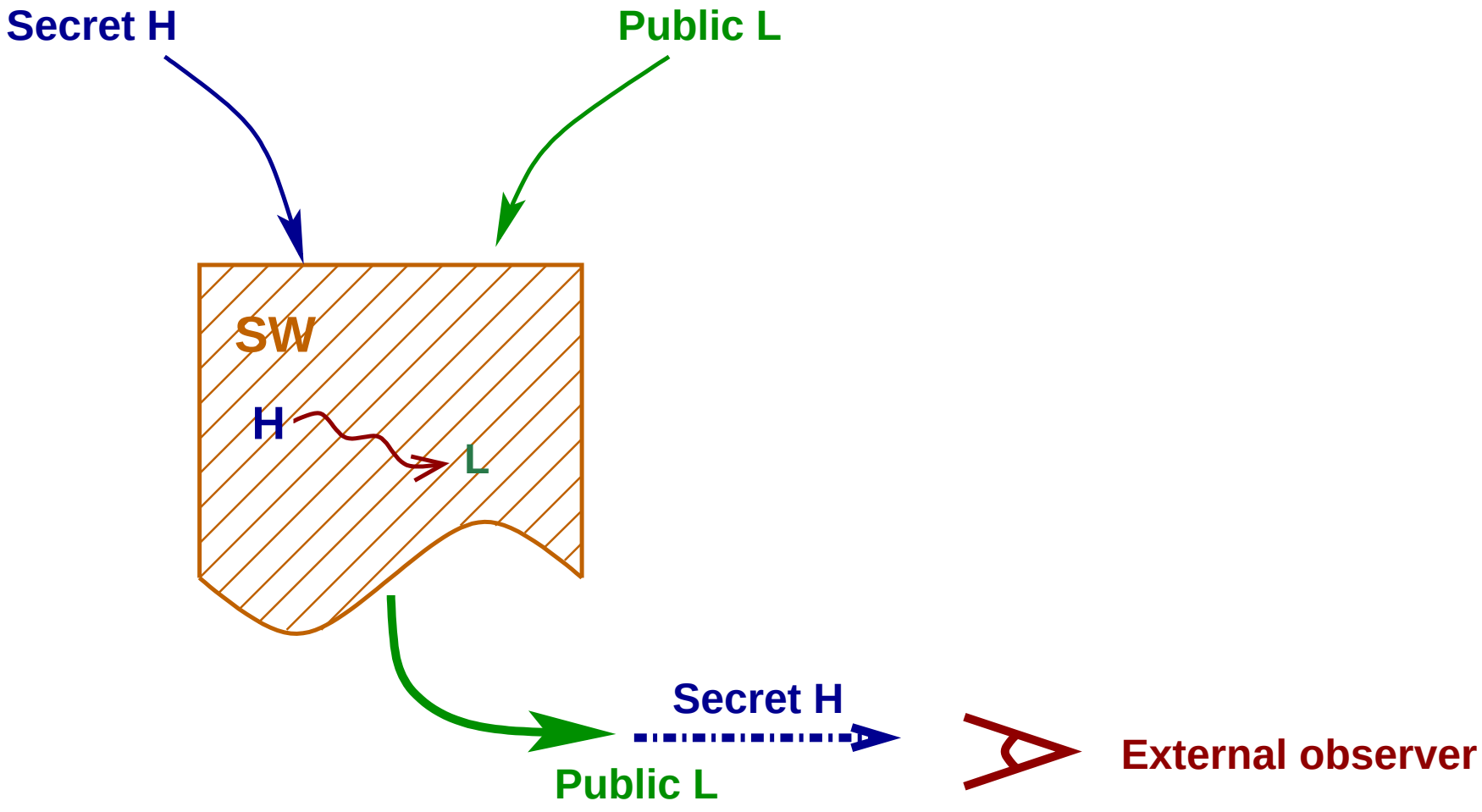
- PER model [Sabelfeld & Sands'01]:
  Let $\forall x, x'.\ x\ \mathtt{All}\ x'$ and $x\ \mathtt{Id}\ x' \Leftrightarrow x = x'$.

$$\mathtt{P} \text{ is } \textit{secure} \quad \text{iff}$$

$$\forall s, t\ .\ \langle s^{\mathtt{H}}, s^{\mathtt{L}}\rangle\ \mathtt{All} \times \mathtt{Id}\ \langle t^{\mathtt{H}}, t^{\mathtt{L}}\rangle \Rightarrow [\![\mathtt{P}]\!](s)\ \mathtt{All} \times \mathtt{Id}\ [\![\mathtt{P}]\!](t)$$

Abstracting Non-Interference



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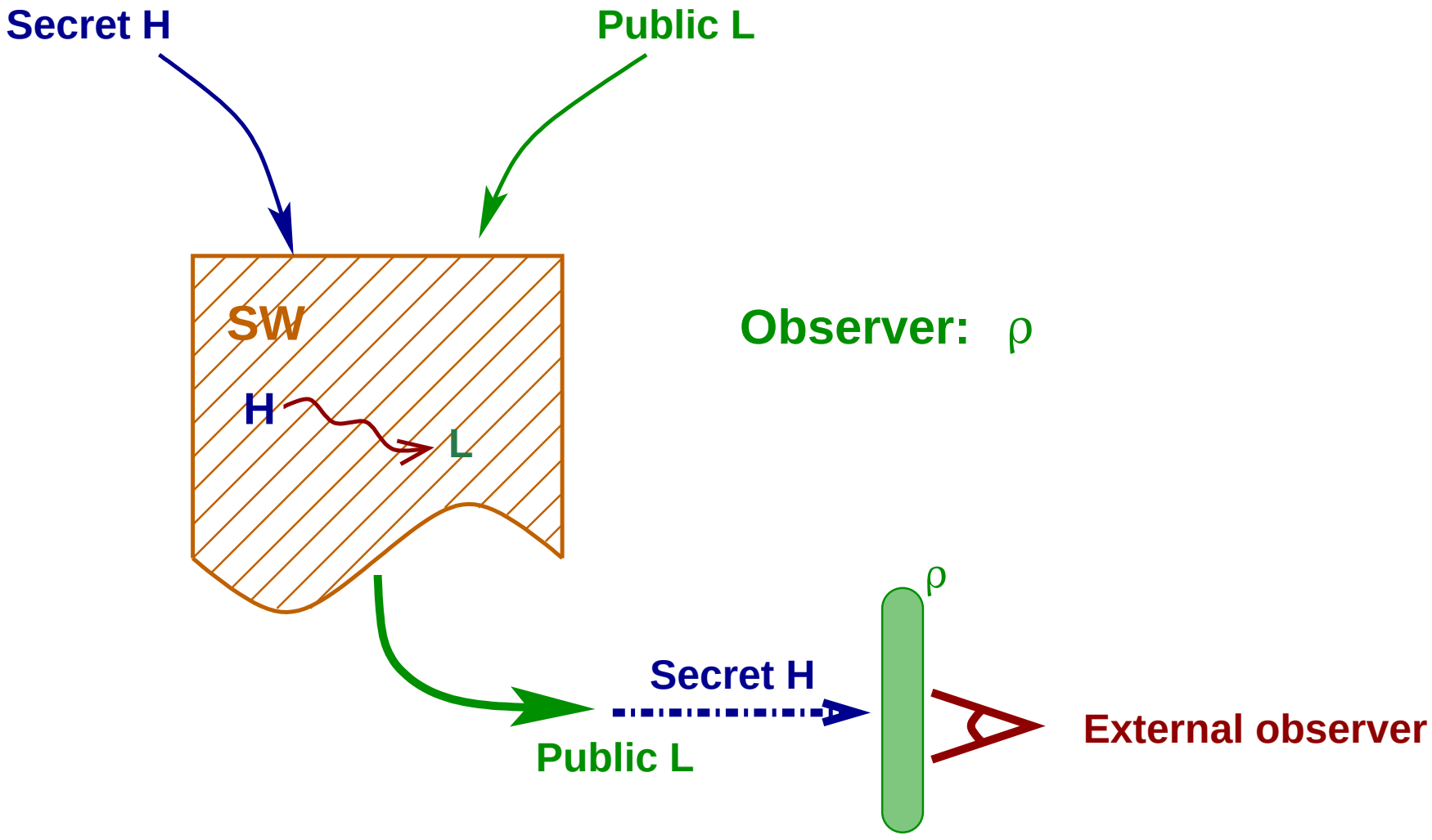

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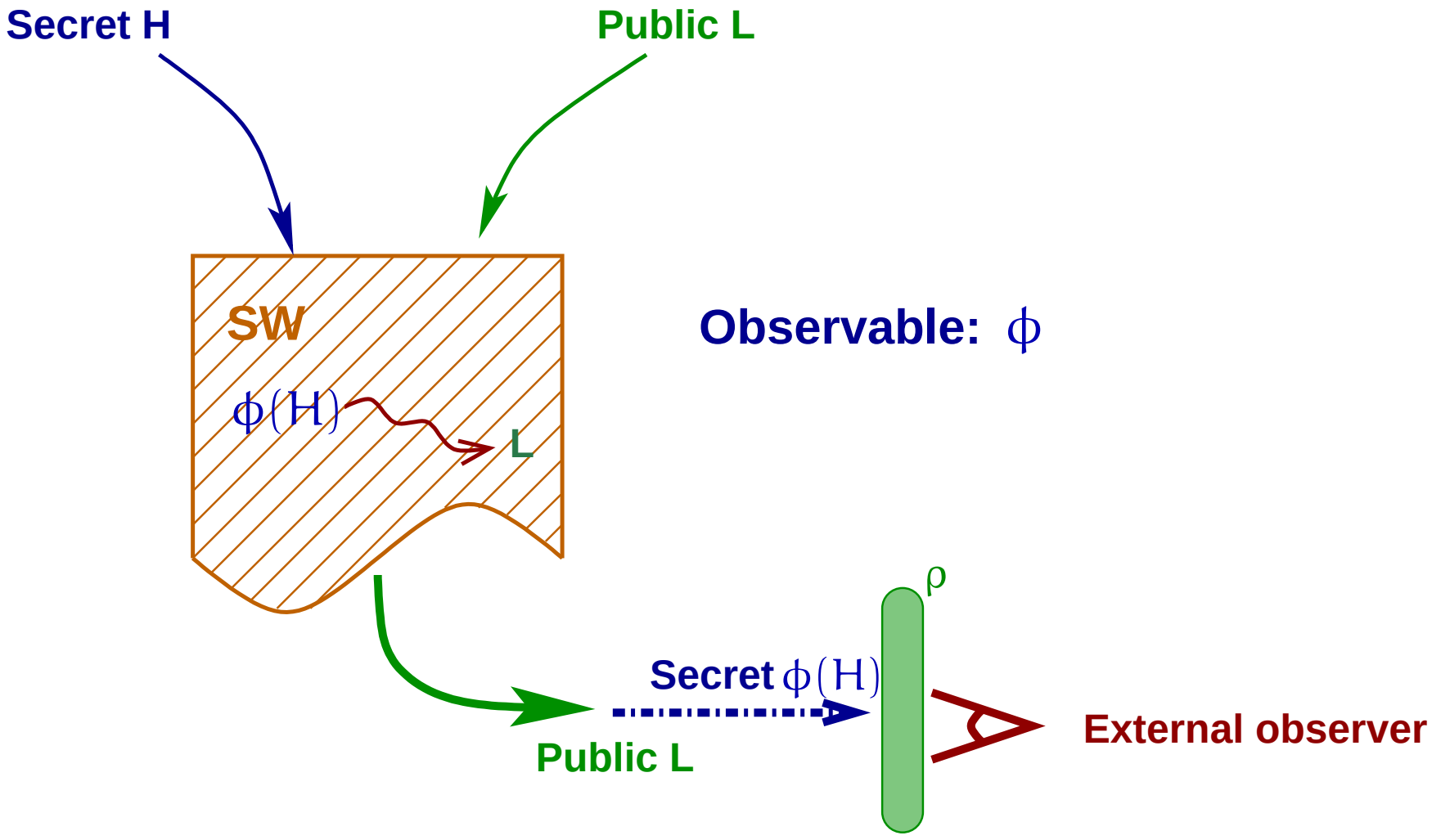
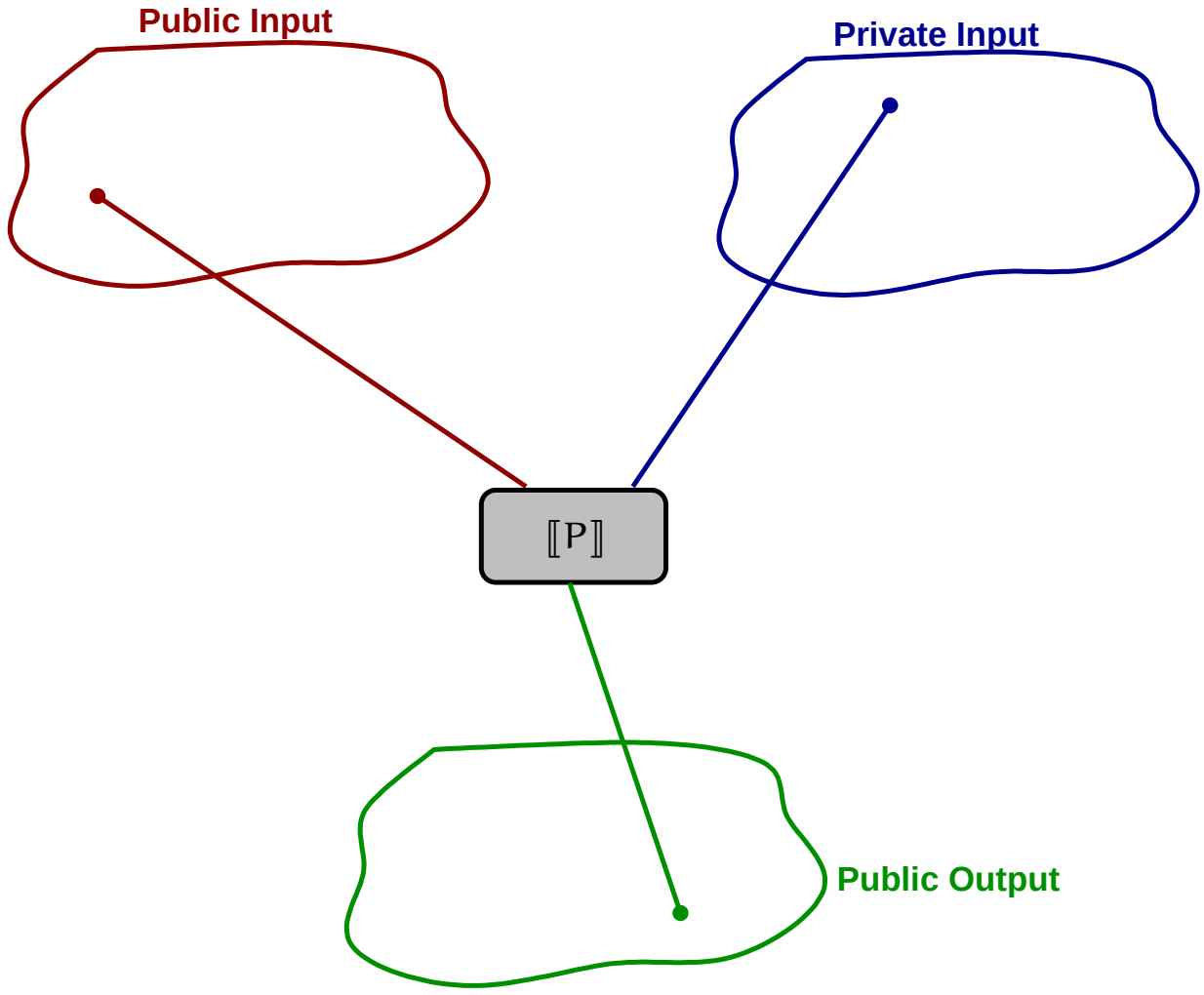


Standard non-interference



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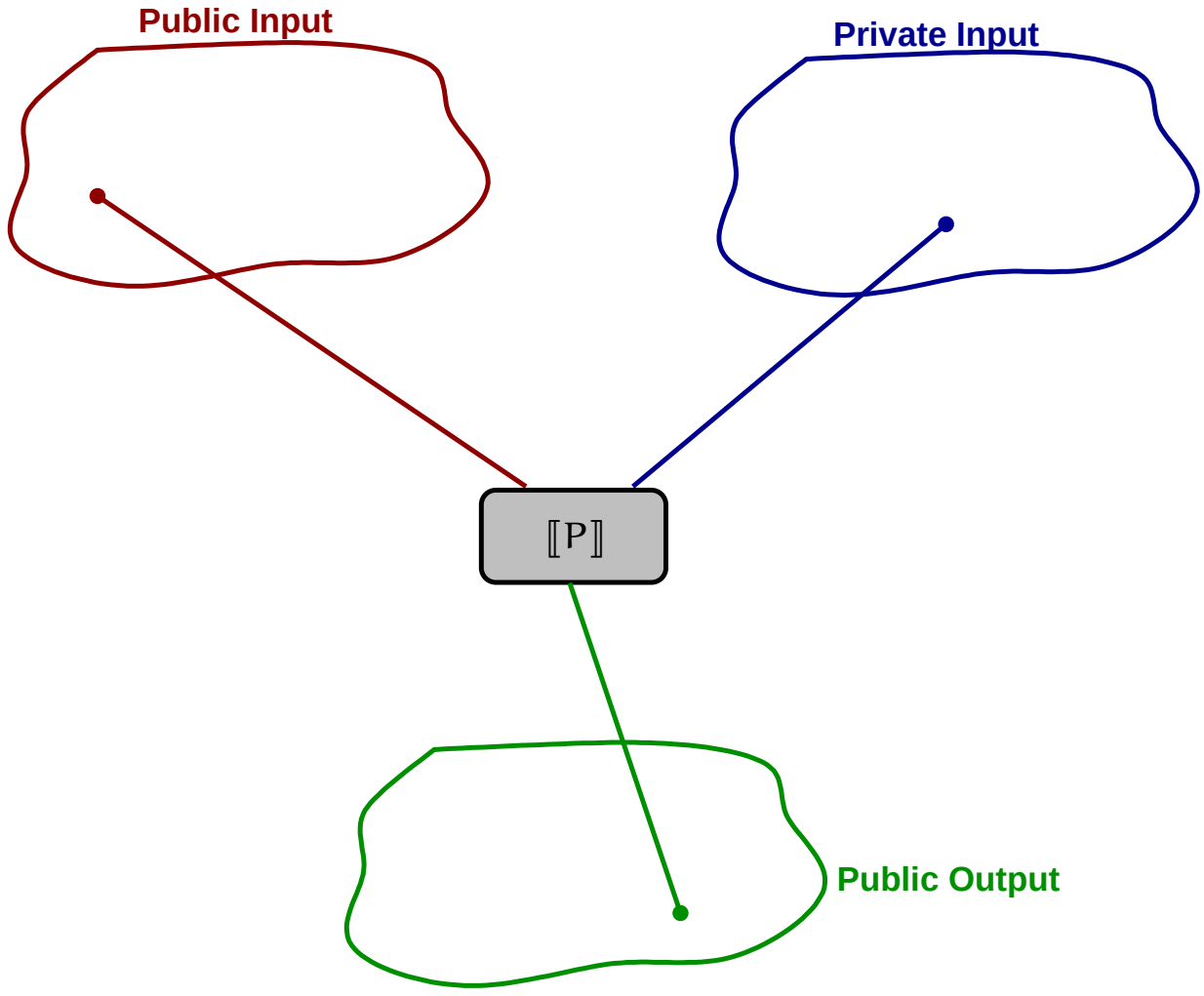

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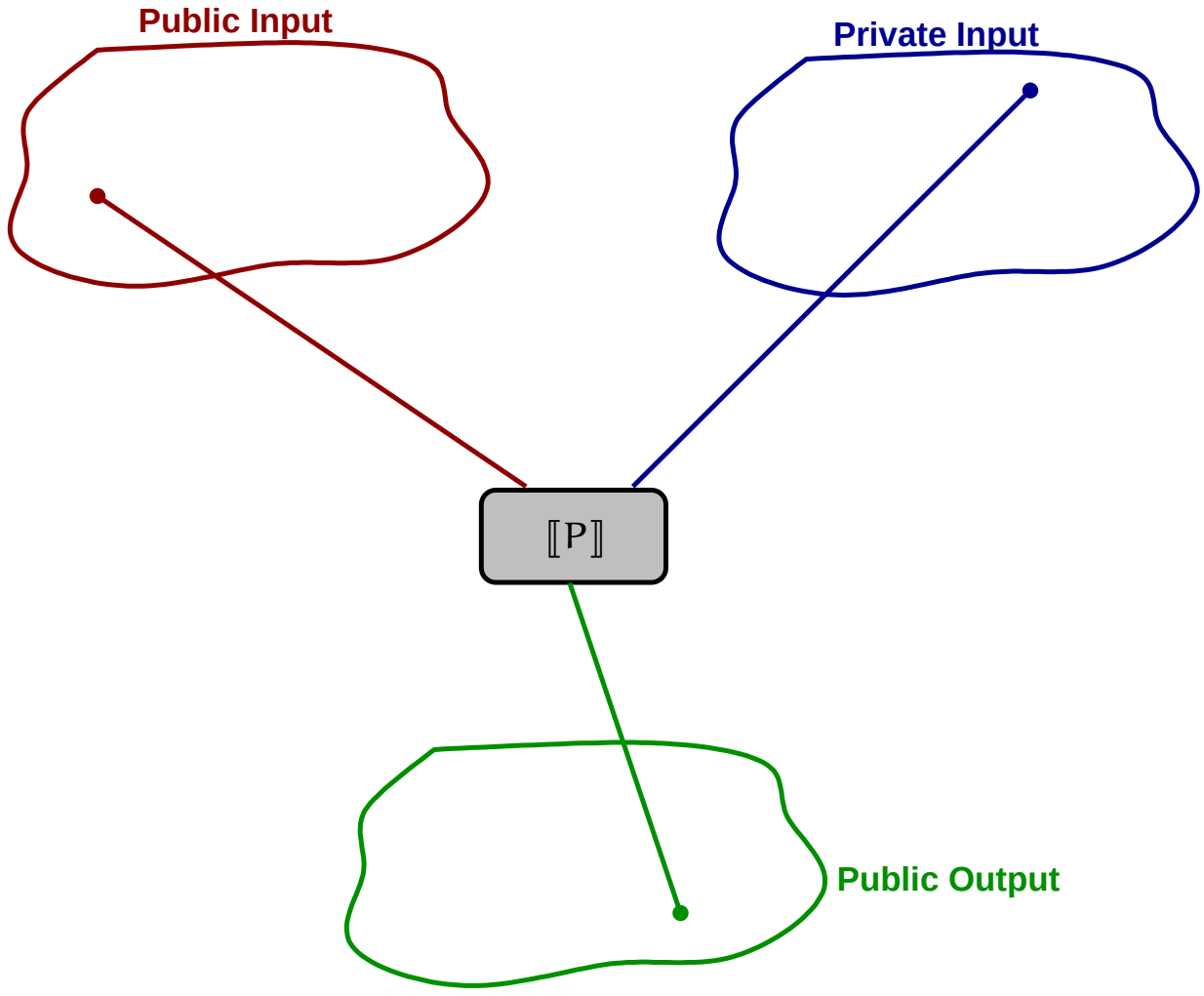

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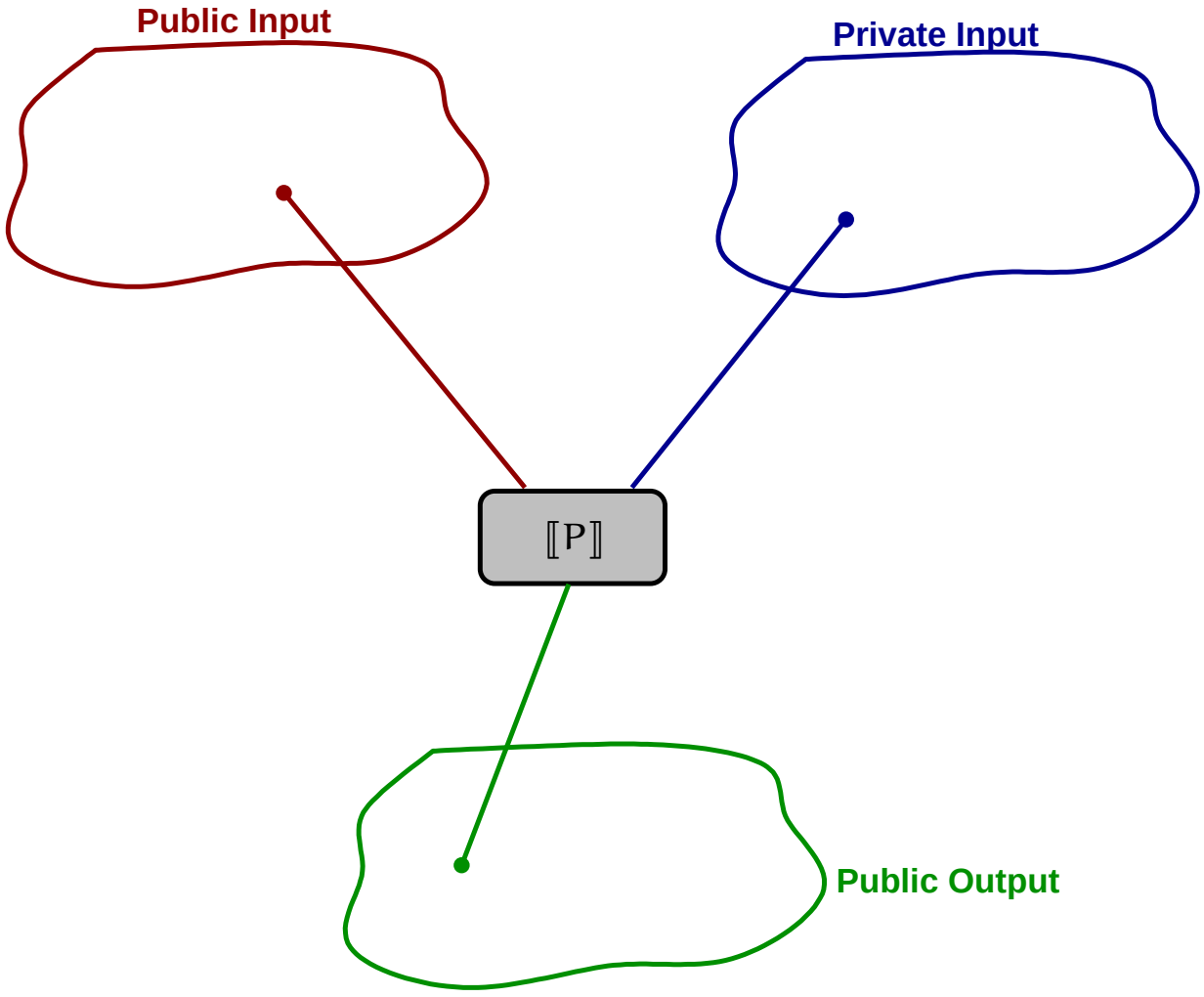

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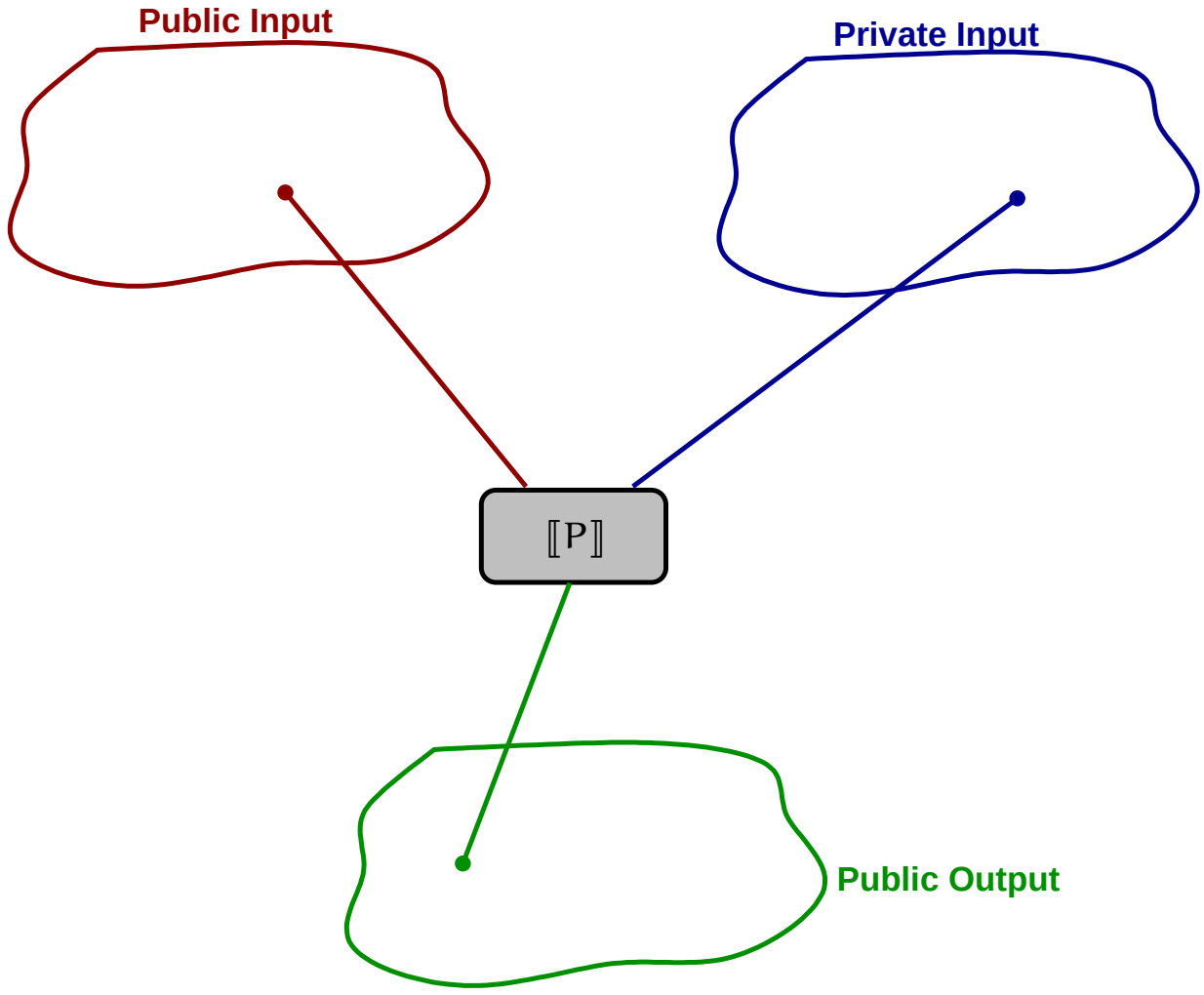

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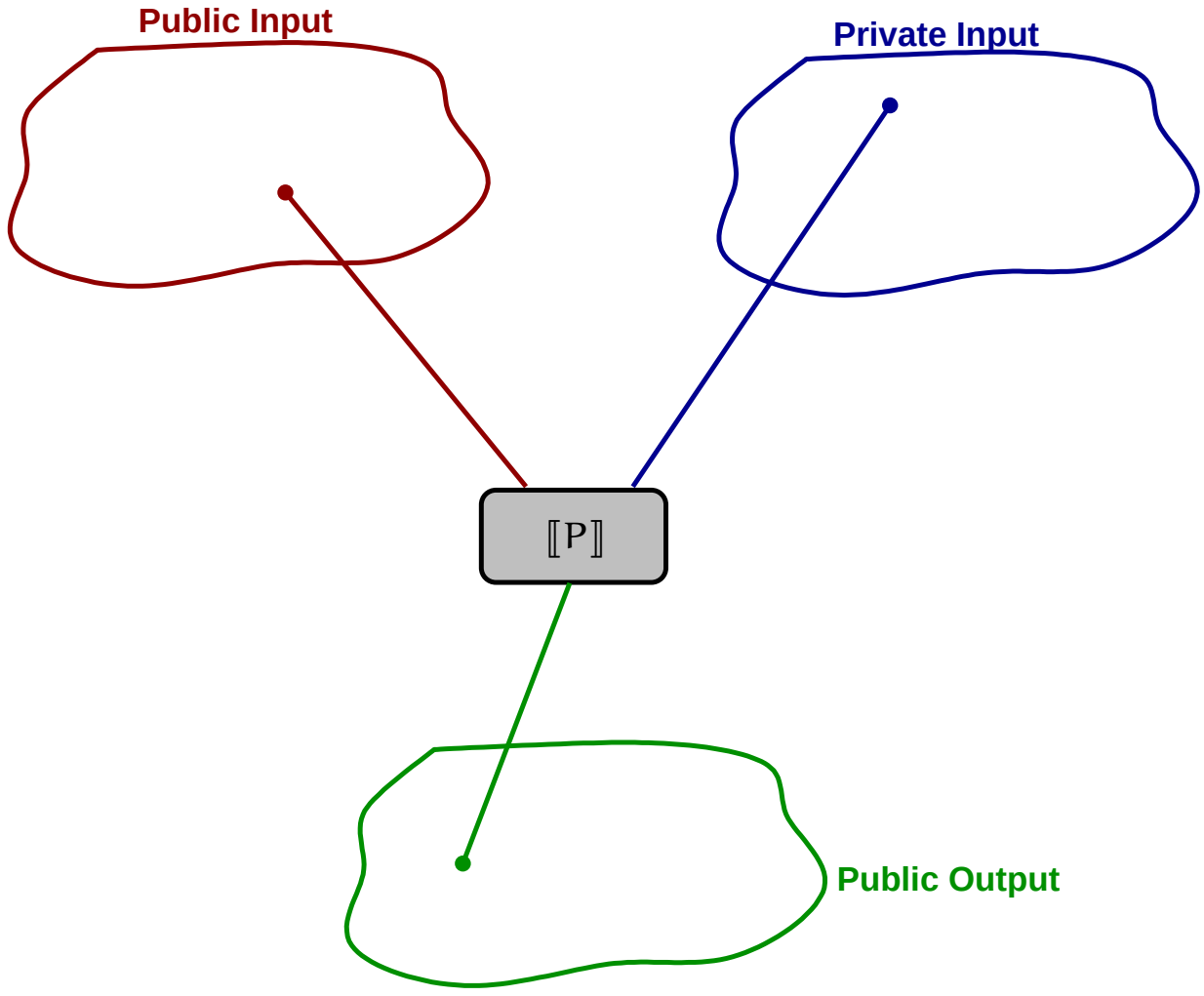

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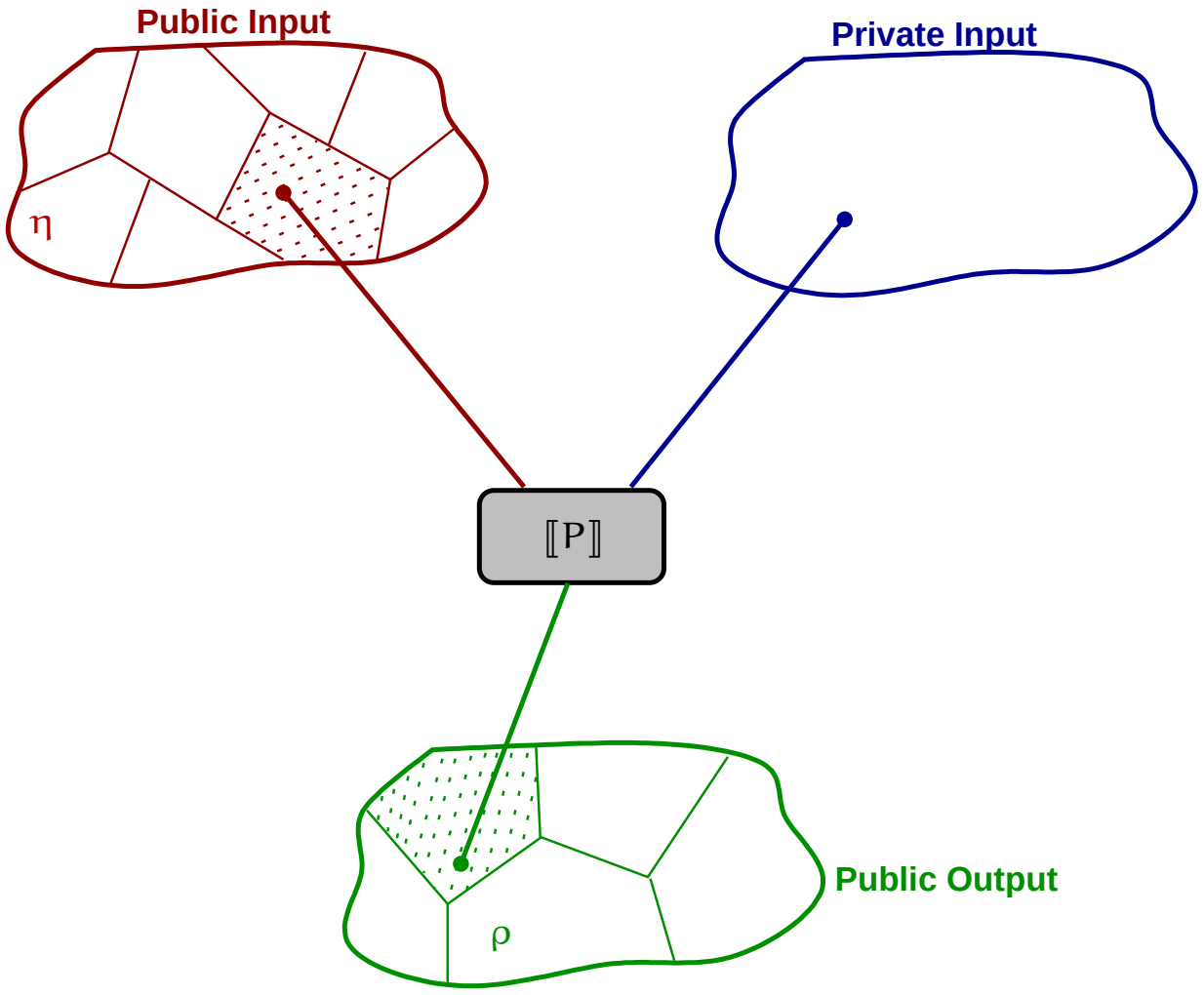

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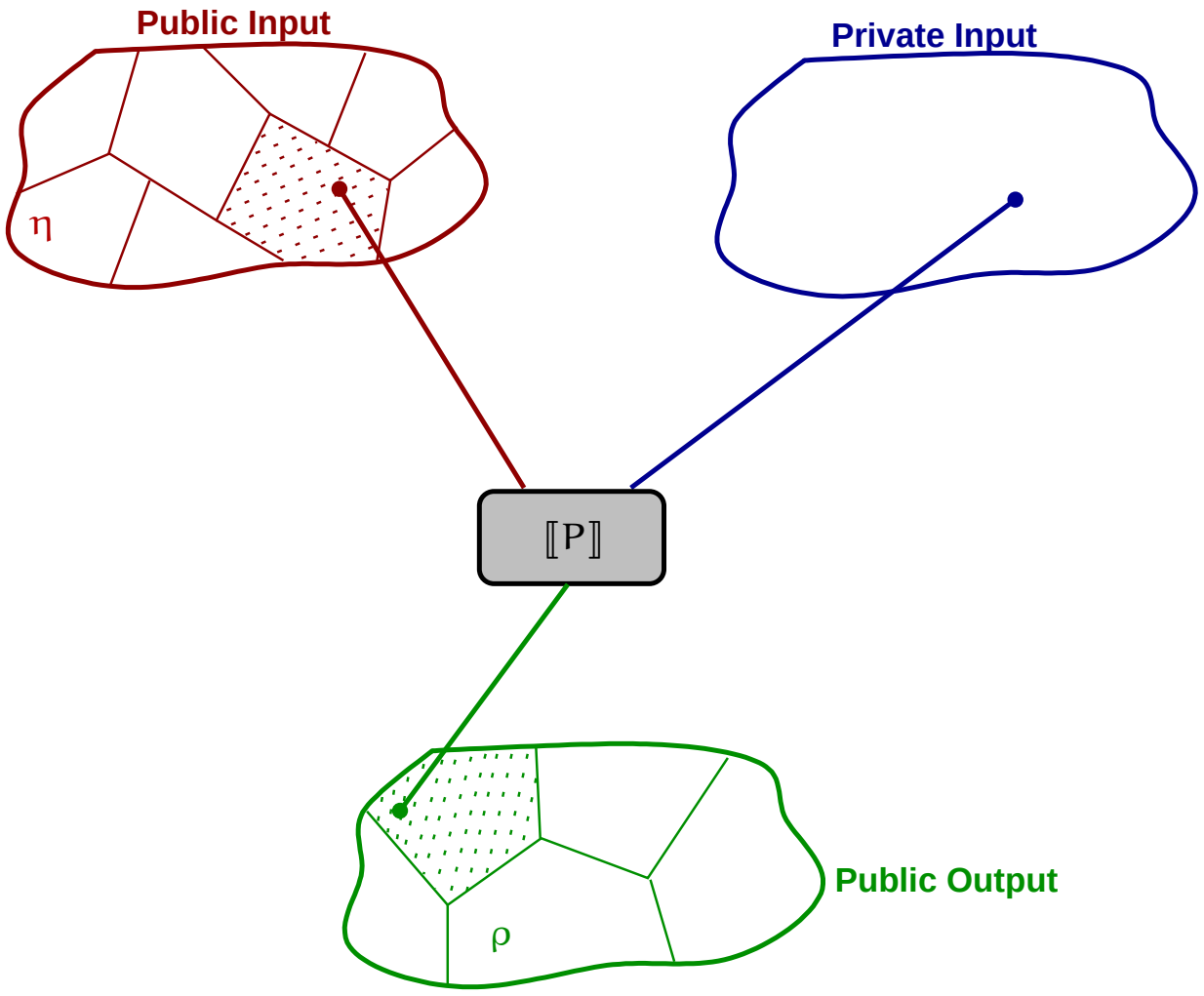

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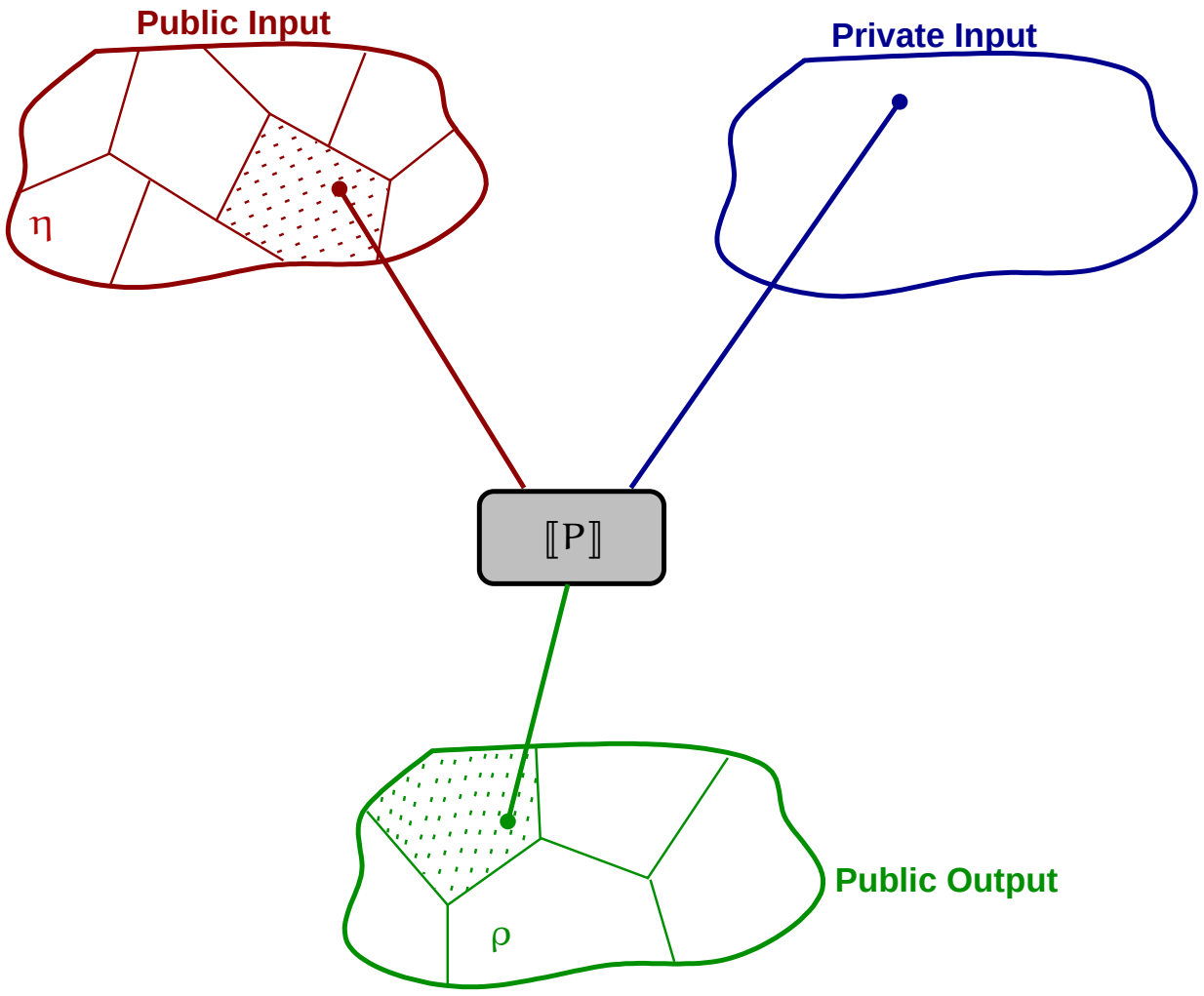

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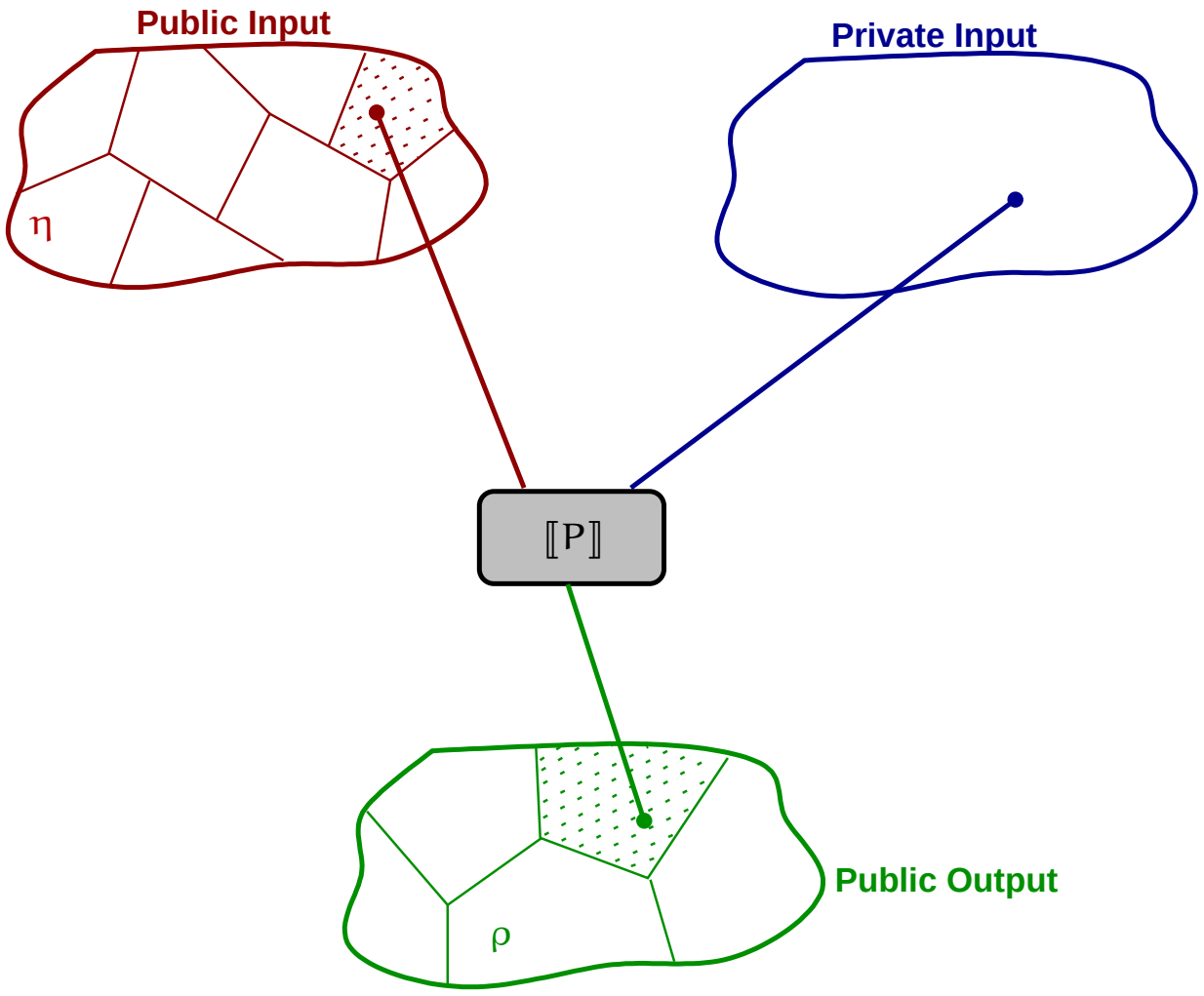

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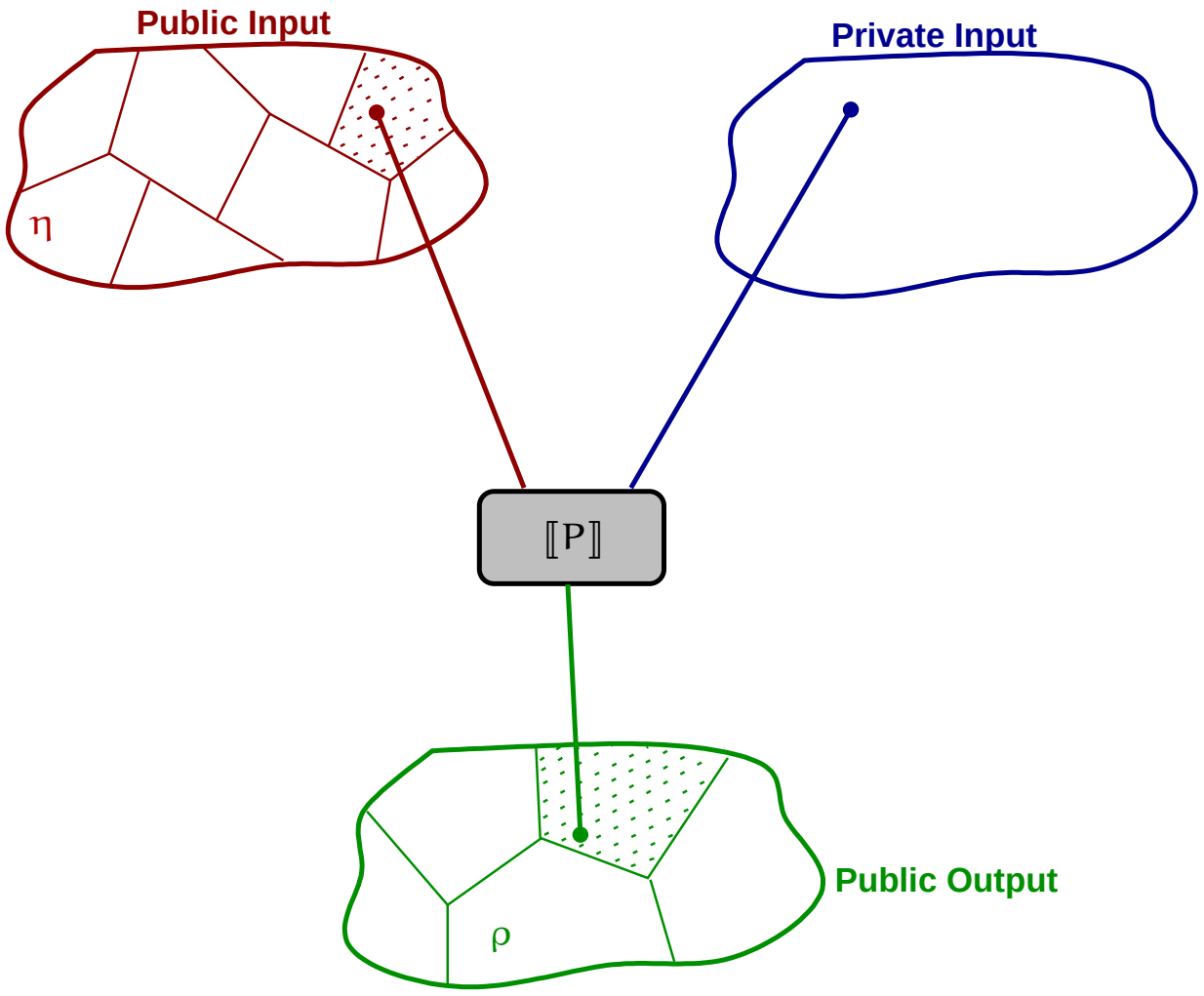

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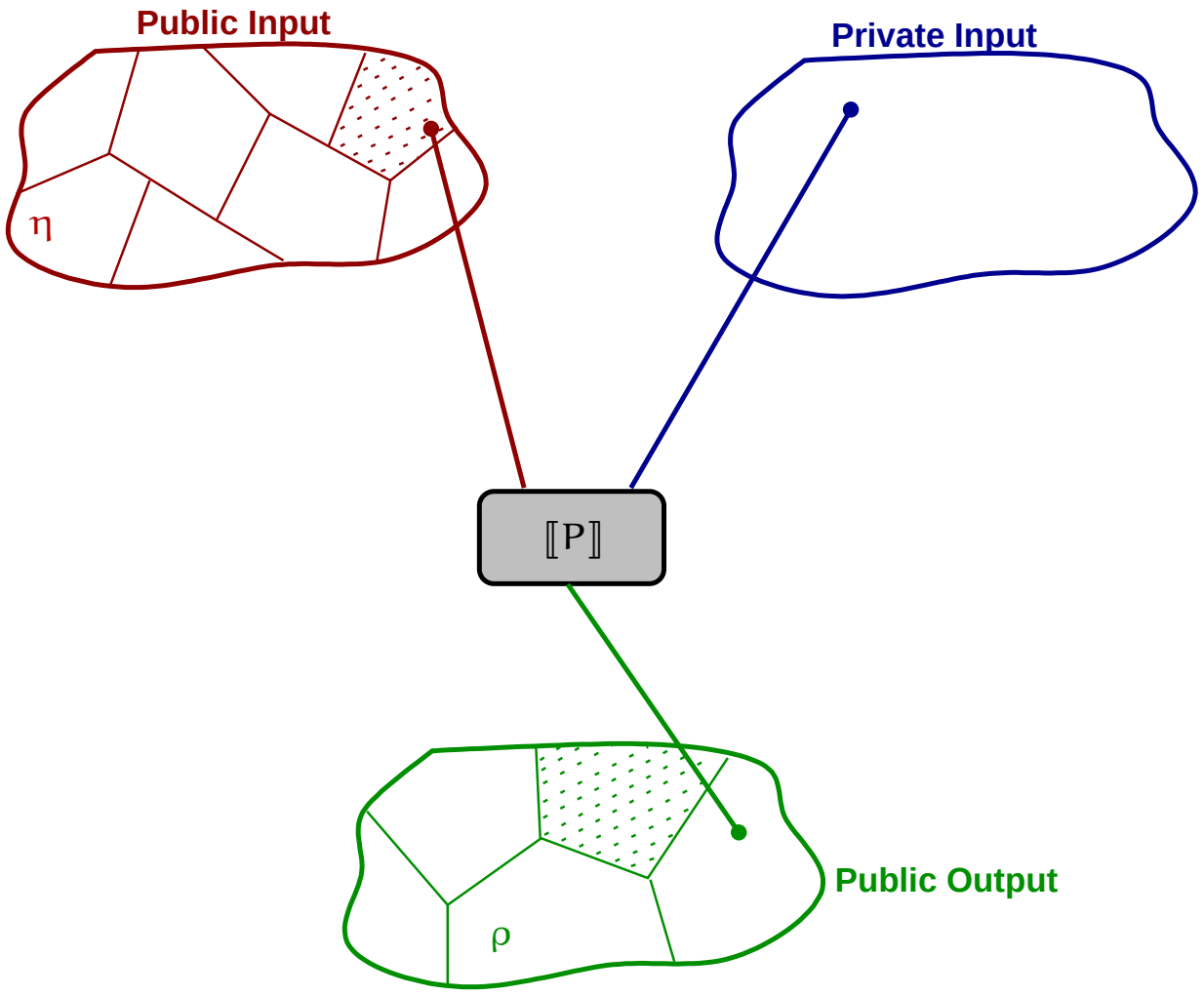

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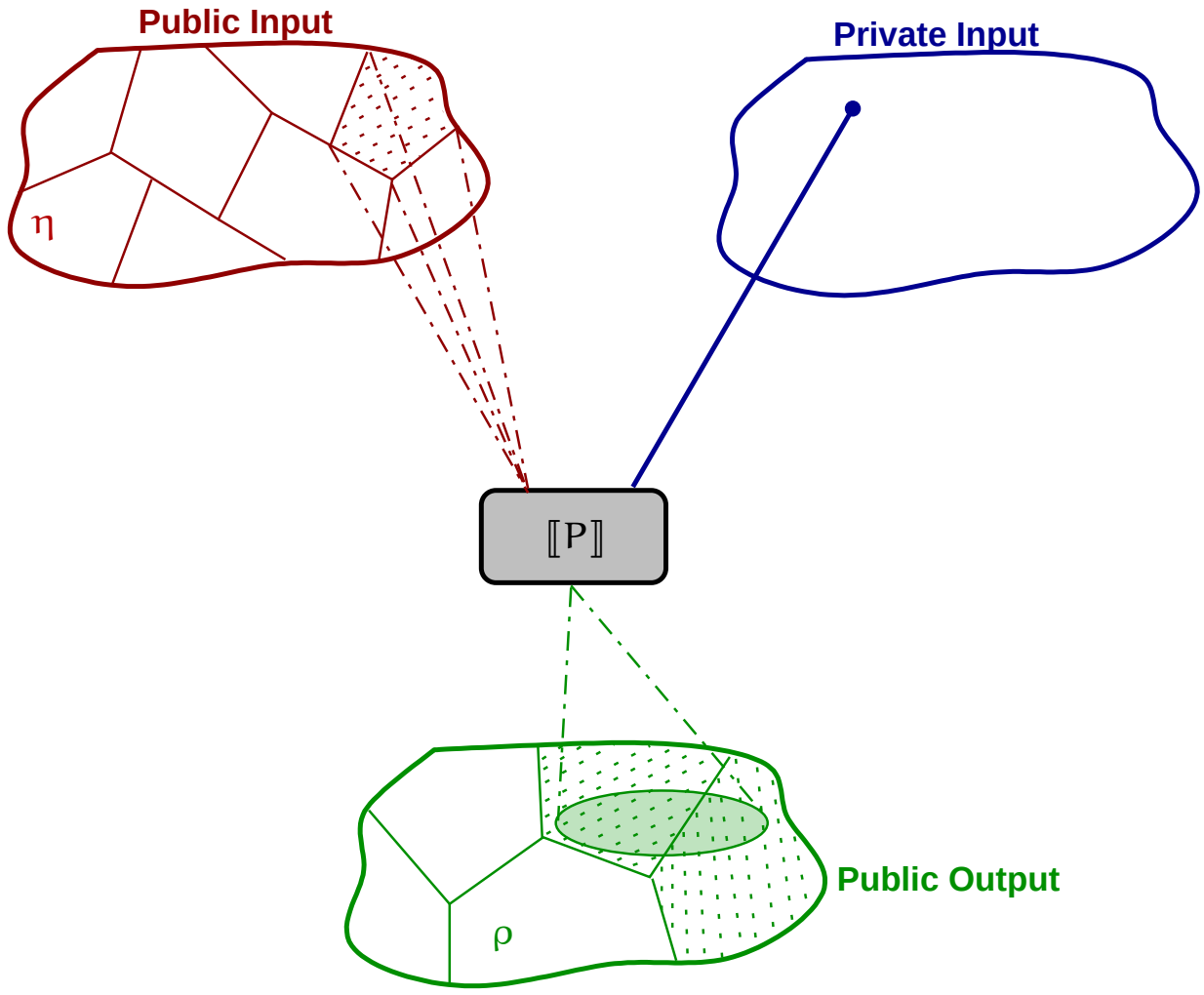

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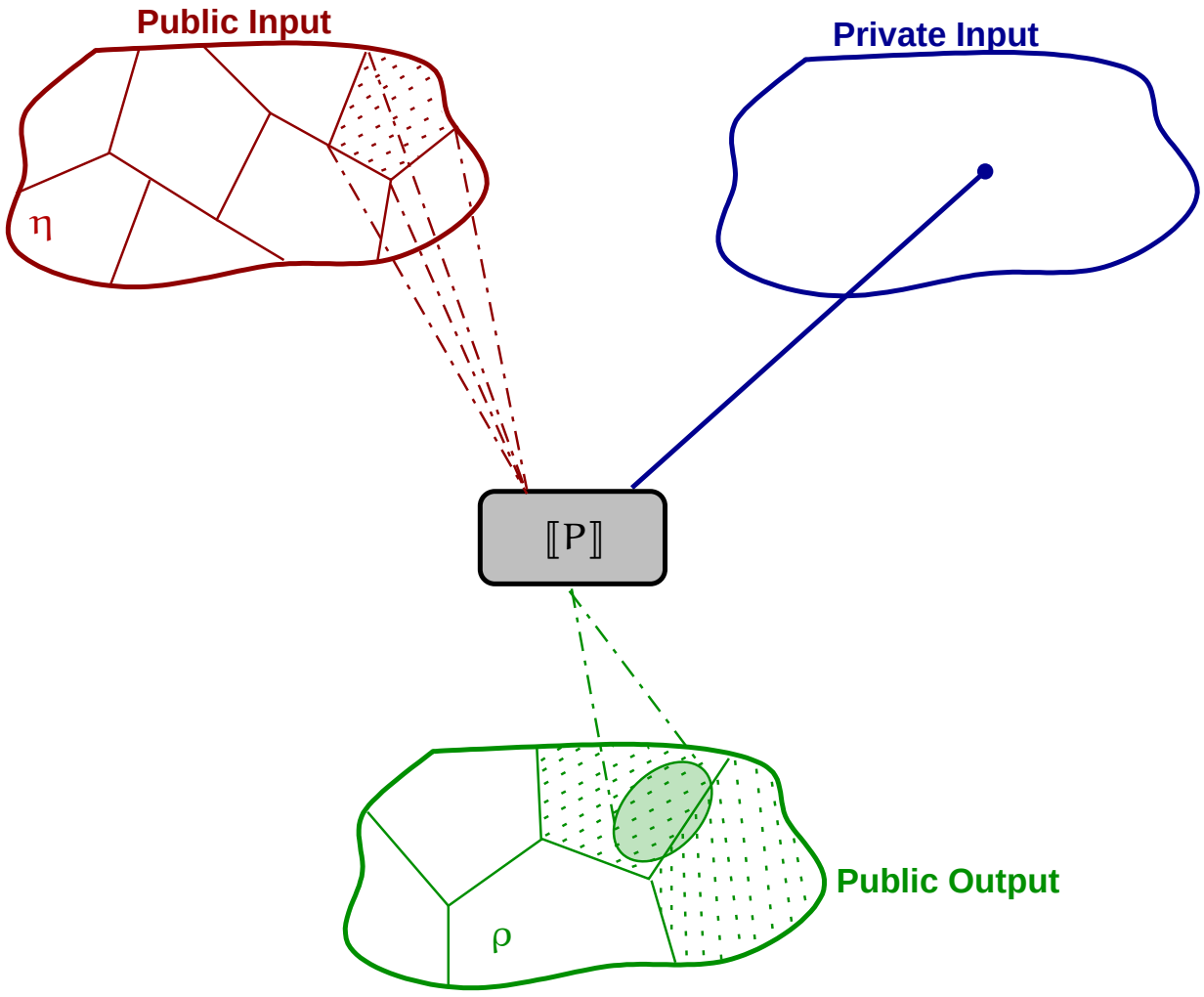

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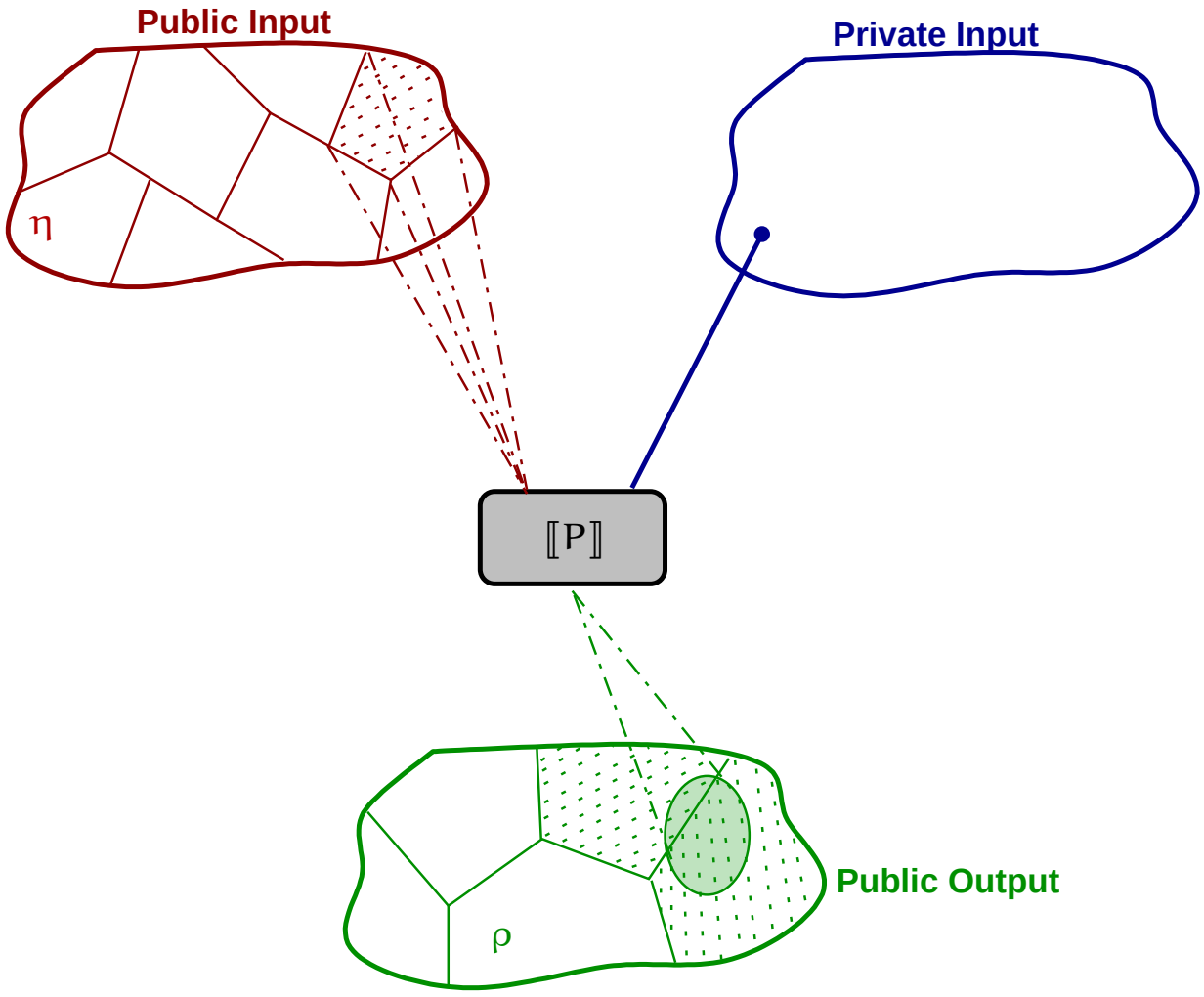

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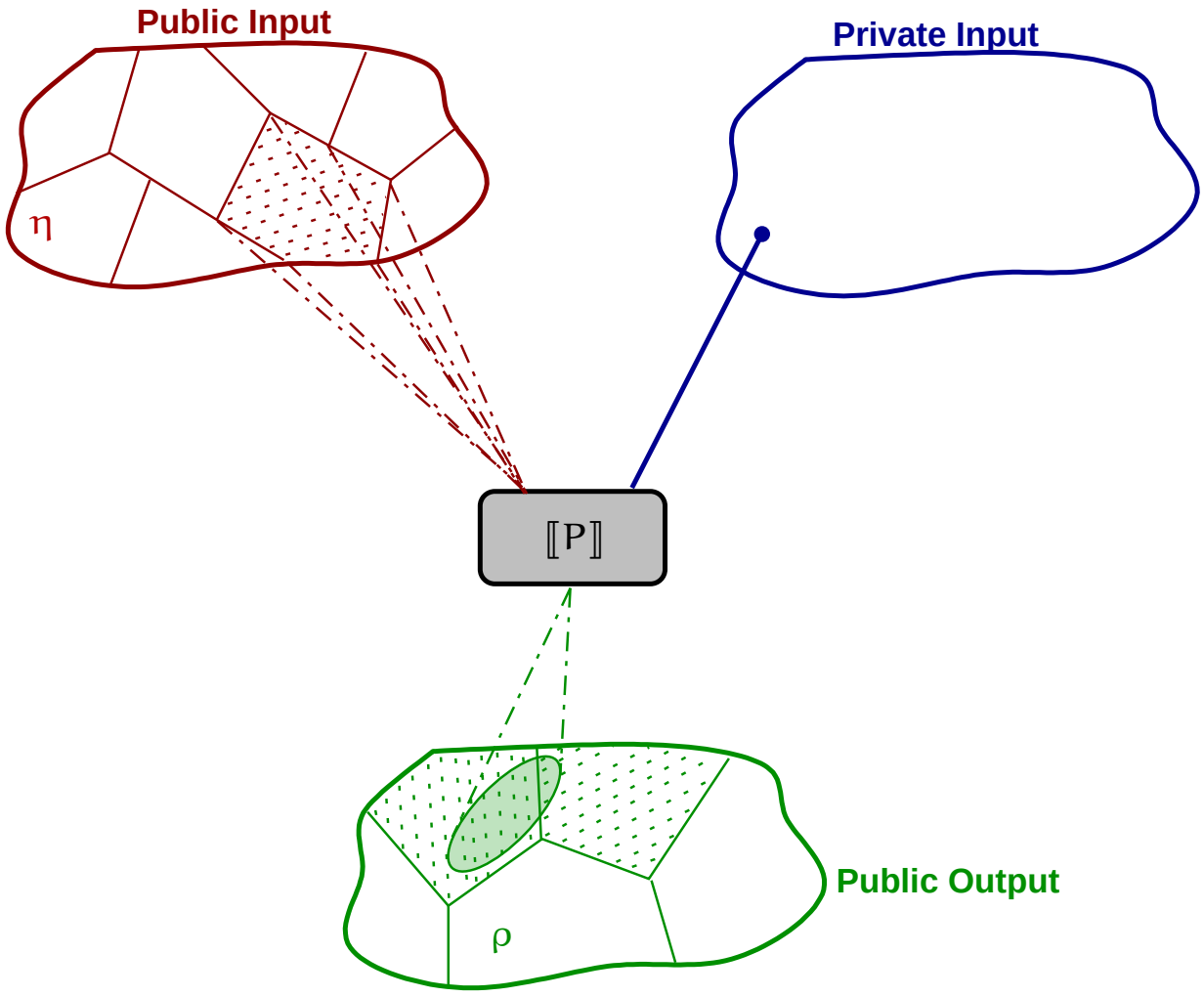

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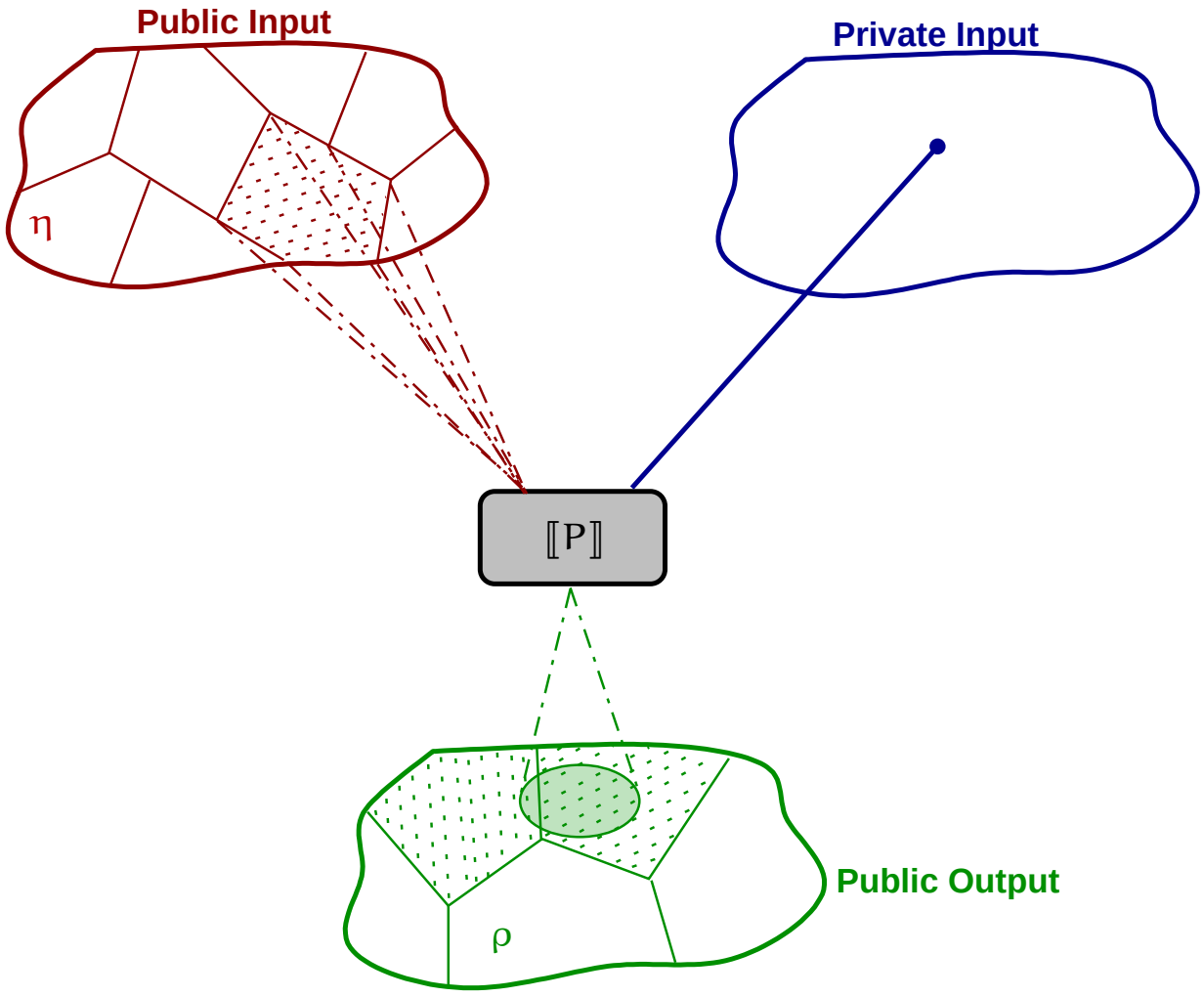

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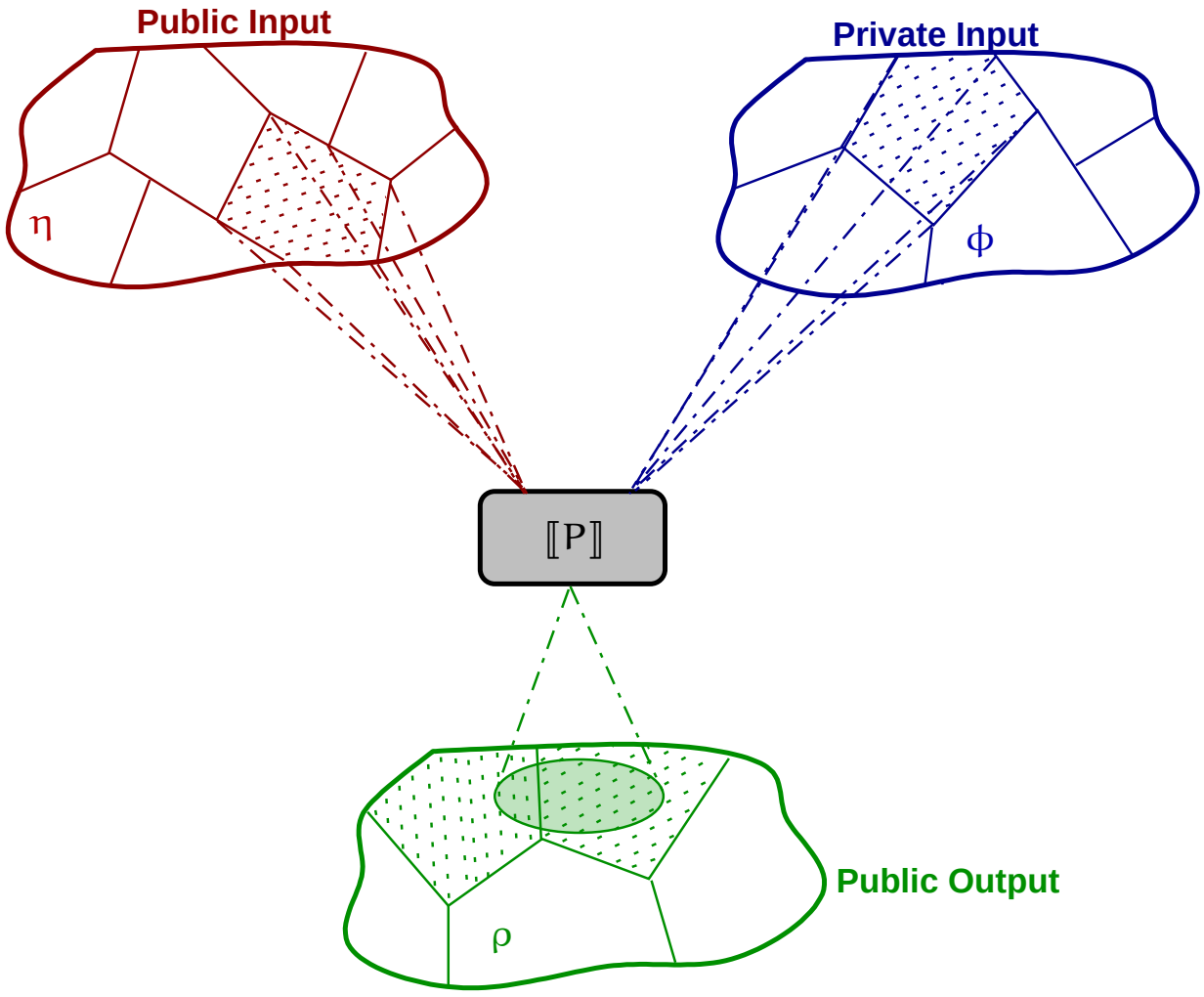

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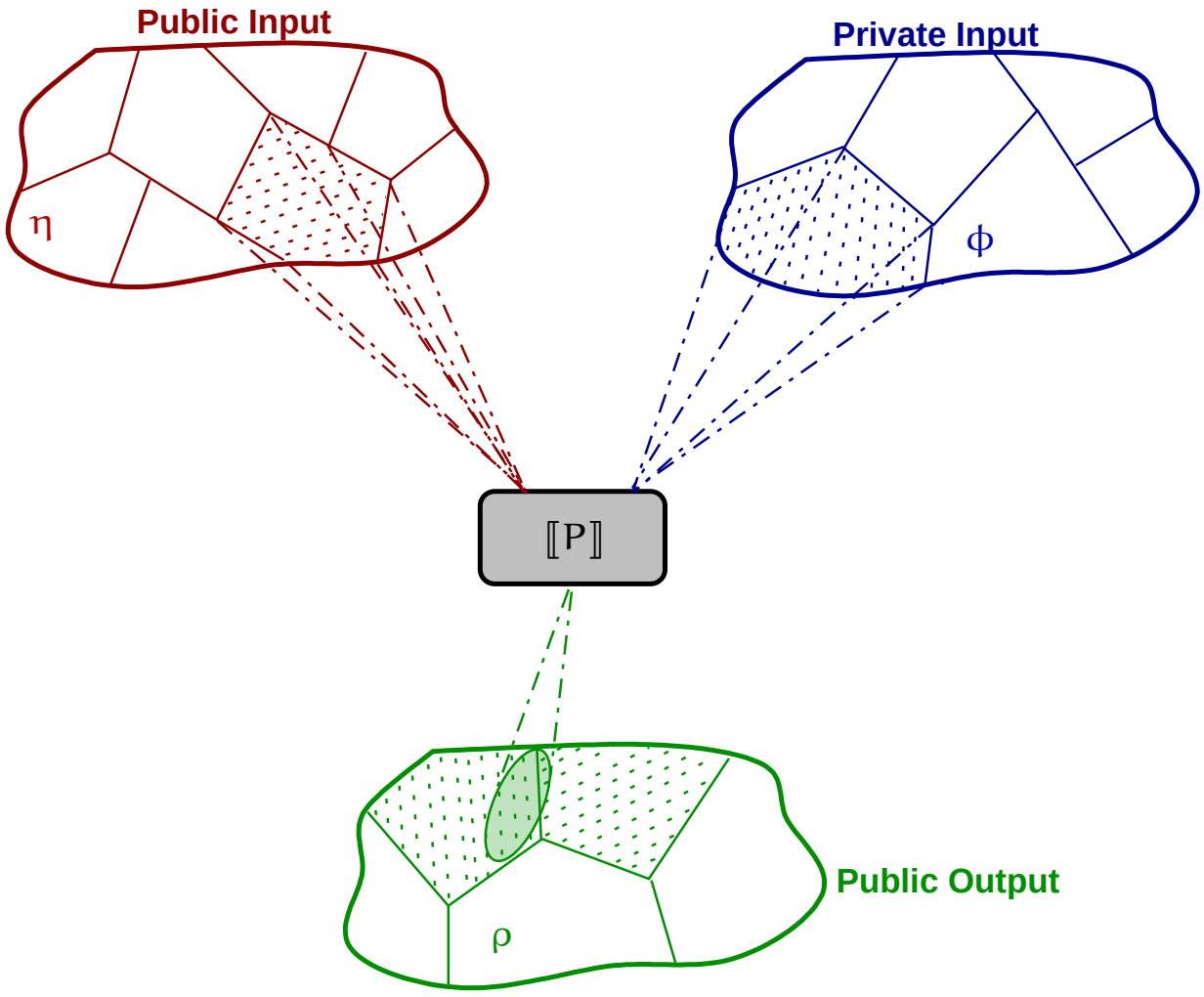

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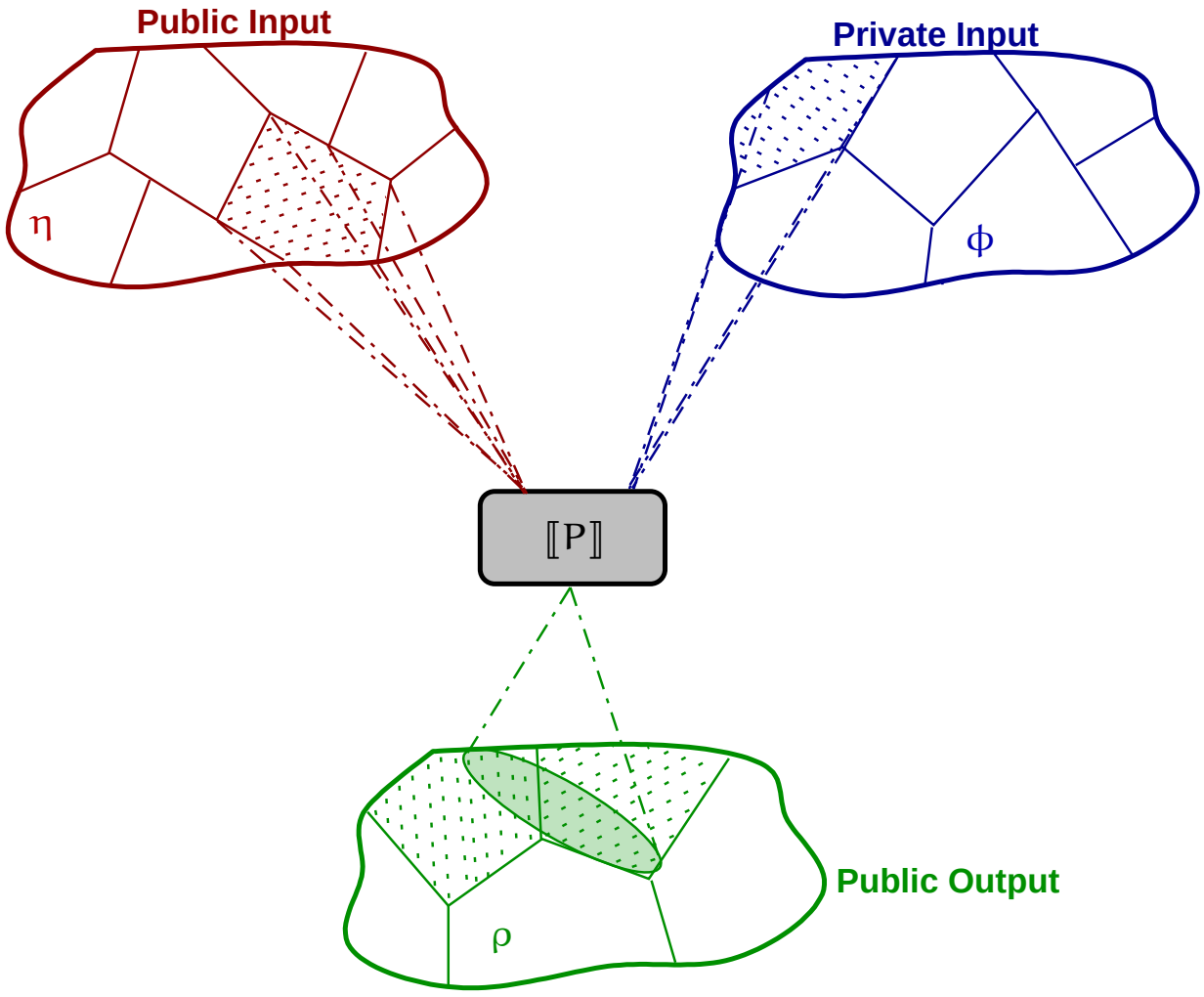

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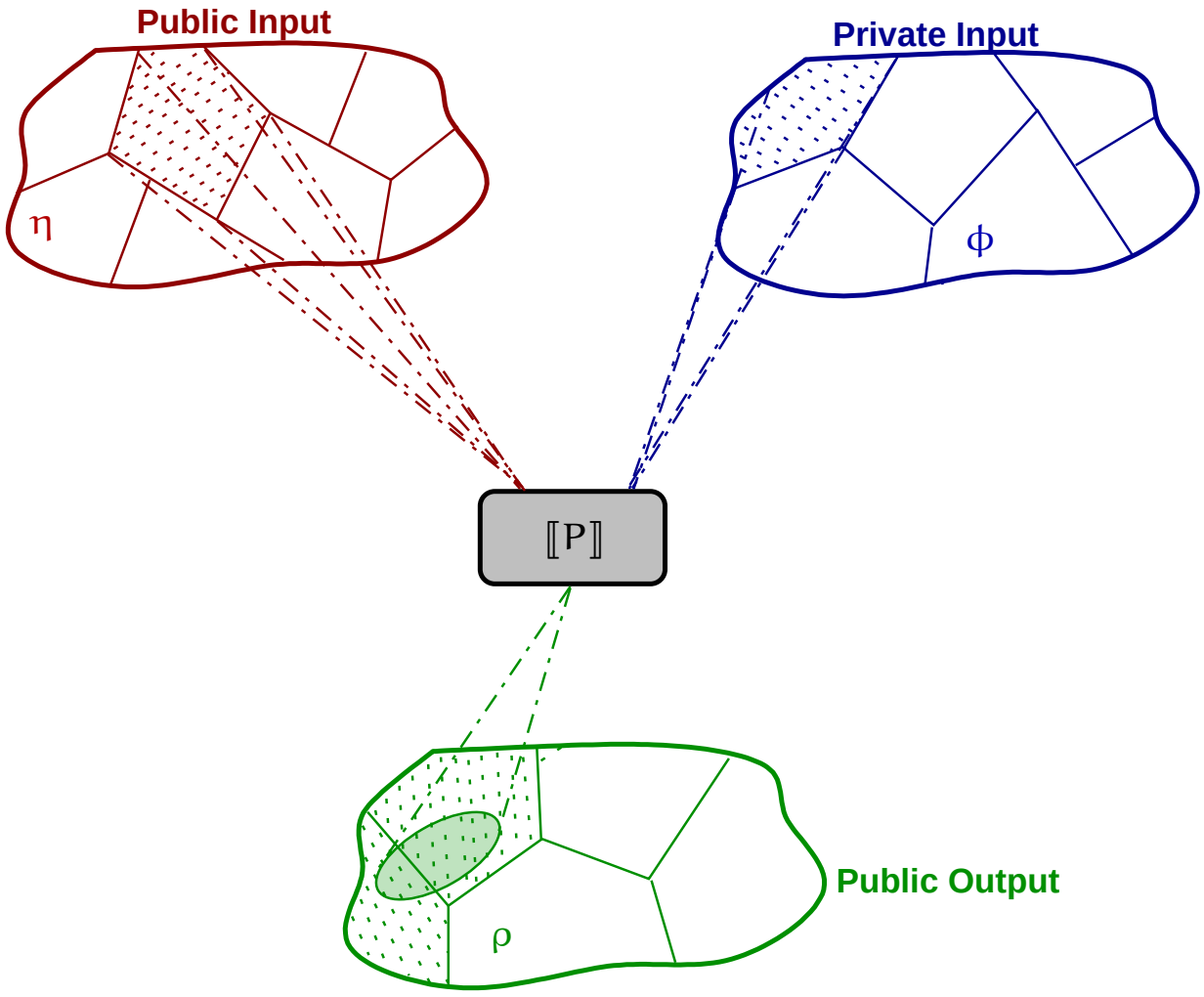

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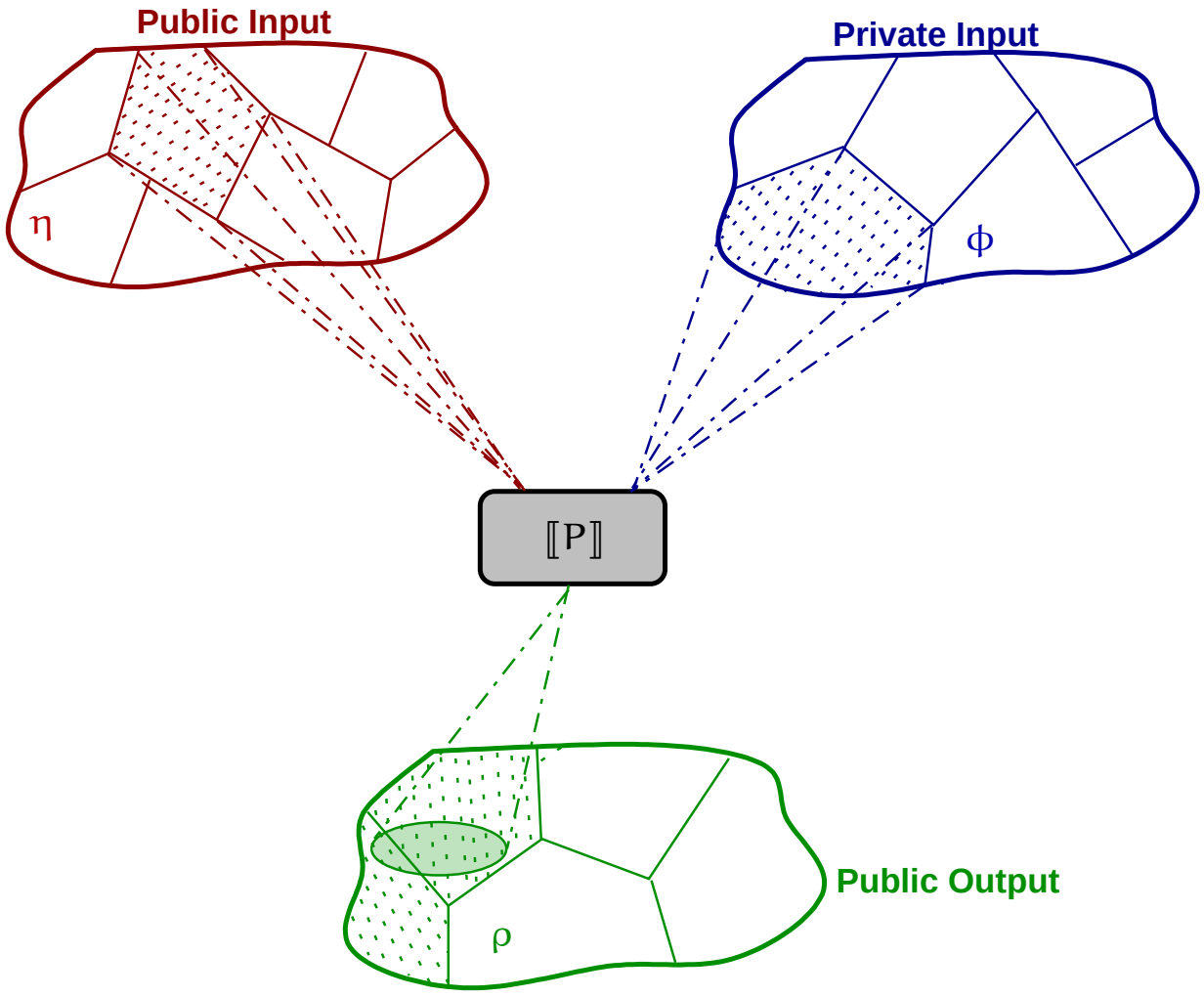

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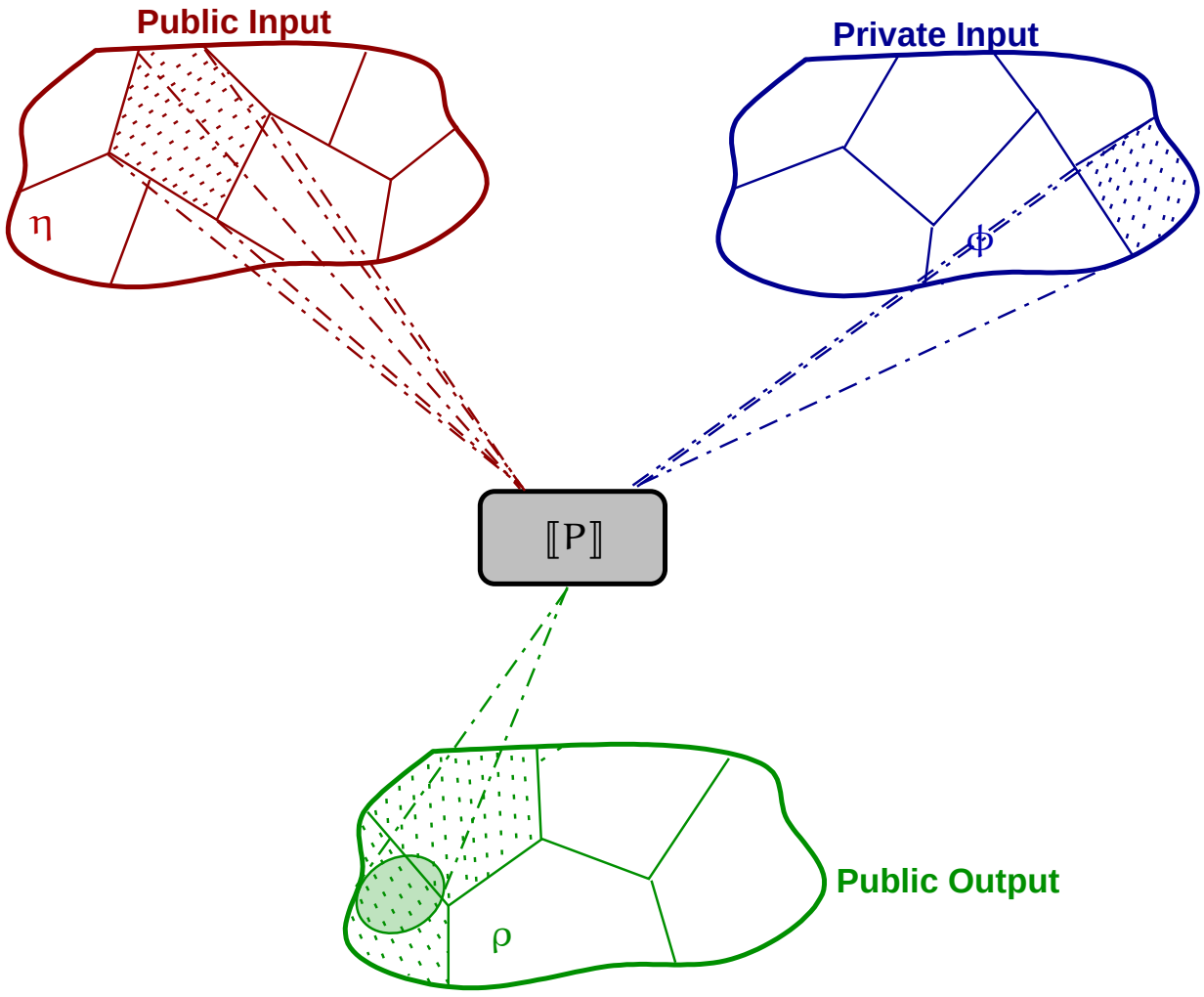

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Deriving output attackers

Abstract interpretation provides advanced methods for designing abstractions (refinement, simplification, compression ...) [Giacobazzi & Ranzato ’97]

Designing abstractions = designing attackers

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  [The most powerful *public observer*]

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  [The most powerful *public observer*]

$\Rightarrow$ This would provide a certificate for security with a fixed input observation.

Equivalence relation vs Closure Operators

Equivalence relations uniquely correspond to particular upper closure operators:

PARTITIONING
[Ranzato and Tapparo'04]

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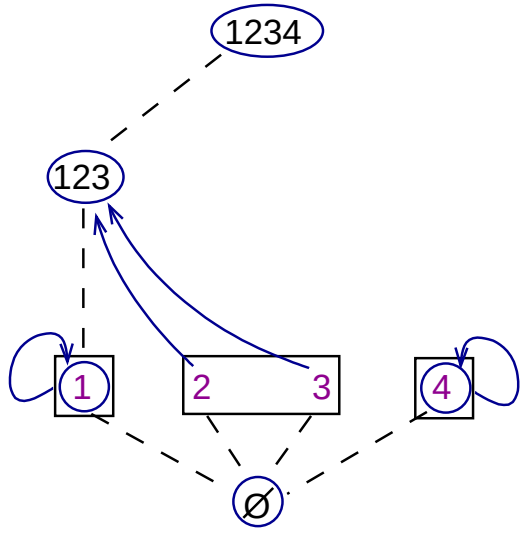

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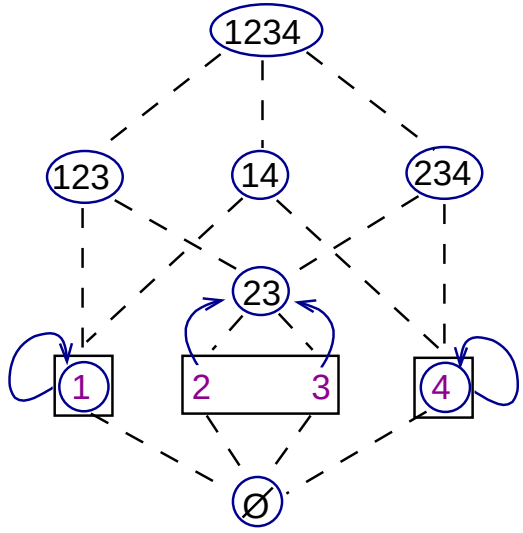

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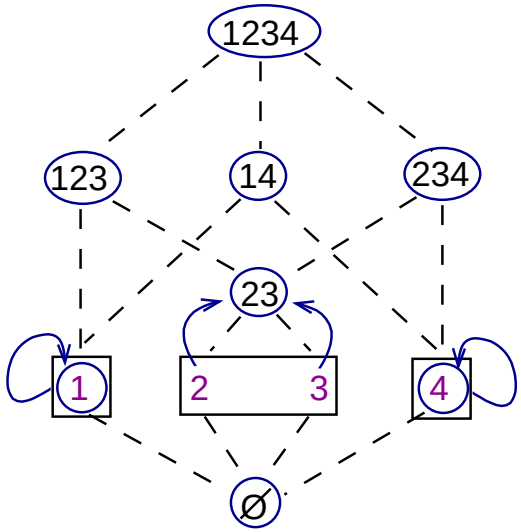

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- $\Pi(\eta) \stackrel{\text{def}}{=} \mathit{Clo}^{\mathit{Rel}^{\eta}} \sqsubseteq \eta$

PER model vs Narrow ANI

PROPOSITION:

$$[\eta]\mathrm{P}(\rho) \quad \text{iff} \quad [\![\mathrm{P}]\!] : \mathit{All} \times \mathit{Rel}^{\eta} \to \mathit{All} \times \mathit{Rel}^{\rho}$$
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$$\Downarrow$$

$$[\mathrm{R}]\mathrm{P}(\mathrm{S}) \text{ iff } [\![\mathrm{P}]\!] : \mathit{All} \times \mathrm{R} \to \mathit{All} \times \mathrm{S}$$

PER model vs ANI

PROPOSITION 2:

$$(\eta)\mathrm{P}(\phi \rightsquigarrow\!\!\!\!\!\rightarrow\!\!\!] \rho) \quad \text{if} \quad (\Pi(\eta))\mathrm{P}(\Pi(\phi) \rightsquigarrow\!\!\!\!\!\rightarrow\!\!\!] \Pi(\rho))$$

PER model vs ANI

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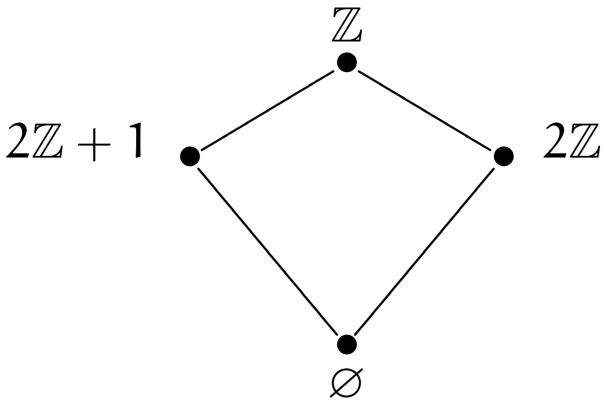
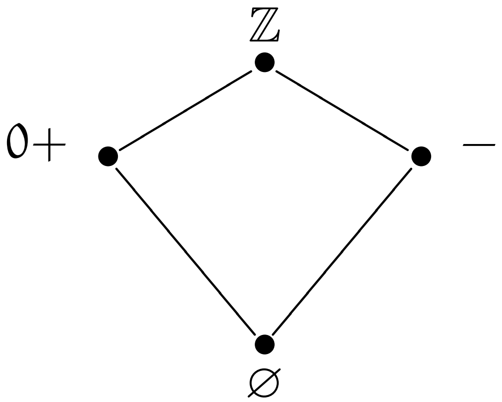
EXAMPLE:

$$P \stackrel{\text{def}}{=} \textbf{if } h = 0 \textbf{ then } l := l \; mod \; 6 + 2; \textbf{else if } l < 0 \textbf{ then } l := 2 \textbf{ else } l := 7;$$



$\eta = \mathit{Par}$



$\phi = \mathit{Sign}$

$\rho = \mathit{Int}$

PER model vs ANI

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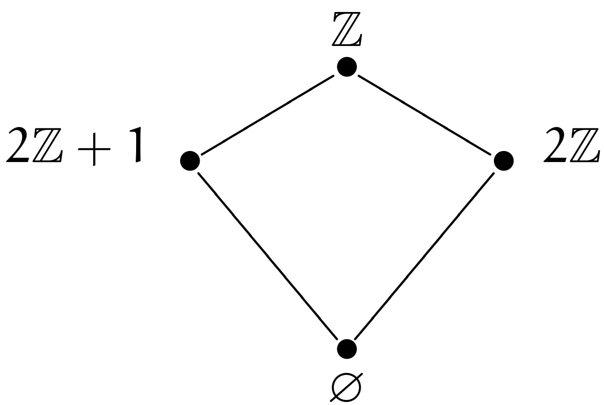

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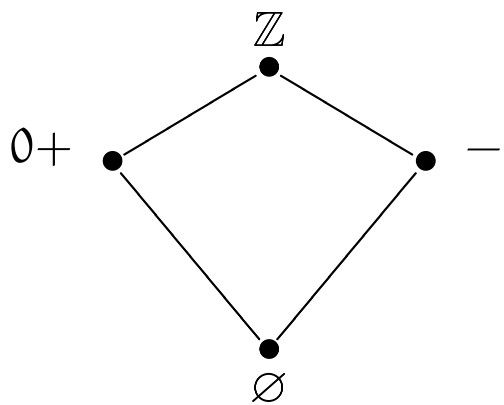

$\phi = \textit{Sign}$

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$$\eta(l) = 2\mathbb{Z}: \quad \rho(\llbracket P \rrbracket(0+, 2\mathbb{Z})^L) = \rho(\{2,3,5,7\}) = [2,7] = \rho(\llbracket P \rrbracket(-, 2\mathbb{Z})^L) = \rho(\{2,7\})$$

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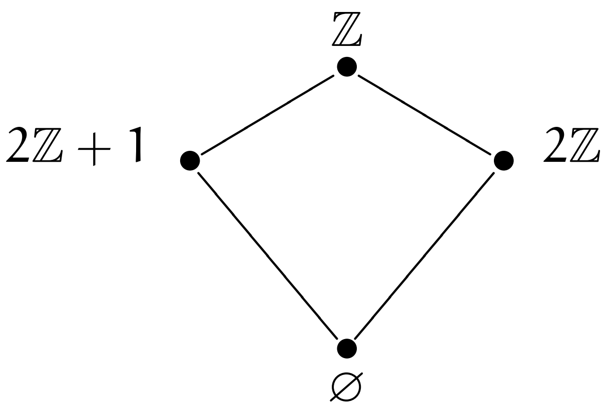
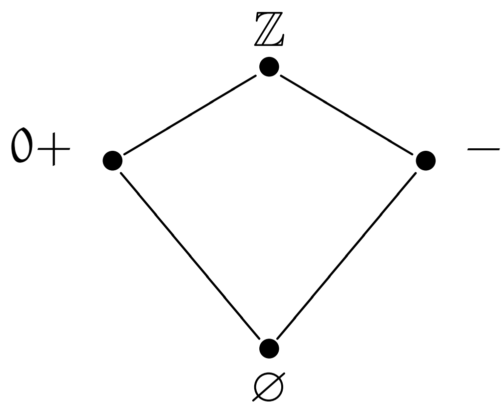


$\eta = Par$



$\phi = Sign$

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$$\eta(l) = 2\mathbb{Z}: \quad \Pi(\rho)(\llbracket P \rrbracket(0+, 2\mathbb{Z})^{L}) = \Pi(\rho)(\{2,3,5,7\}) = \{2,3,5,7\} \neq \{2,7\}$$

$$\Pi(\rho)(\llbracket P \rrbracket(-, 2\mathbb{Z})^{L}) = \Pi(\rho)(\{2,7\})$$

Deriving un-constrained attackers

Input Observation: R;

Output Observation: S;

Non-Interference: $f : R \rightarrowtriangle S$ iff $x \, R \, y \;\Rightarrow\; f(x) \, S \, f(y)$

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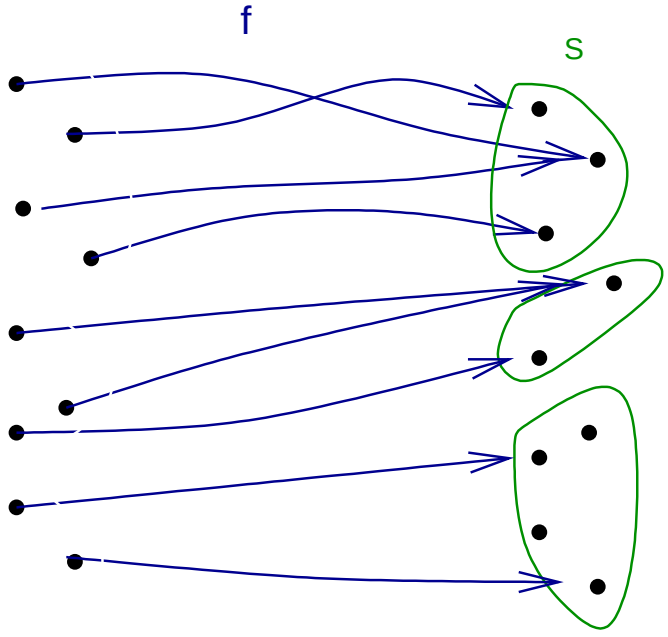

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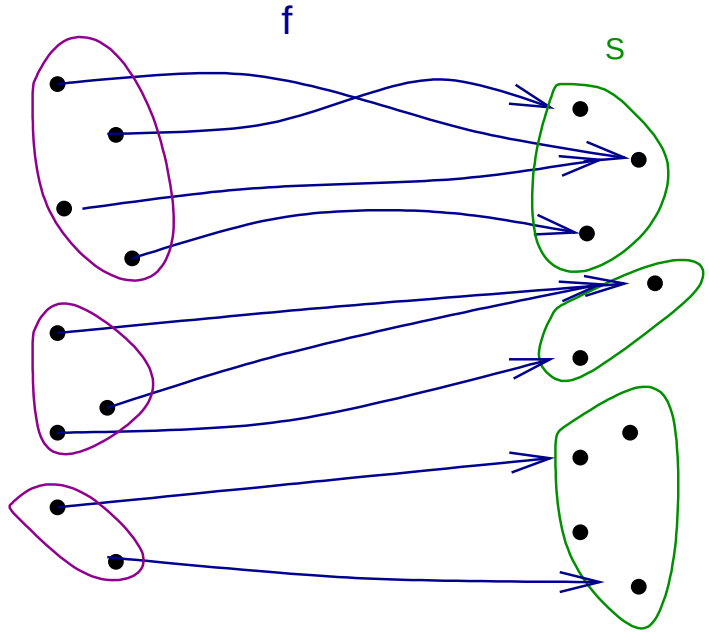

$x \, \widehat{f^{-1}}(S) \, y$ iff $f(x) \, S \, f(y)$

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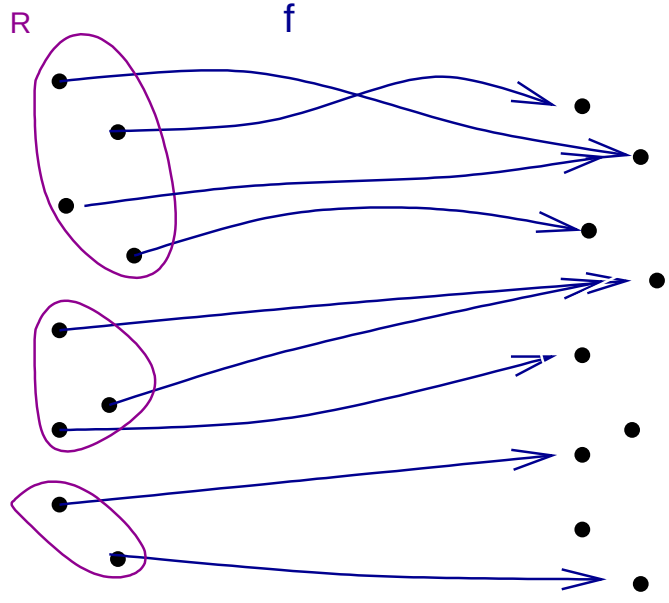

$$x' \, R \, y'$$

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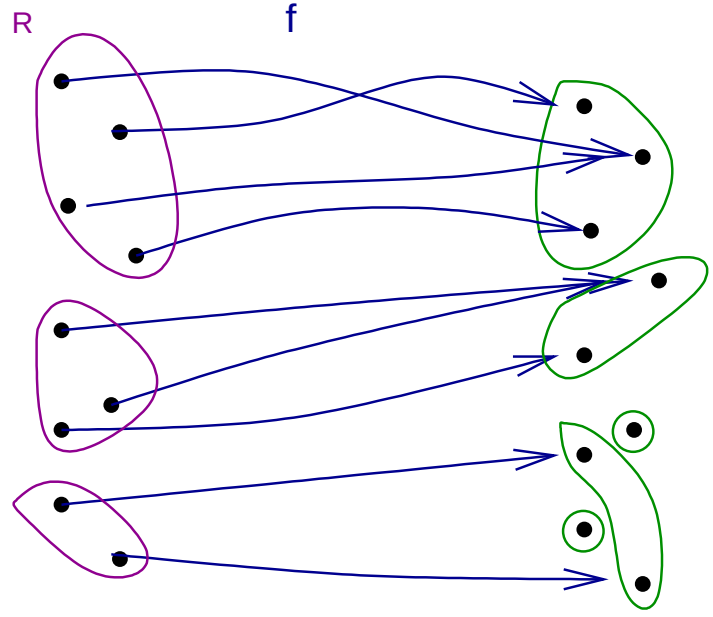

$$x\, \widehat{f}(R)\, y \text{ iff } x = y \text{ or } x = f(x'),\ y = f(y'),\ x'\, R\, y'$$

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Input Observation: R;

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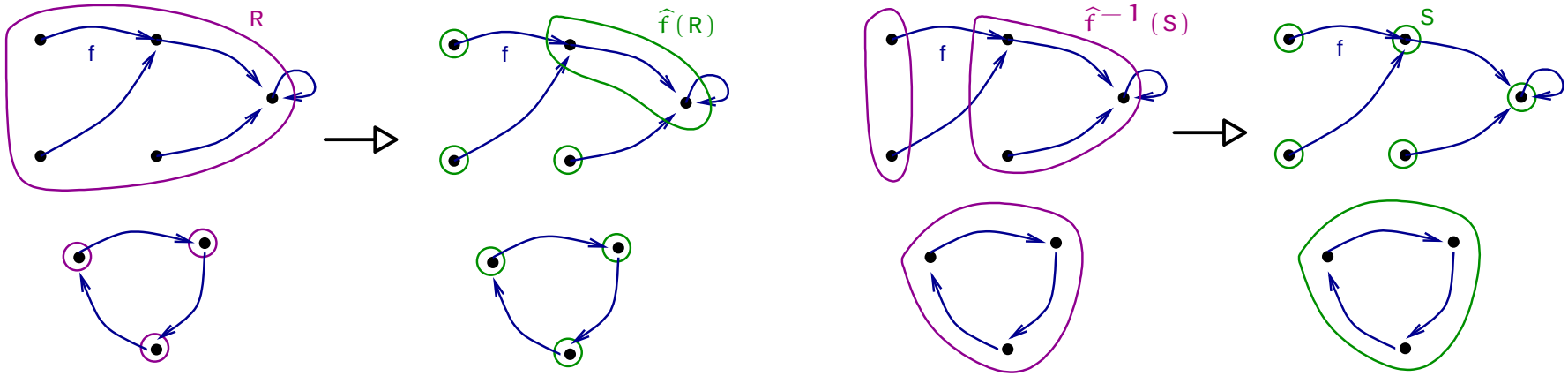
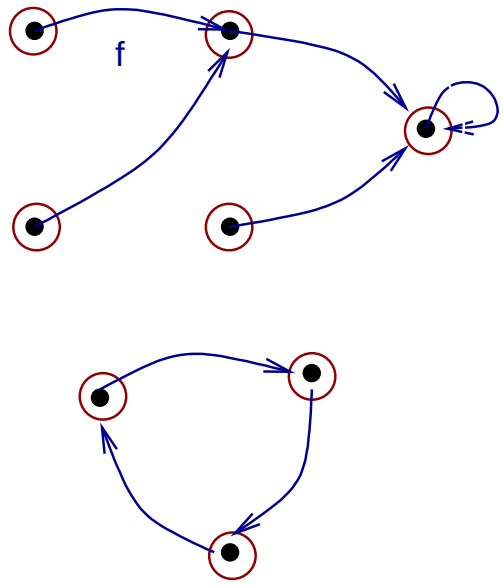


Fix point attackers

- Least fix point of $\widehat{f}^{-1}$



(a)

Fix point attackers

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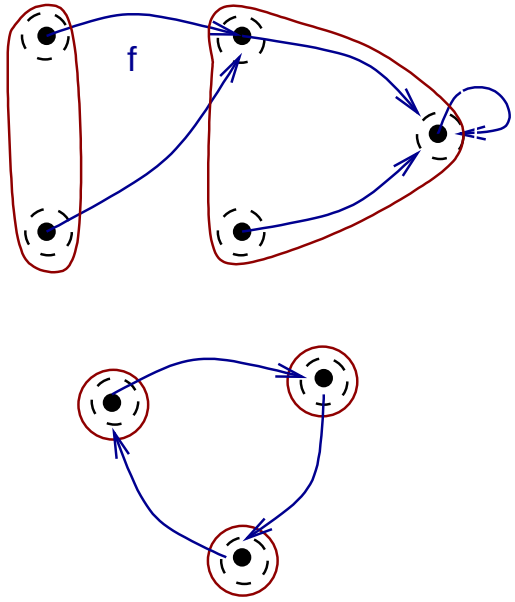

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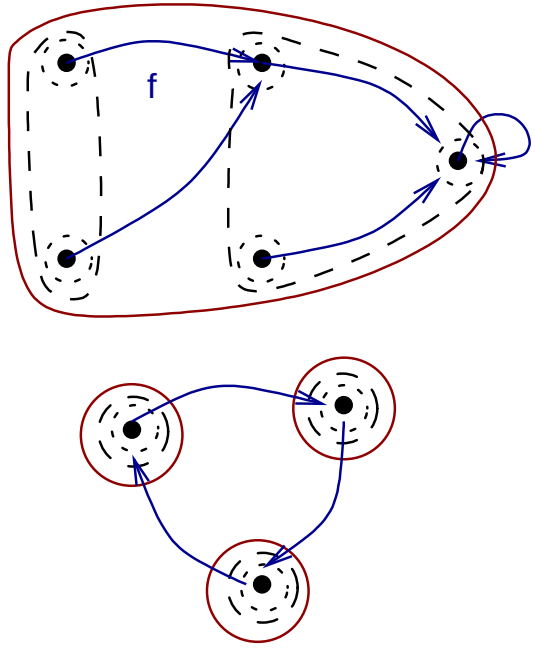

(a)

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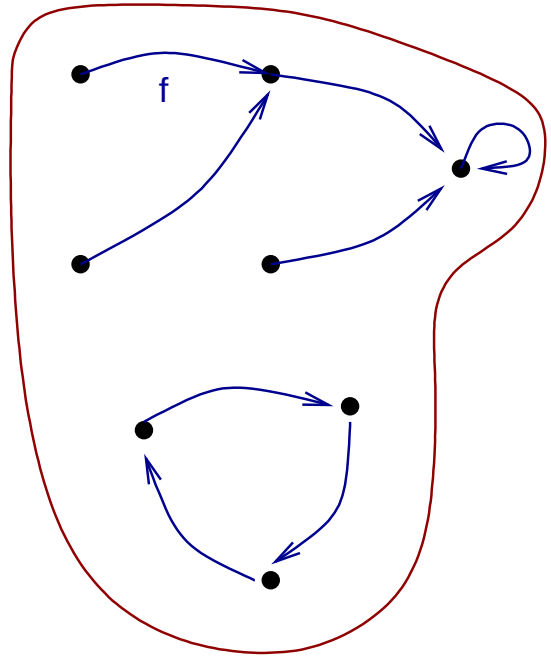

(b)

Fix point attackers

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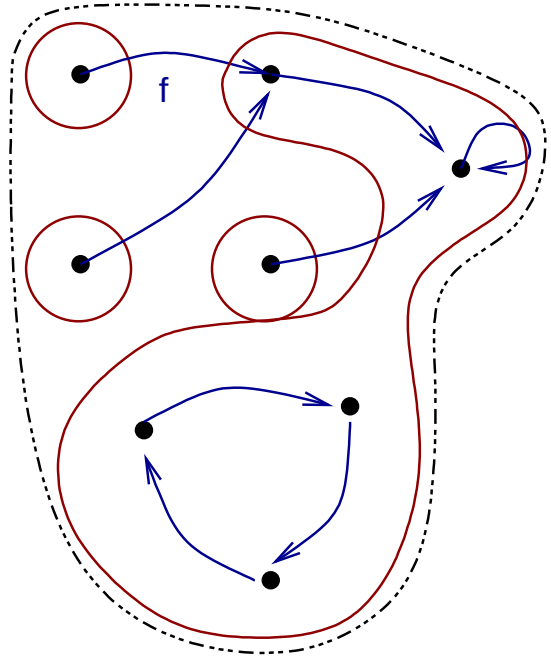

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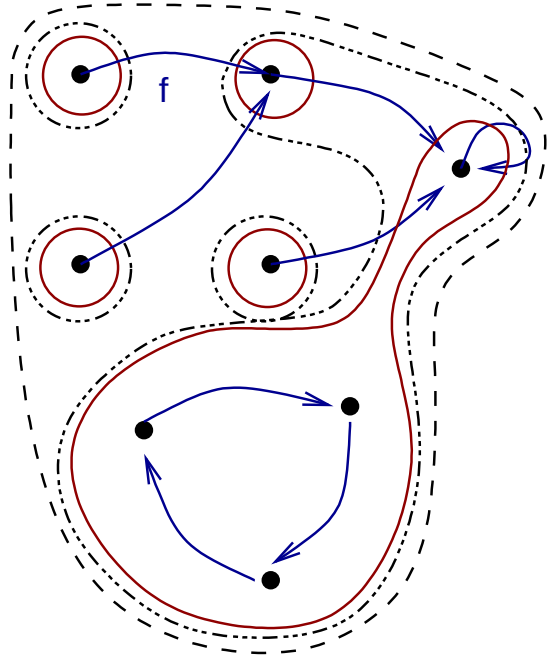

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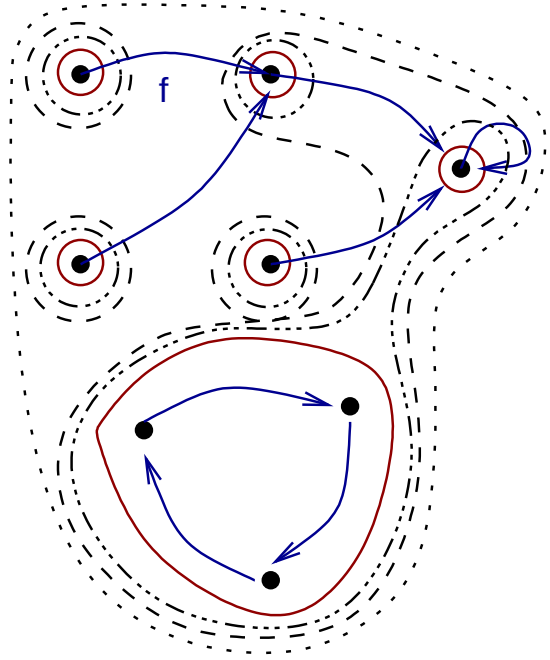

(b)

Fix point attackers

- Least fix point of $\widehat{f}^{-1}$
- Greatest fix point of $\widehat{f}$
- The smallest R such that $[R]P(R)$ is the least fix point of $\widehat{f}(\mathit{All} \times R)$.

PER model and completeness

Completeness in abstract interpretation can be formalized as follows:

$$\rho \circ f \circ \eta = \rho \circ f$$

[Giacobazzi et al. '00]

iff

$$\forall x, y.\, \eta(x) = \eta(y) \;\Rightarrow\; \rho(f(x)) = \rho(f(y))$$

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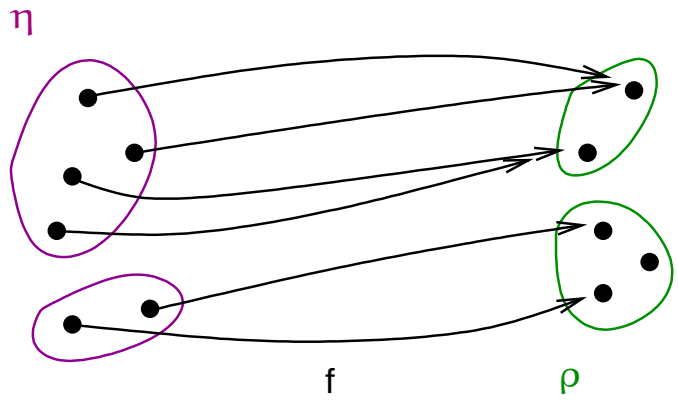

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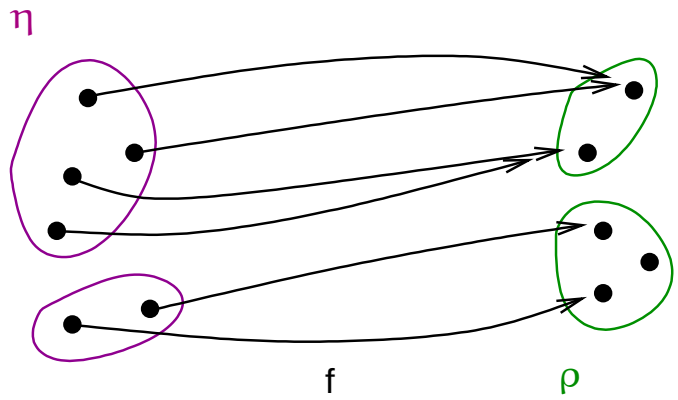


$$\Downarrow$$

$$f : R \rightarrowtriangle S$$

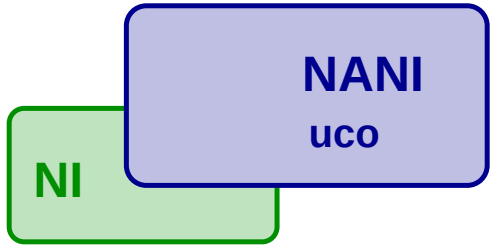
iff

$$Clo^S \circ f \circ Clo^R = Clo^S \circ f$$

Conclusion

NI

Conclusion



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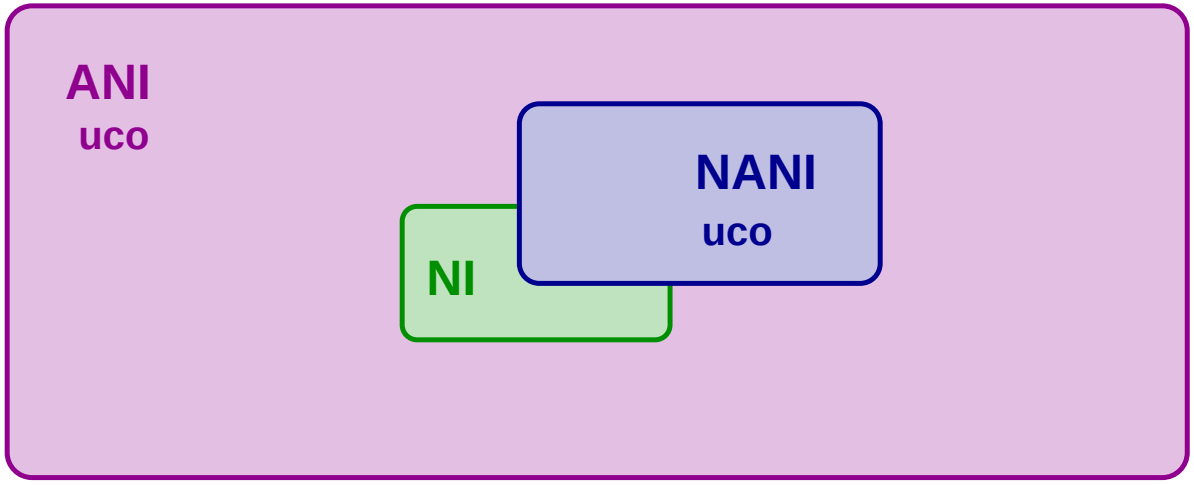

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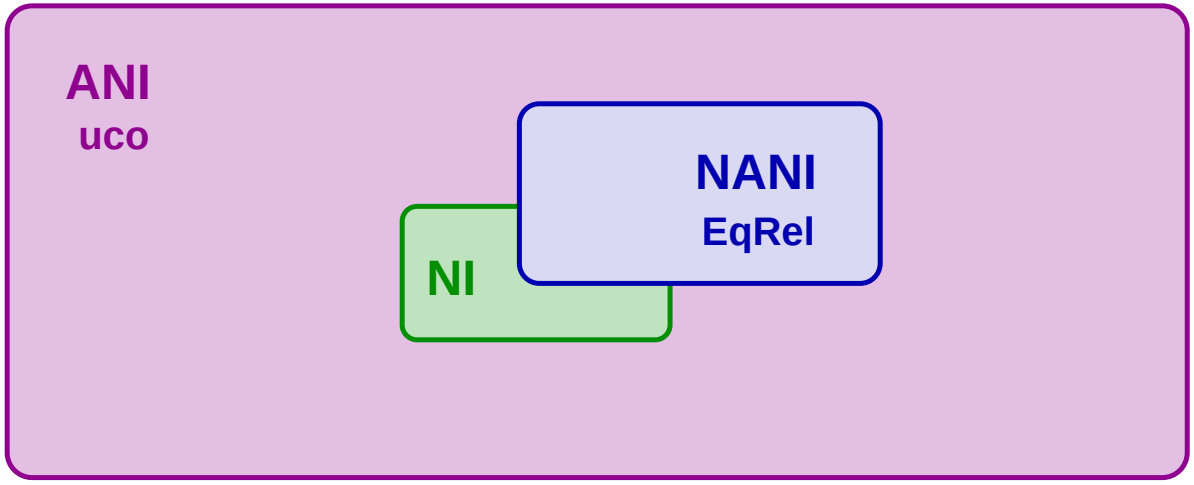

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