DATA DEPENDENCIES AND PROGRAM SLICING:

FROM SYNTAX TO ABSTRACT SEMANTICS

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PEPM 2008

Slicing vs dependencies

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- Slicing by means of a calculus for independencies [Amtoft & Banerjee ’07];
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- Abstract dependencies [Rival ’05];
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  - Applied to Alarm diagnosis;
- Abstract Slicing [Hong et al. ’05]
  - Only for predicate abstractions;
  - Considers a subset of possible executions

Abstract Interpretation

Consider the complete lattice $< C, \leq, \wedge, \vee, \bot, \top >$, $A_i \in \mathsf{uco}(C)$

Lattice of Abstract Domains $\equiv$ Lattice uco

$\mathsf{A} \equiv \rho(C)$

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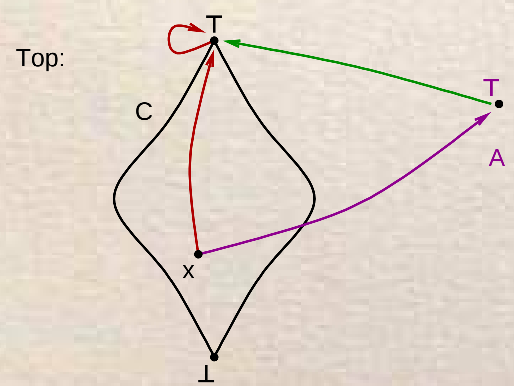

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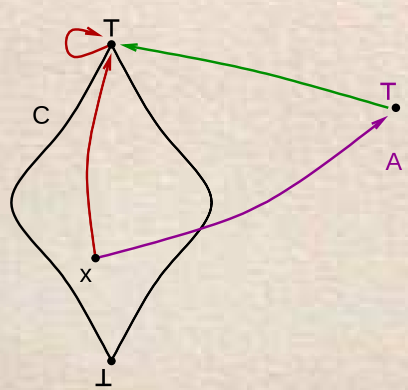

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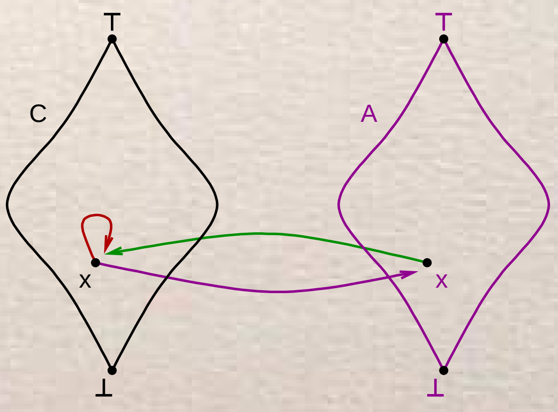

DEPENDENCIES IN PDG

Program Dependency Graphs (PDG) are a standard way for modelling dependencies for slicing. They are defined by two kind of edges (s_1, s_2):

CONTROL FLOW EDGE: s_1 represents a control predicate and s_2 represents a component of the program immediately nested within the predicate s_1;

FLOW DEPENDENCE EDGE: s_1 defines a variable x which is used in s_2
i.e., $x \in \mathbf{def}(s_1) \cap \mathbf{ref}(s_2)$,
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Flow dependence edges = DIRECT FLOWS=DEF-REF dependencies

Control flow edges = INDIRECT FLOWS

SLICING VS DEPENDENCIES

- SLICING $\Rightarrow$ Requires the same I/O behaviour, i.e., no *semantic* dependencies
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There is a clear gap: Semantics vs Syntax

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THERE IS A CLEAR GAP: SEMANTICS VS SYNTAX

PDG $\Rightarrow$ Generate a slicing considering more dependencies (syntactic)

(SEMANTIC) SLICING $\Leftarrow$ Needs a *weaker* notion of dependence.

SLICING PARAMENTRIC ON THE CHOSEN DEPENDENCE NOTION!

A LOGIC FOR (IN)DEPENDENCIES

Formalization of notion of (in)dependence $[x \ltimes y]$[Amtoft & Banerjee '04]:

$G \vdash \{T_0^\#\}\ x := e\ \{T^\#\}$

if $\forall [y \ltimes w] \in T^\#.\ (x \neq y \;\Rightarrow\; [y \ltimes w] \in T_0^\#)$

$(x = y \;\Rightarrow\; (w \notin G \wedge \forall z \in FV(e).\ [z \ltimes w] \in T_0^\#)$

$$\frac{G_0 \vdash \{T_0^\#\} s_1 \{T^\#\} \quad G_0 \vdash \{T_0^\#\} s_2 \{T^\#\}}{G \vdash \{T_0^\#\} \textbf{if } e \textbf{ then } s_1 \textbf{ else } s_2 \{T^\#\}}$$

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SEMANTIC (ABSTRACT) DEPENDENCIES

$$\boxed{z := w + y + 2x^2 - w}$$

SYNTACTIC DEP. A variable is a free variable in the expression assigned to z?

$\Longrightarrow$ z depends on $\{w, y, x\}$!

SEMANTIC (ABSTRACT) DEPENDENCIES

$$\boxed{z := w + y + 2x^2 - w}$$

SEMANTIC DEP. By varying the *value* of a variable does the expression change?

$$\boxed{x \leadsto_S e \;\Leftrightarrow\; \exists \sigma_1, \sigma_2.\, \forall y \neq x. \sigma_1(y) = \sigma_2(y) \;\wedge\; [\![e]\!](\sigma_1) = [\![e]\!](\sigma_2)}$$

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SEMANTIC (ABSTRACT) DEPENDENCIES

$$\boxed{z := w + y + 2x^2 - w}$$

ABSTRACT SEMANTIC DEP. By varying the *property* of a variable does the *property* of the expression change?

$$\boxed{x \rightsquigarrow_{\mathsf{N}} e \;\Leftrightarrow\; \exists \sigma_1, \sigma_2. \forall y \neq x. \rho(\sigma_1(y)) = \rho(\sigma_2(y)) \;\wedge\; \rho([\![e]\!](\sigma_1)) = \rho([\![e]\!](\sigma_2))}$$

$\Longrightarrow$ If we consider *Parity* z depends on $\{y\}$!

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$\Longrightarrow$ If we consider *Sign* z depends on $\{y, x\}$!

PRUNING DEPENDENCIES

We have two kind of dependencies:

- Data dependencies (Assignments);
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$$\boxed{\text{We propose a PRUNING of } \textit{data dependencies}!}$$

$\Downarrow$

STILL WE LOSE SOMETHING ABOUT CONTROL DEPENDENCIES!

Pruning dependencies

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- Data dependencies (Assignments) $\Rightarrow$ Direct flows;
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$$\textbf{if } (y + 2x \; mod \; 2) == 0 \textbf{ then } w := 0 \textbf{ else } w := 0$$

$\Rightarrow$ The guard DOES NOT DEPEND on x: OK

$\Rightarrow$ The variable w DOES NOT DEPEND on y: NO!

DERIVING ABSTRACT DEPENDENCIES (1)

The definition of narrow deps contains quantifiers on variables and states

This means that, even abstracting states,
the number of comparisons between $\rho\left(\llbracket e \rrbracket \sigma_1\right)$ and $\rho\left(\llbracket e \rrbracket \sigma_2\right)$
may be huge or infinite if the domain is non-trivial

Yet, we observe that:

- Some states are not *possible* at a given program point
- What is computed in a *broader* state can be valid in *narrower* states (monotonicity)

$$\rho(\llbracket e \rrbracket \langle [\top] \rangle) \leq \mathsf{U} \;\Leftrightarrow\; \rho(\llbracket e \rrbracket \langle [\mathbf{even}] \rangle) \leq \mathsf{U} \wedge \rho(\llbracket e \rrbracket \langle [\mathbf{odd}] \rangle) \leq \mathsf{U}$$

DERIVING ABSTRACT DEPENDENCIES (2)

- A systematic way to go through the (variables and states)-space:
  - incrementally find the set X of variables which are enough to determine the value of e
  - X determines e if any change to other variables can be ignored (needs to go into the state space)

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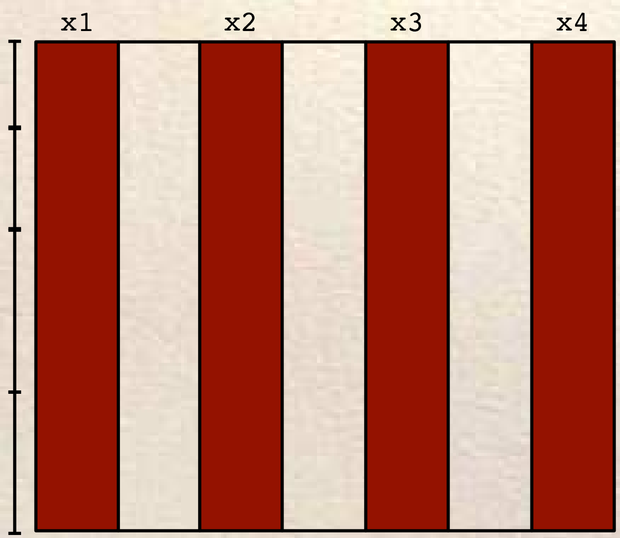

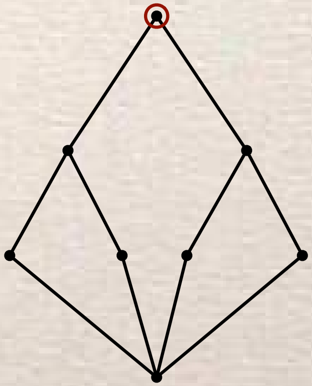

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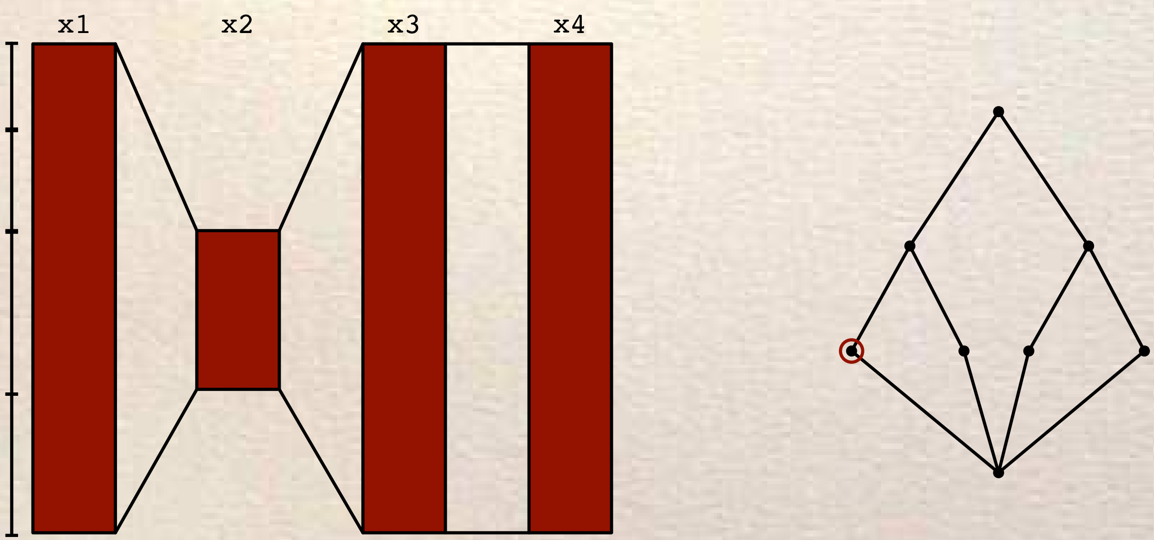

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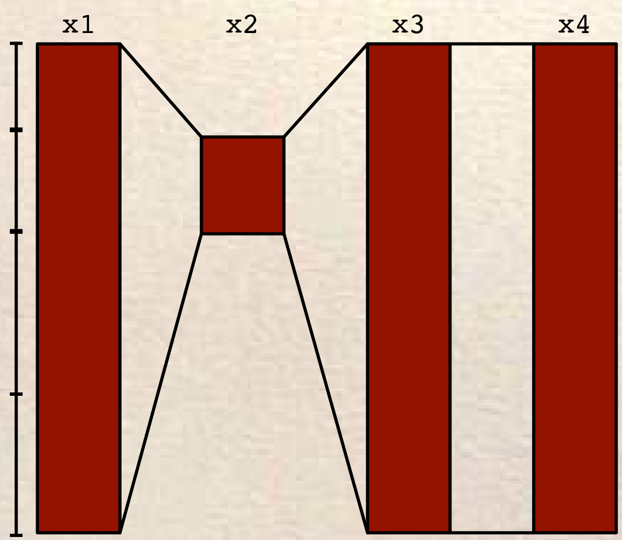

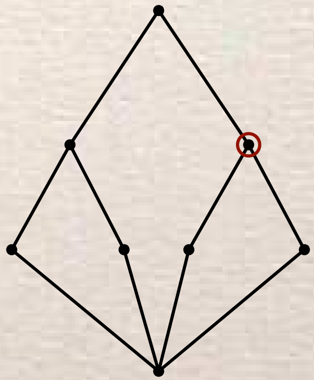

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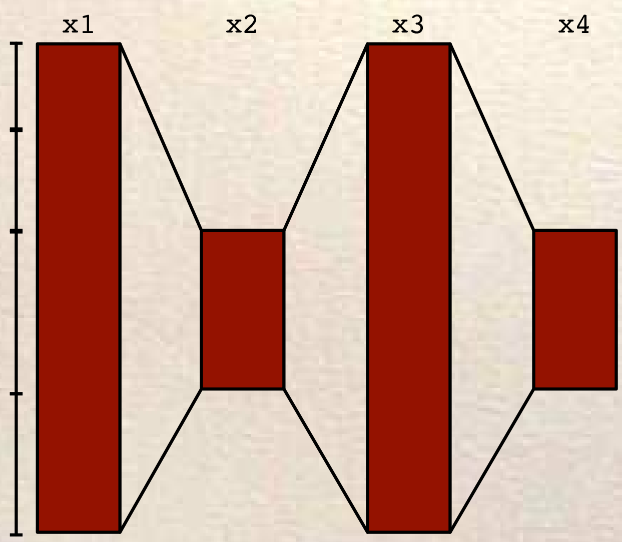

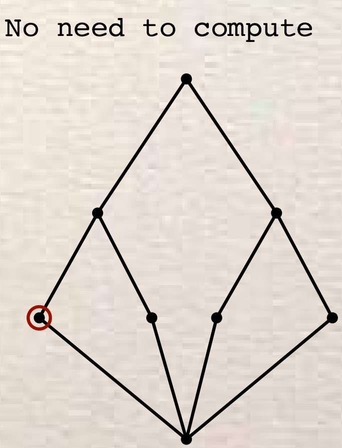

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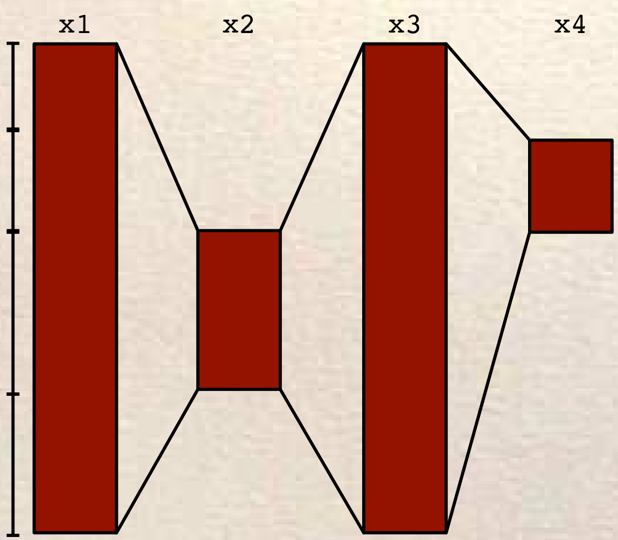
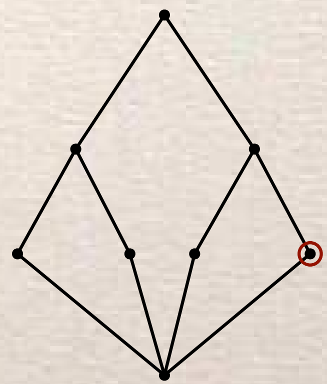
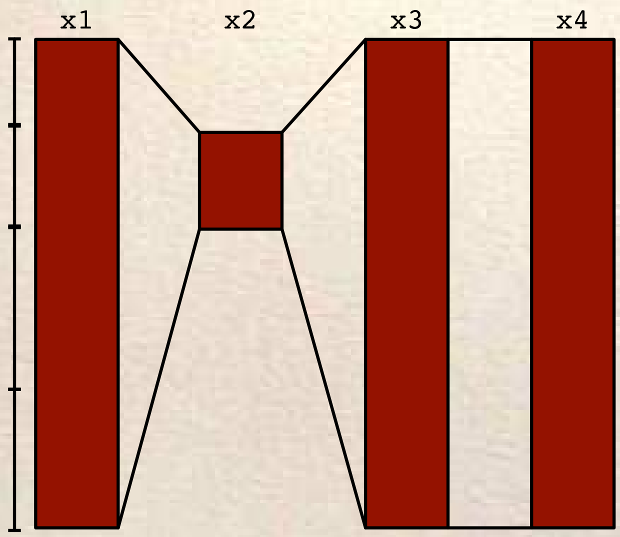


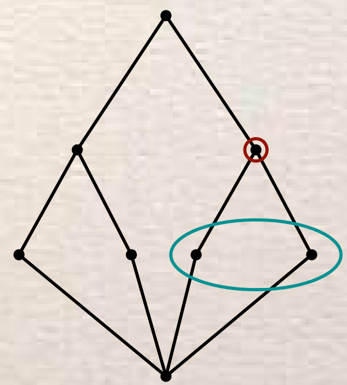


REMOVING ABSTRACT DEPENDENCIES

Another application: simplifying a domain in order to remove dependencies on a set of variables

- Basically, systematically removing from ρ the abstract values which are responsible for the distinguishability of two states





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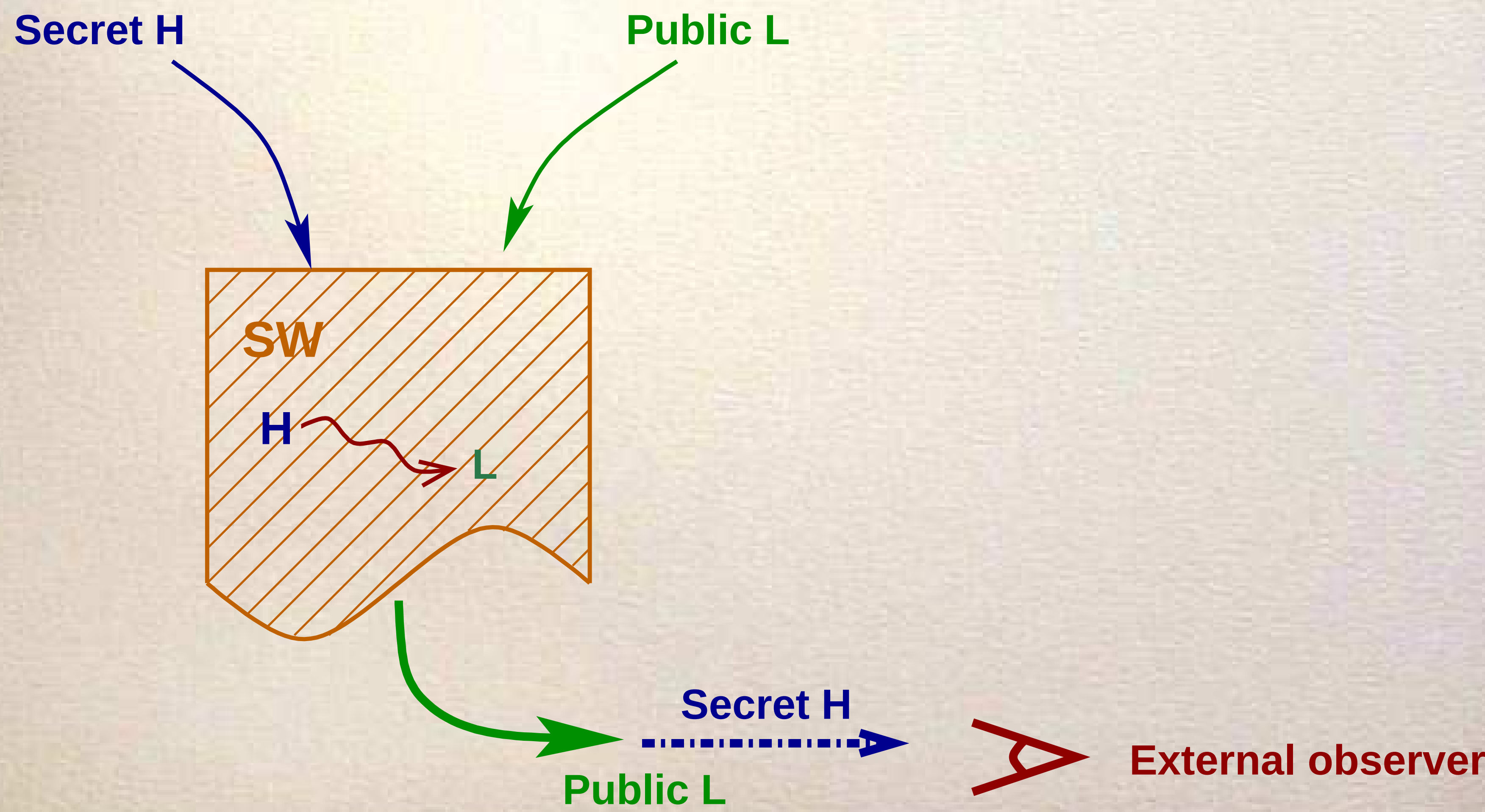
[Giacobazzi & Mastroeni ’04]



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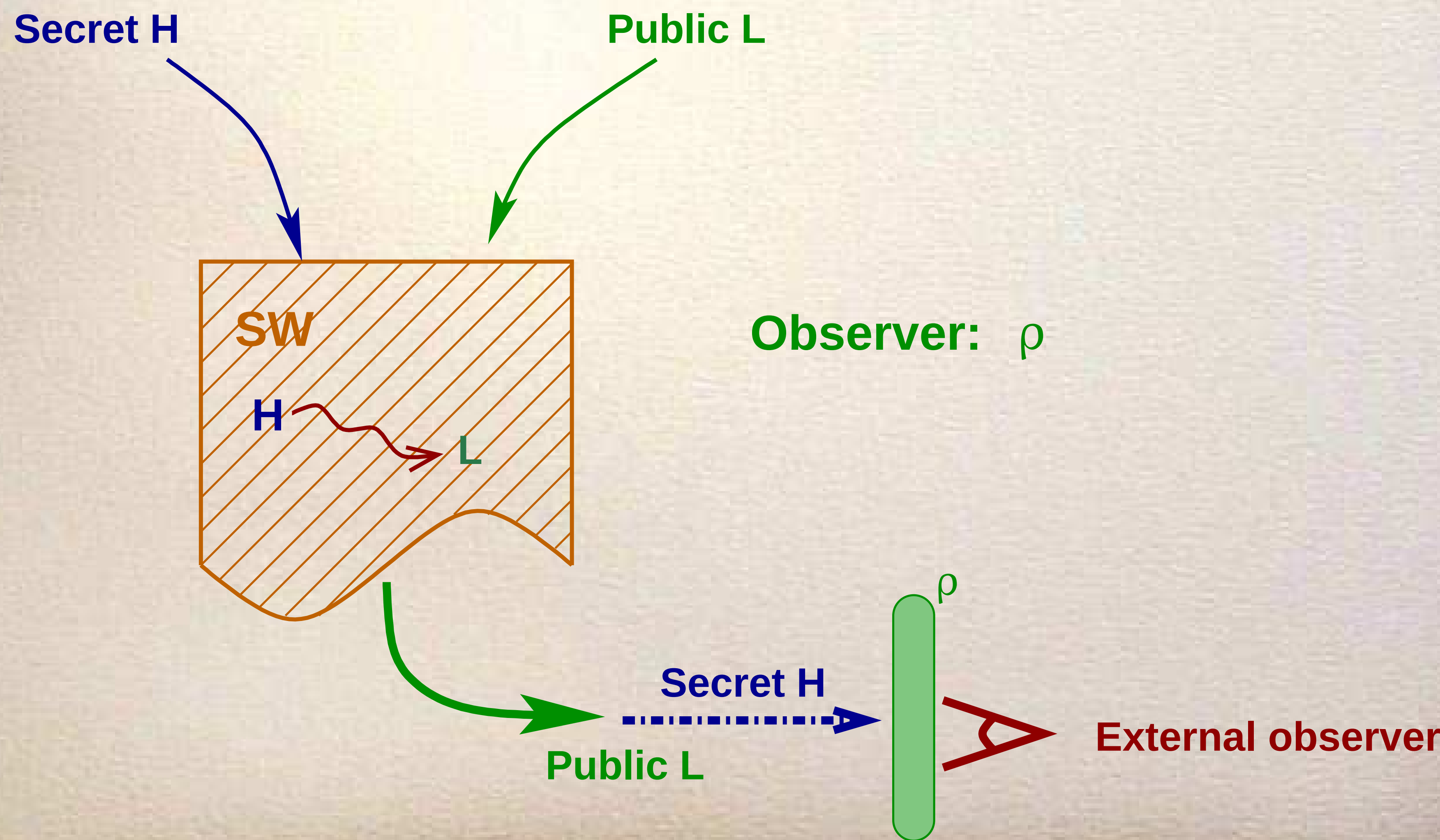

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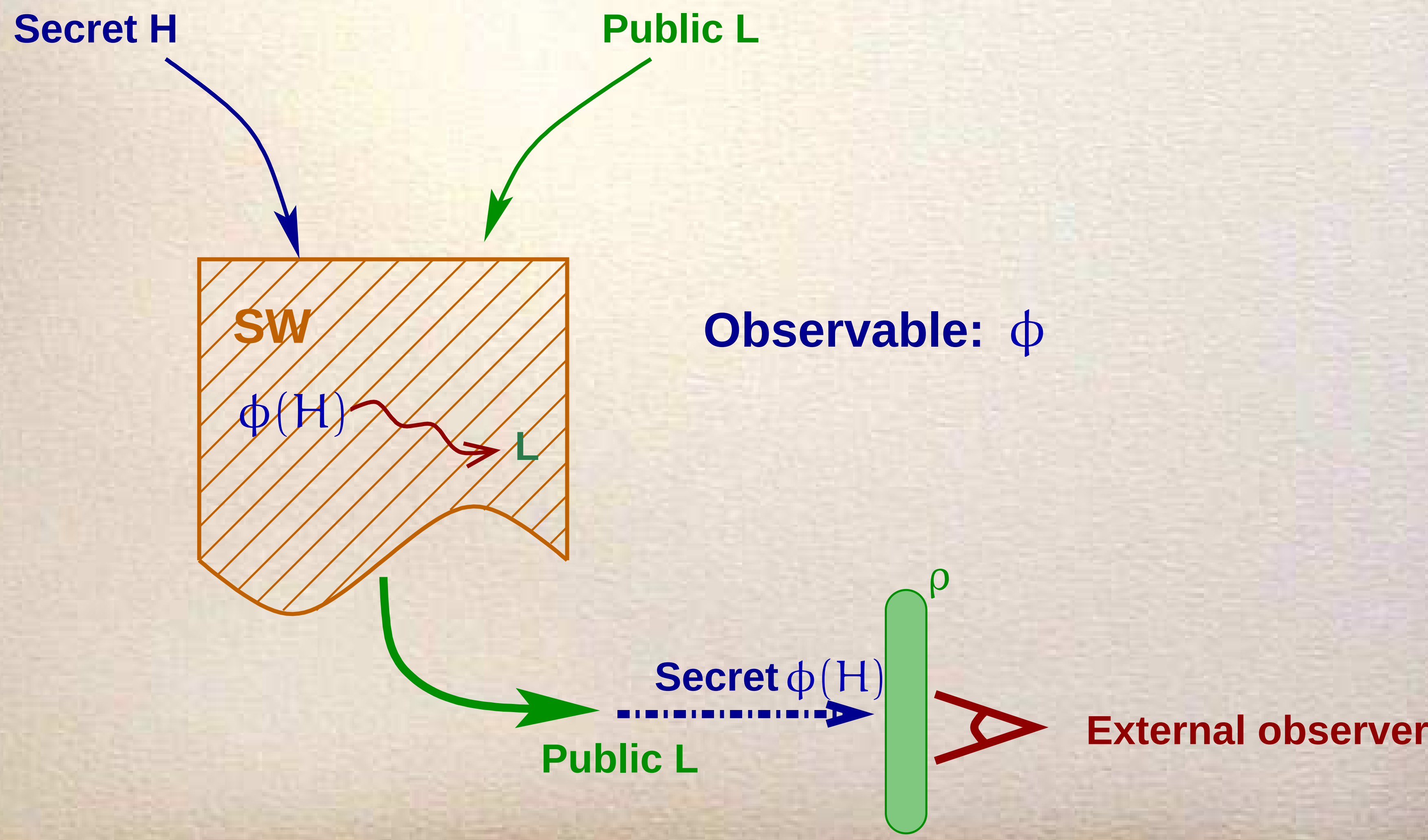

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- We certify the security degree of programs relatively to an output observation [Giacobazzi & Mastroeni].
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We can avoid the semantic rule by computing ABSTRACT DEPENDENCIES for assignment!

CONCLUSIONS

- We provide an insight on the strong relation between slicing and dependency;
- A new point of view: Slicing parametric on a notion of dependency;
- Still we are not able to get the most precise semantic slicing;
- Still there is a lot of work to do towards a real implementation.