

WHAT YOU LOSE IS WHAT YOU LEAK

INFORMATION LEAKAGE IN DECLASSIFICATION POLICIES

A. Banerjee, R. Giacobazzi and I. Mastroeni

Kansas State University
Manhattan (KS), USA

Università di Verona
Verona, Italy

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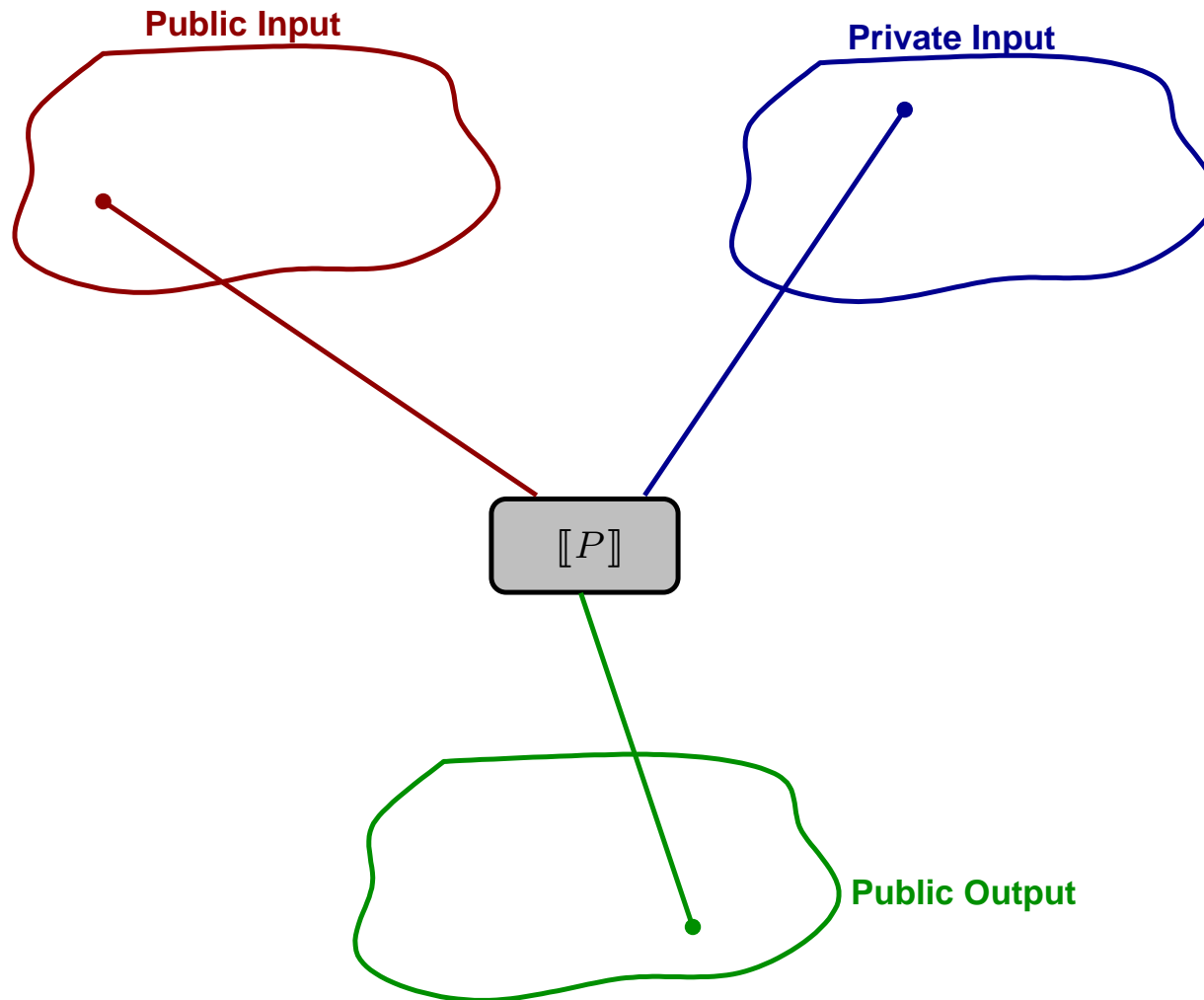
Overview

- ⑥ By exploiting the strong relation between completeness and non-interference we can obtain the following results:
 - ▣ Model declassification as a forward completeness problem for the weakest precondition semantics;
 - ▣ Derive counterexamples to a given declassification policy;
 - ▣ Refine a given declassification policy;

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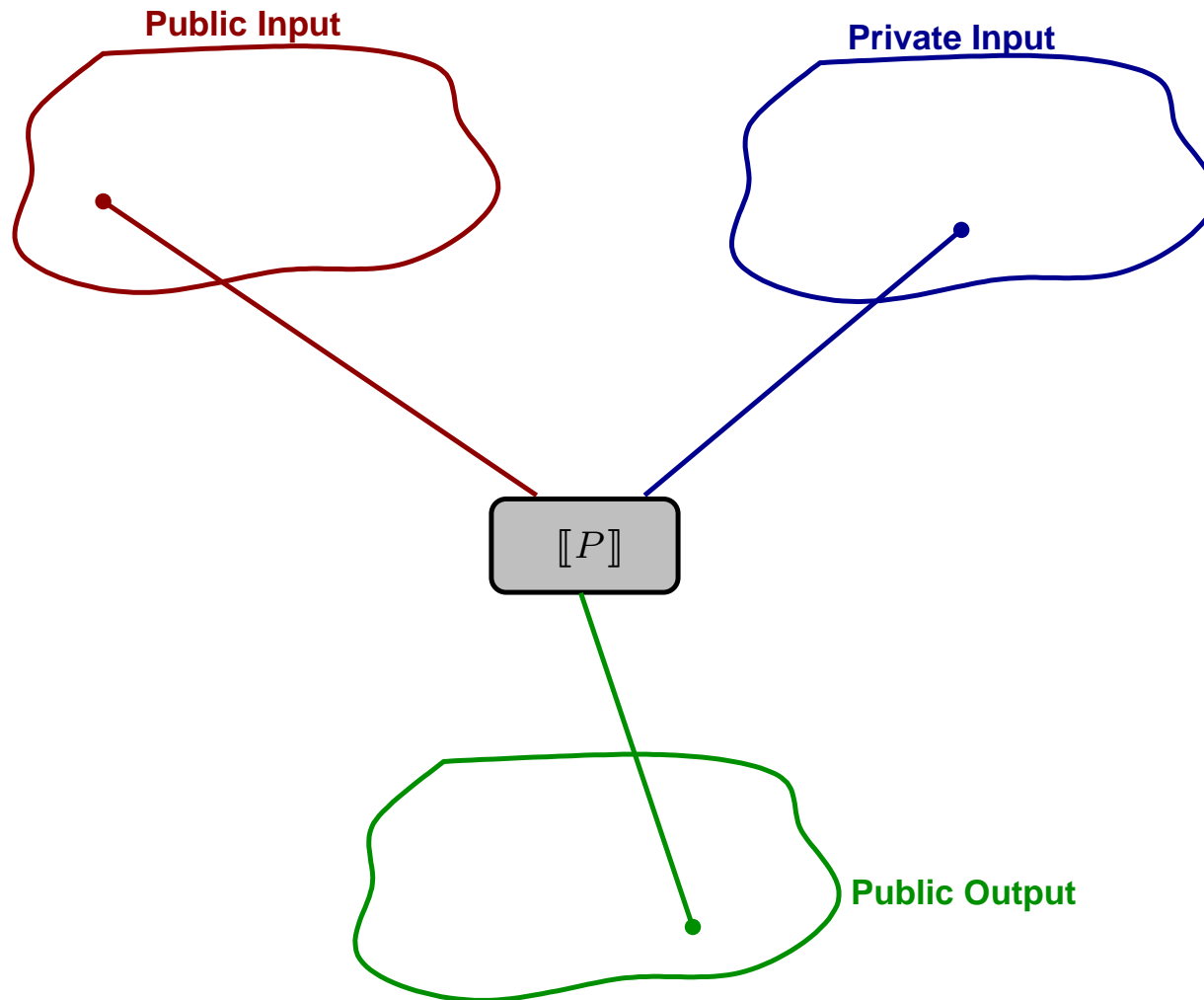
- ⑥ By exploiting the strong relation between completeness and non-interference we can obtain the following results:
 - ▣ Model declassification as a forward completeness problem for the weakest precondition semantics;
 - ▣ Derive counterexamples to a given declassification policy;
 - ▣ Refine a given declassification policy;
- ⑥ We can model declassification as a model checking problem (see the relation with robust declassification)

Standard Non-Interference



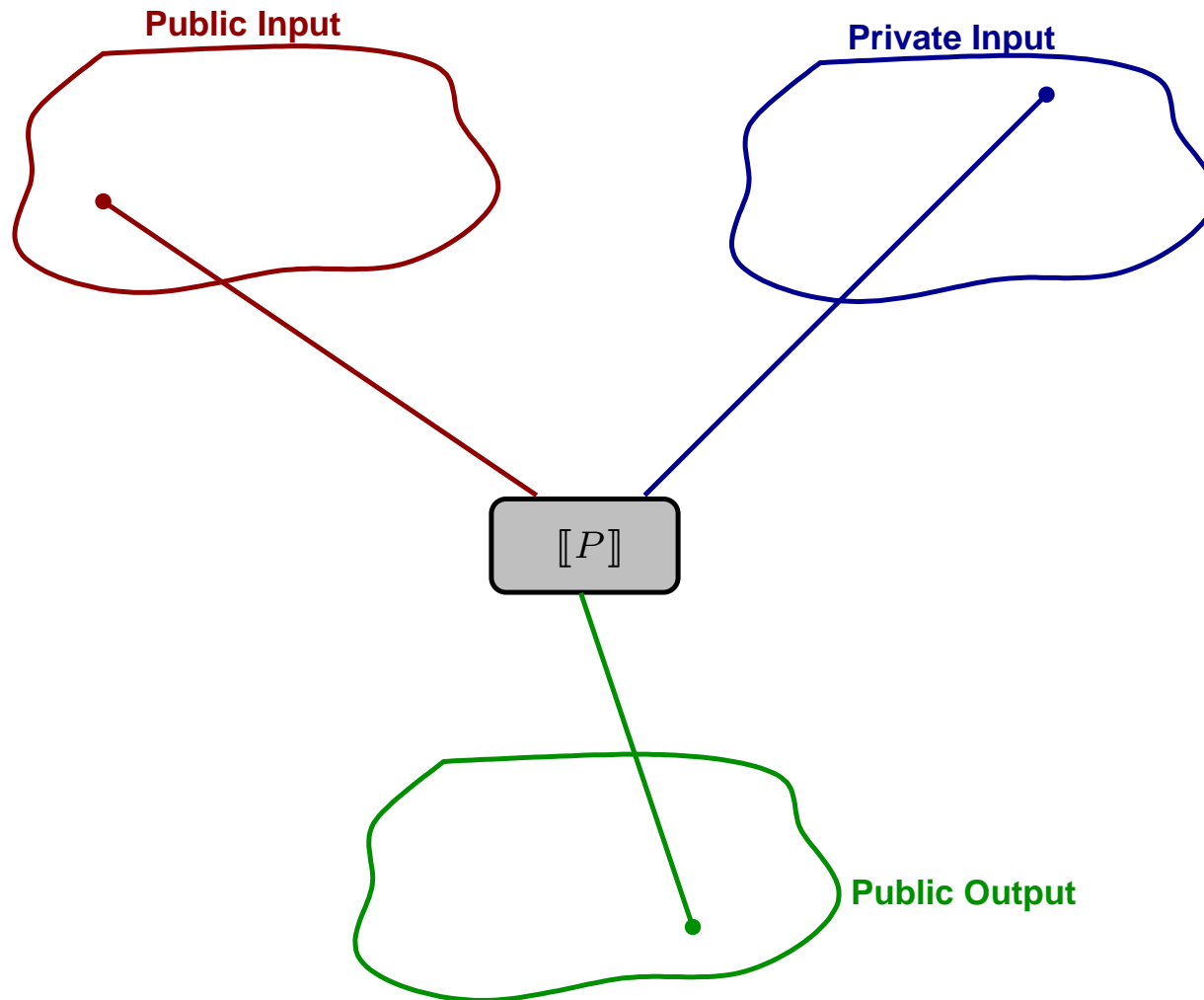
$$\forall l : \mathbb{L}, \forall h_1, h_2 : \mathbb{H}. \llbracket P \rrbracket(h_1, l)^{\mathbb{L}} = \llbracket P \rrbracket(h_2, l)^{\mathbb{L}}$$

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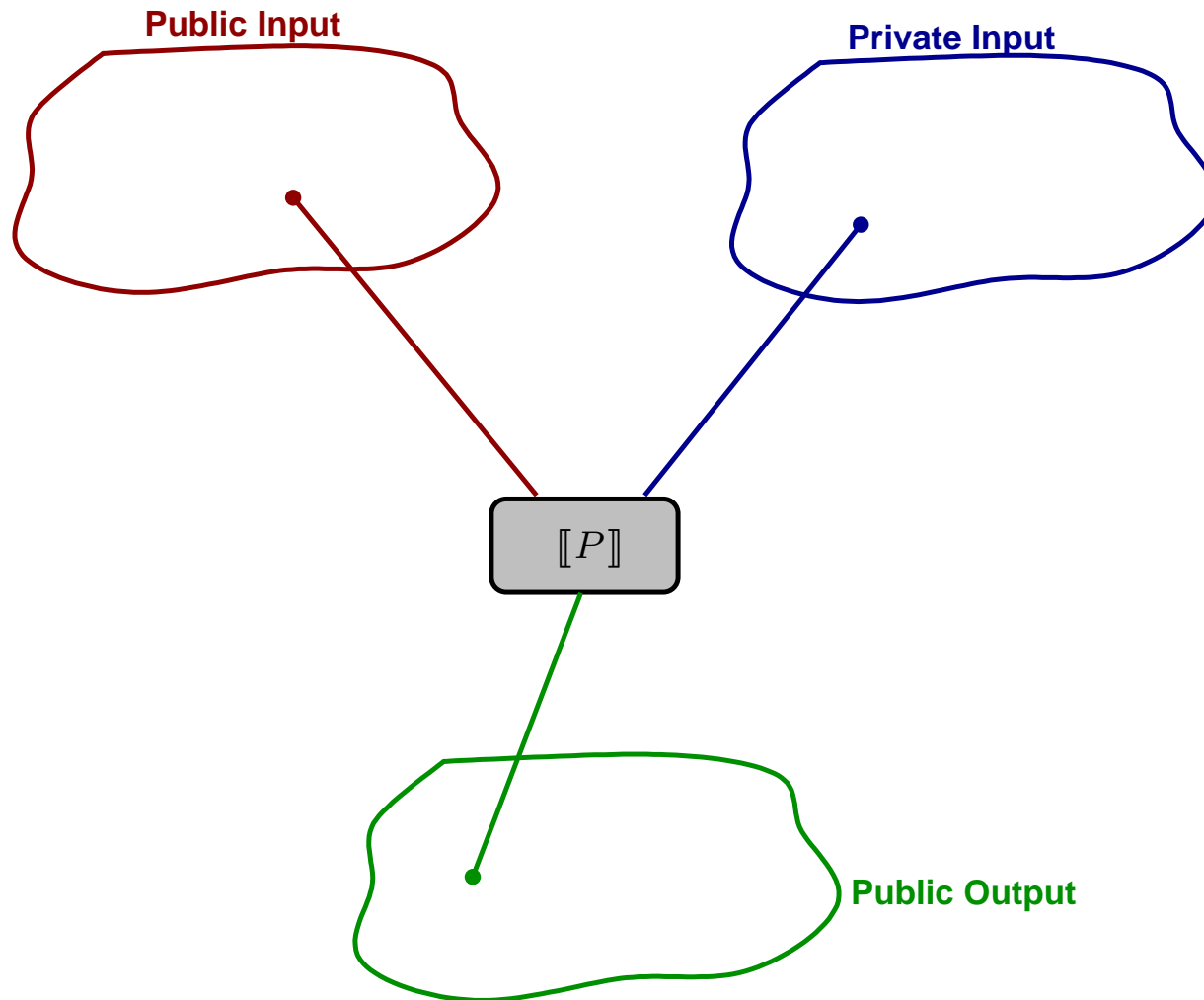
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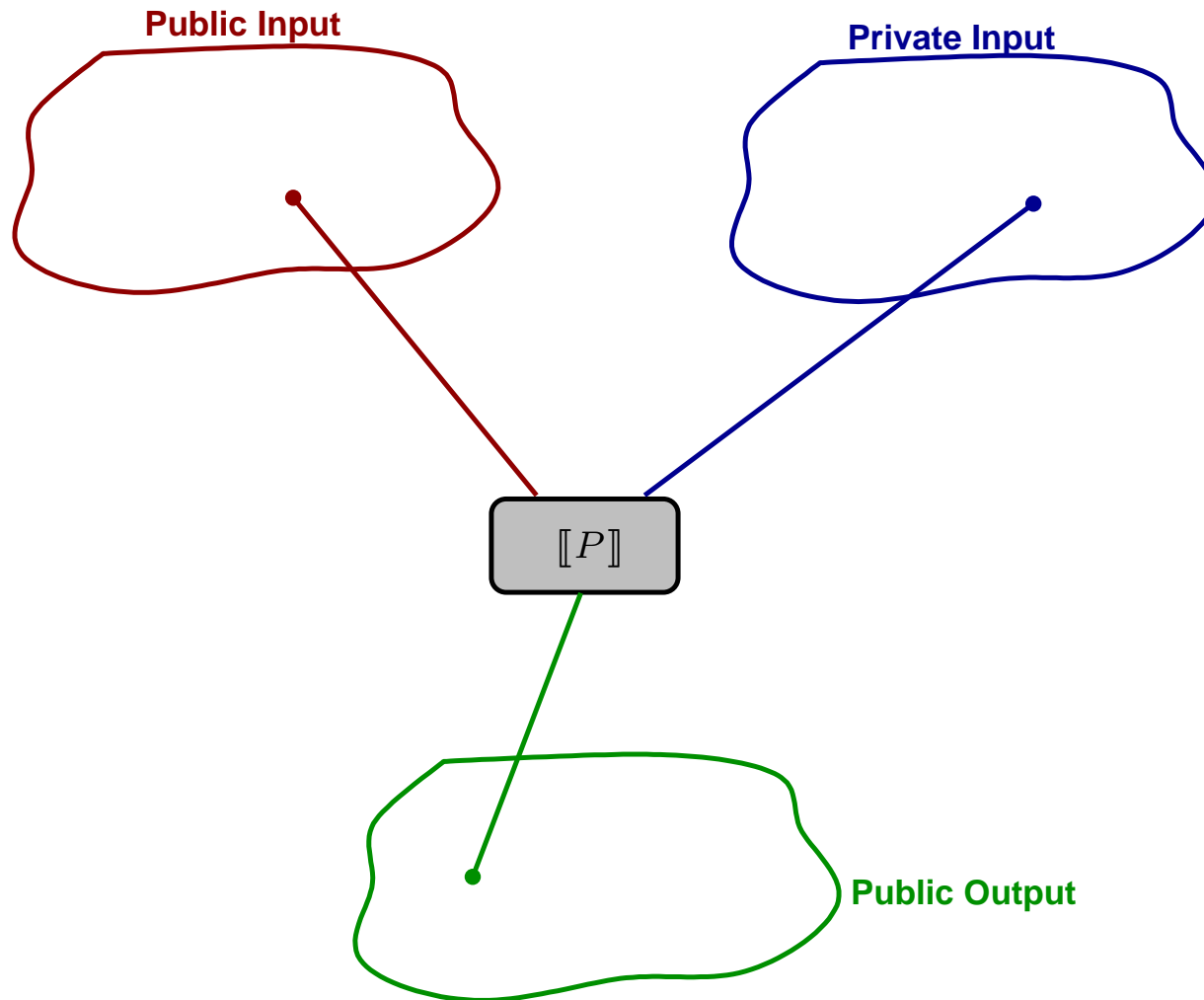
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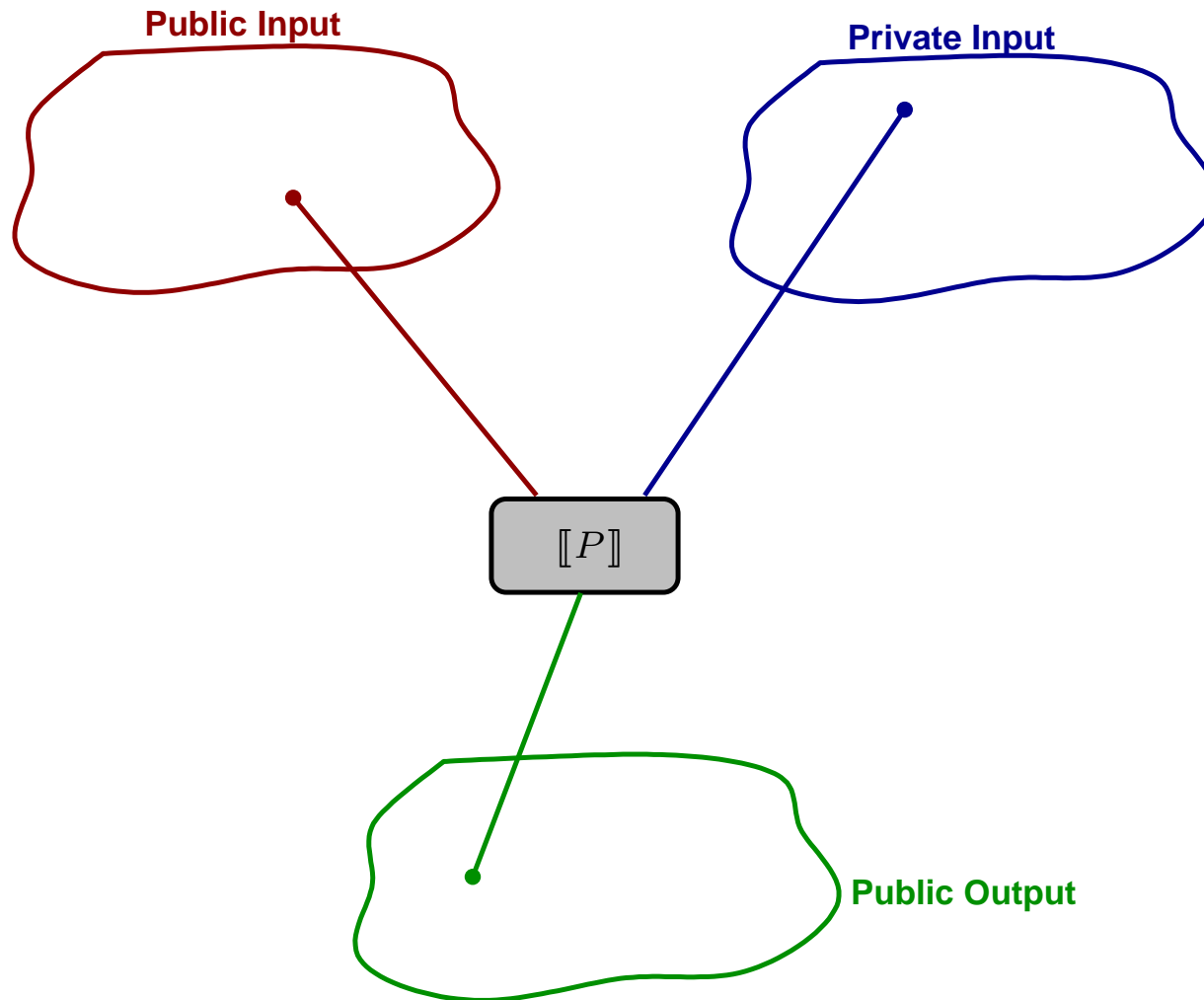
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$$\forall l : \mathbb{L}, \forall h_1, h_2 : \mathbb{H}. [[P]](h_1, l)^{\perp} = [[P]](h_2, l)^{\perp}$$

NI: A completeness problem

Recall that [Joshi & Leino'00]

P is *secure* iff $\text{HH} ; P ; \text{HH} \doteq P ; \text{HH}$

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Let $X = \langle X^H, X^L \rangle \Rightarrow \mathcal{H}(X) \stackrel{\text{def}}{=} \langle \top^H, X^L \rangle \in \text{uco}(\wp(\mathbb{V}))$

$$\mathcal{H}\mathcal{H}; P; \mathcal{H}\mathcal{H} \doteq P; \mathcal{H}\mathcal{H}$$

\Downarrow

$$\mathcal{H} \circ \llbracket P \rrbracket \circ \mathcal{H} = \mathcal{H} \circ \llbracket P \rrbracket$$

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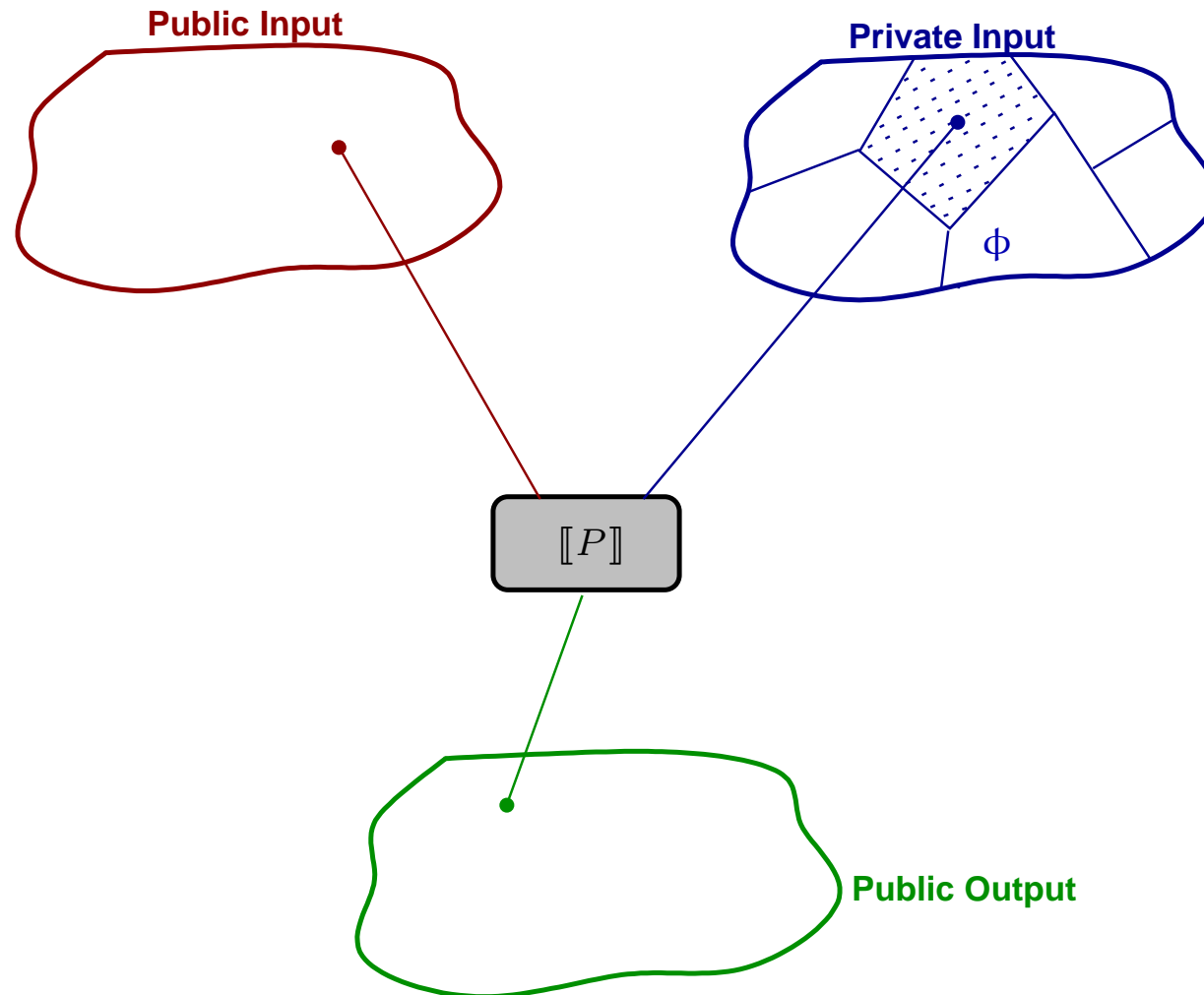
\Downarrow

$\mathcal{H} \circ \llbracket P \rrbracket \circ \mathcal{H} = \mathcal{H} \circ \llbracket P \rrbracket$

\Rightarrow A COMPLETENESS PROBLEM

[Giacobazzi & Mastroeni '05]

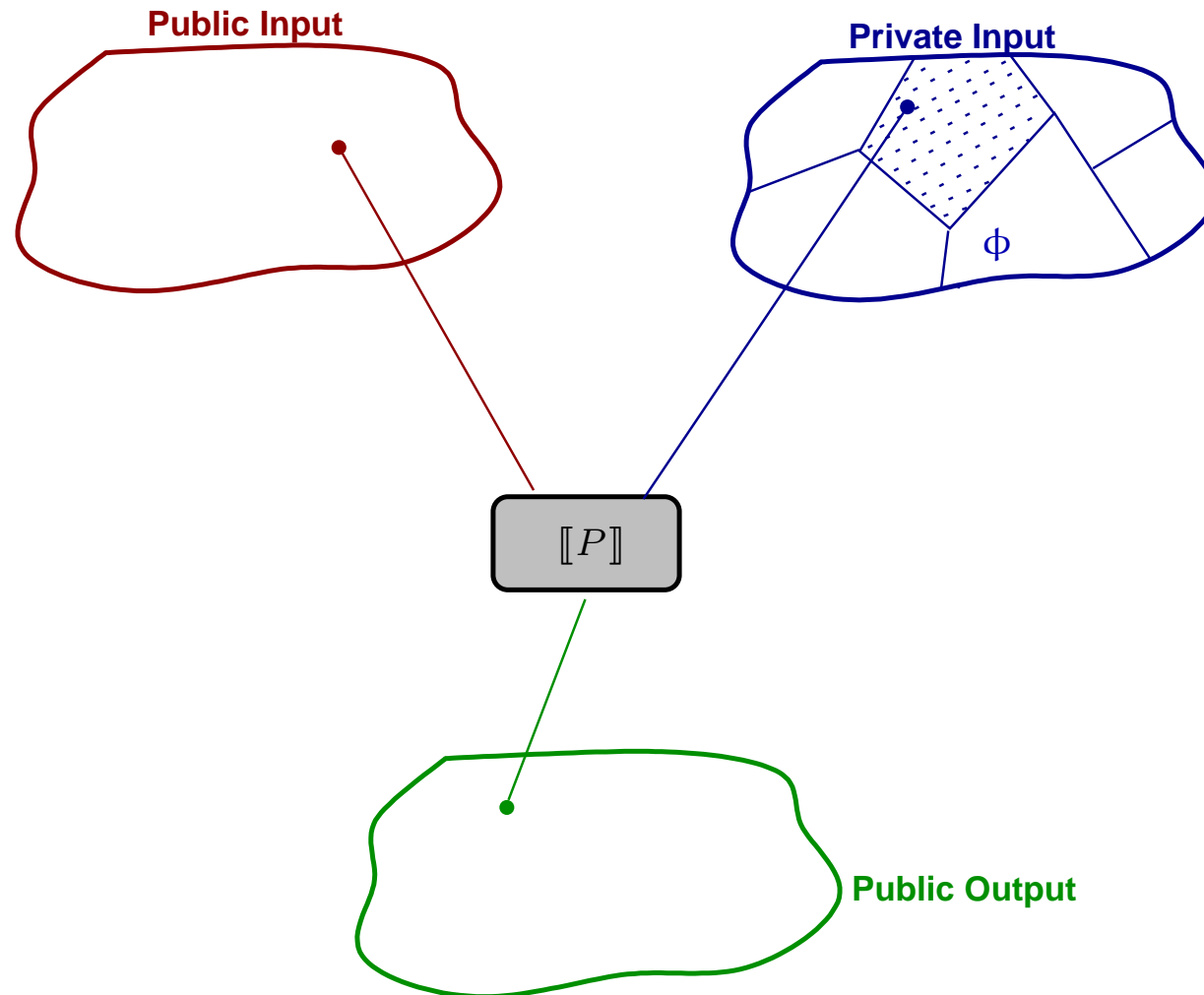
Declassified NI



[Mastroeni '05]

$$\phi \in \mathit{Abs}(\wp(\mathbb{V}^H)): \phi(h_1) = \phi(h_2) \Rightarrow [[P]](h_1, l)^\perp = [[P]](h_2, l)^\perp$$

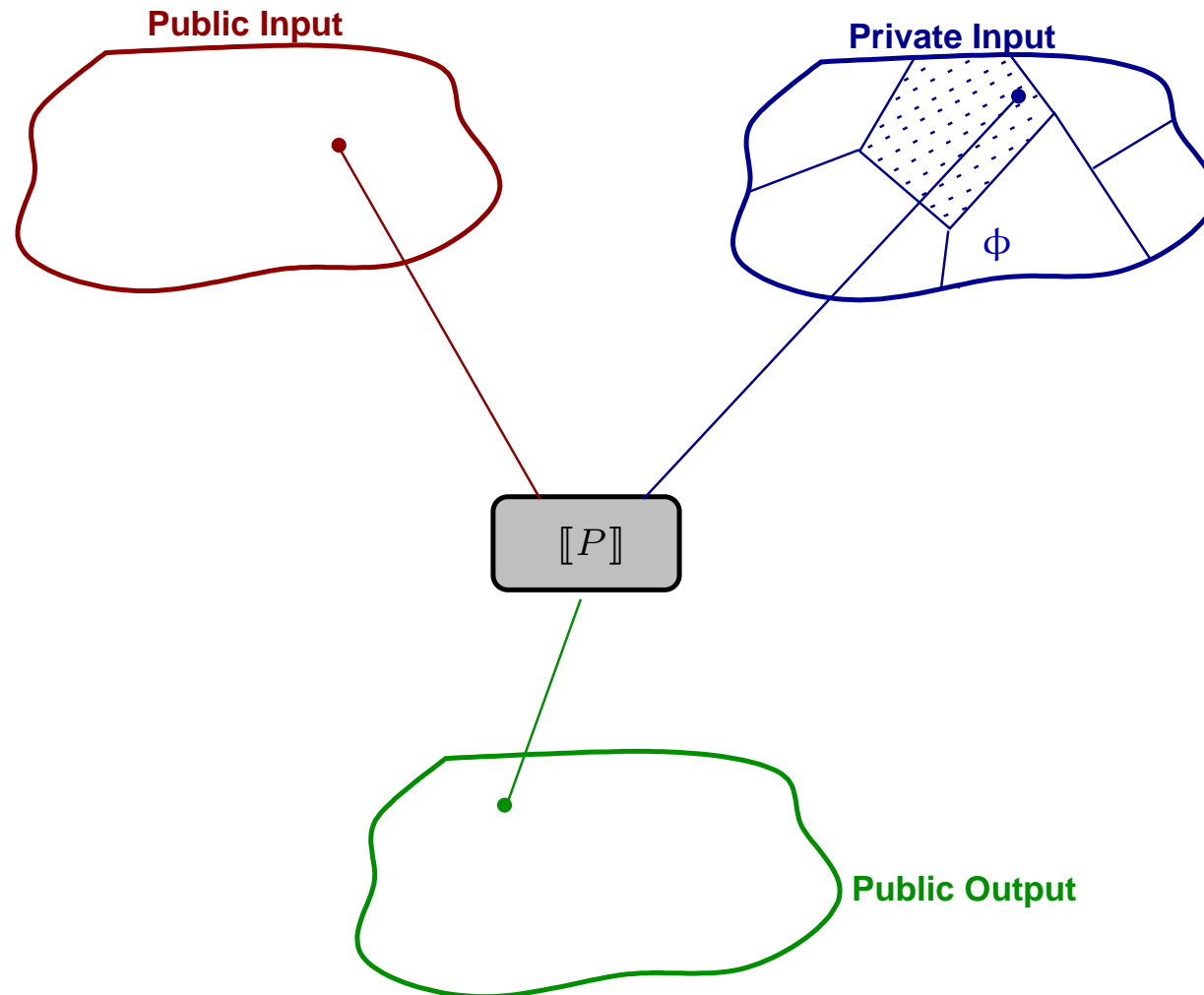
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Modelling declassification: A running example

Let $\phi = \text{Parity} \stackrel{\text{def}}{=} \{\top, \text{Even}, \text{Odd}, \emptyset\}$,

$$P = \left[\begin{array}{l} h := |h|; \\ \mathbf{while} (h > 0) \mathbf{do} (h := h - 1; l := h) \mathbf{endw} \end{array} \right.$$

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```
    {h ∈ ℤ}
h := |h|;
    {(h = 0 ∧ l = 0) ∨ h > 0}
while (h > 0) do (h := h - 1; l := h) endw
    {l = 0}
```

$\mathbb{Z} \in \phi \Rightarrow \phi$ is ok!

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```
    {h = 0}
h := |h|;
    {h = 0 ∧ l = a}
while (h > 0) do (h := h - 1; l := h) endw
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$\{0\} \notin \phi \Rightarrow \phi$ is not ok!

Modelling declassification: A running example

Let $\phi = \text{Parity} \stackrel{\text{def}}{=} \{\top, \text{Even}, \text{Odd}, \emptyset\}$,

$H_a = \{ \langle h, l \rangle \mid h \in \mathbb{Z}, l = a \}$ (a value observed in output).

$$P = \left[\begin{array}{l} h := |h|; \\ \mathbf{while} (h > 0) \mathbf{do} (h := h - 1; l := h) \mathbf{endw} \end{array} \right.$$
$$\text{Wlp} : \left\{ \begin{array}{ll} H_0 & \mapsto \{ \langle h, l \rangle \mid h \neq 0, l \in \mathbb{Z} \} \cup \{ \langle 0, 0 \rangle \} \\ H_a & \mapsto \{ \langle 0, a \rangle \} \quad (a \neq 0) \end{array} \right.$$

P secure with ϕ declassified $\Leftrightarrow \mathcal{H}^\phi \circ \text{Wlp}_P \circ \mathcal{H} = \text{Wlp}_P \circ \mathcal{H}$

DNI: A completeness problem (1)

Let \mathcal{H}^ϕ the abstract domain declassifying the property ϕ of the *private input*:

$$\mathcal{H} \circ \llbracket P \rrbracket \circ \mathcal{H}^\phi = \mathcal{H} \circ \llbracket P \rrbracket \Leftrightarrow \mathcal{H}^\phi \circ \text{Wlp}_P \circ \mathcal{H} = \text{Wlp}_P \circ \mathcal{H}$$

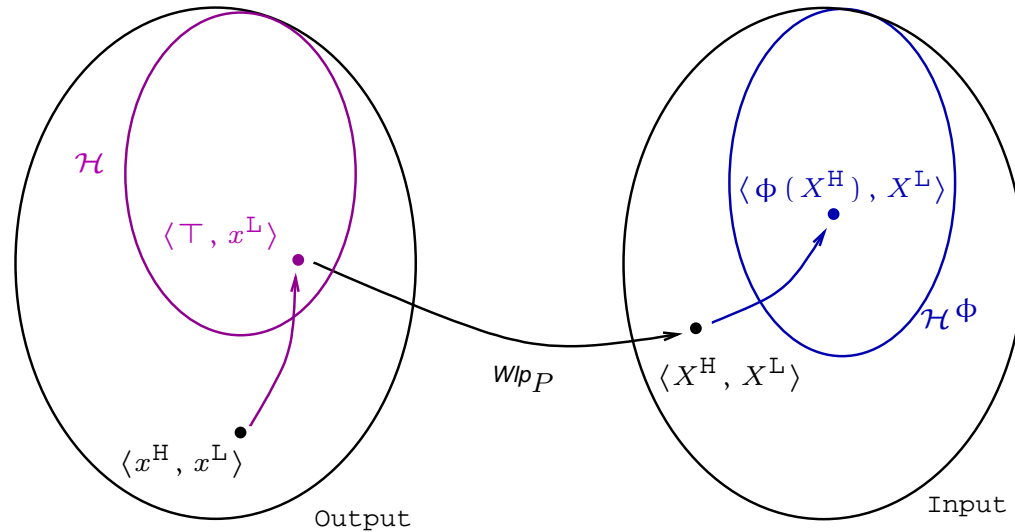


To release ϕ *means* to distinguish between elements in ϕ !

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Deriving counterexamples: A running example

Consider again $\phi = \text{Parity} \stackrel{\text{def}}{=} \{\top, \text{Even}, \text{Odd}, \emptyset\}$,

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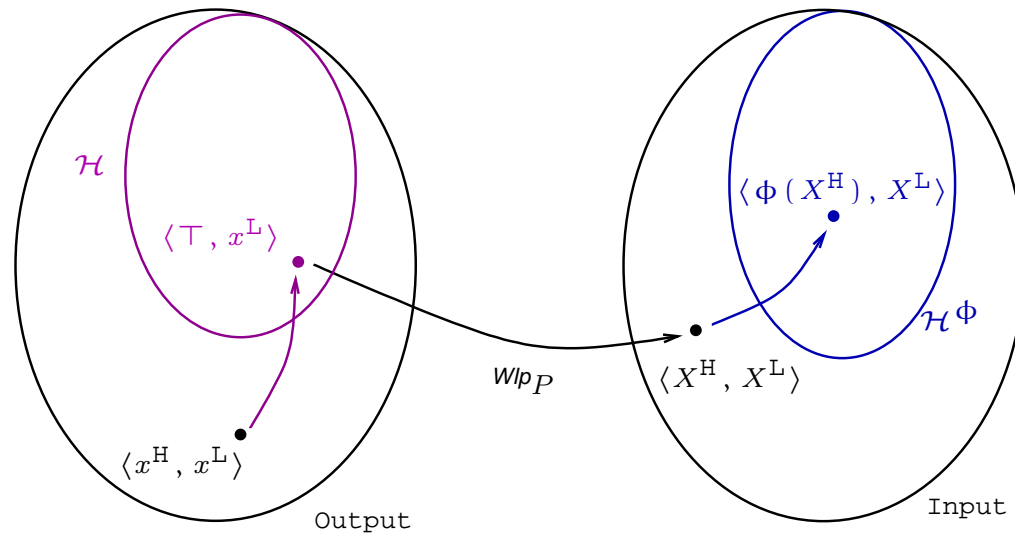
$\{h = 0\} \Rightarrow \text{Even split in } \{0\} \text{ and } \text{Even} \setminus \{0\}$
 $h := |h|;$
 $\{h = 0 \wedge l = a\}$
while $(h > 0)$ **do** $(h := h - 1; l := h)$ **endw**
 $\{l = a \neq 0\}$

Let $l = 5$, $h_1 = 0 \in \text{Even}$ and $h_2 = 2 \in \text{Even}$:
 $\llbracket P \rrbracket(\langle 0, 5 \rangle) = \langle 0, 5 \rangle \neq \langle 0, 0 \rangle = \llbracket P \rrbracket(\langle 2, 5 \rangle)$

DNI: A completeness problem (2)

Let \mathcal{H}^ϕ the abstract domain declassifying the property ϕ of the *private input*:

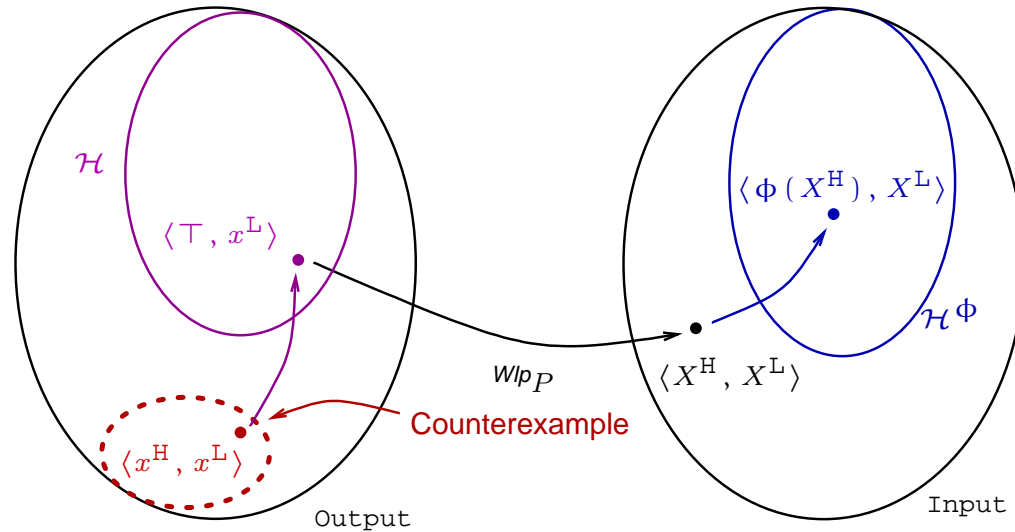
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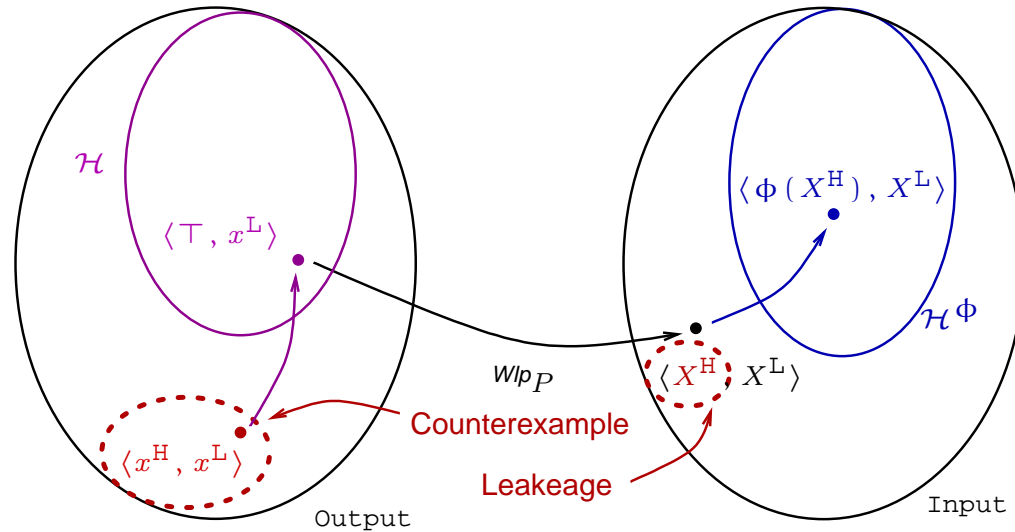
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Refining policies: An example

Consider $\phi = \left\{ \left\{ \langle h_1, h_2, \dots, h_n \rangle \mid (h_1 + h_2 + \dots + h_n)/n = a \right\} \mid a \in \mathbb{Z} \right\}$

$$P = \left[\begin{array}{l} h_1 := h_1; h_2 := h_2; \dots; h_n = h_n \\ avg := \text{declassify}((h_1 + h_2 + \dots + h_n)/n); \end{array} \right.$$

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$$\{h_1 = a\}$$

$$h_1 := h_1; h_2 := h_2; \dots; h_n = h_n$$

$$\{(h_1 + h_2 + \dots + h_n)/n = a\}$$

$$avg := \text{declassify}((h_1 + h_2 + \dots + h_n)/n);$$

$$\{avg = a\}$$

$\left\{ \langle h_1, h_2, \dots, h_n \rangle \mid (h_1 + h_2 + \dots + h_n)/n = a, h_1 = a \right\} \notin \phi \Rightarrow \phi \text{ is not ok!}$

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$H_a = \left\{ \langle h_1, \dots, h_n, avg \rangle \mid avg_{h_i} = a \right\}$ (a value observed in output).

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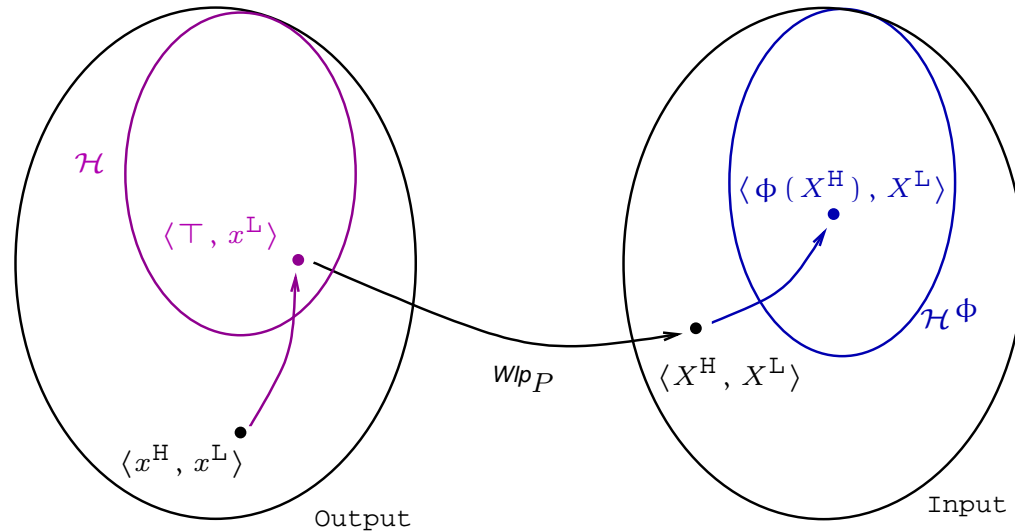
$$Wlp: H_a \mapsto \left\{ \langle a, h_2, \dots, h_n, a \rangle \mid avg = a \right\}$$

P secure with ϕ' declassified $\Leftrightarrow \phi' = \phi \sqcap \left\{ Wlp(H_a) \mid a \in \mathbb{Z} \right\}$

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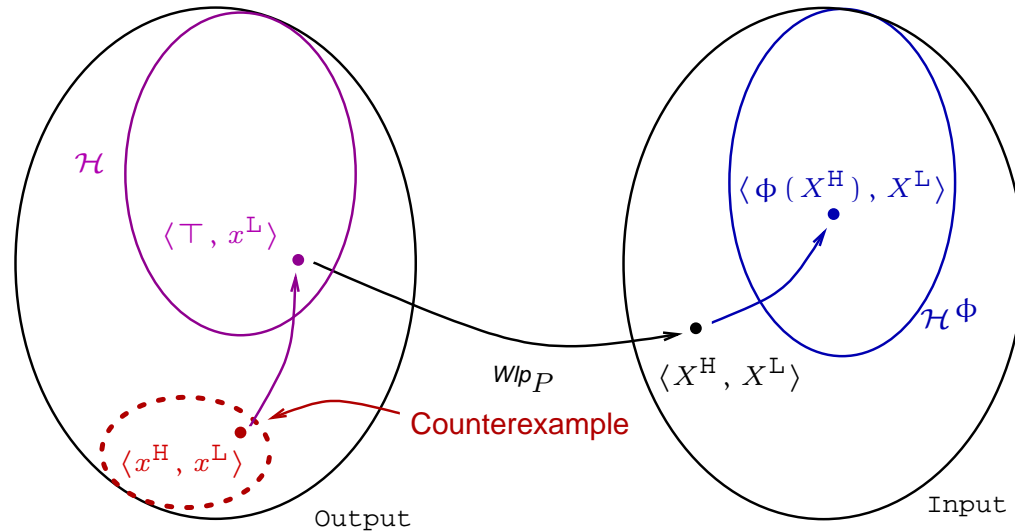
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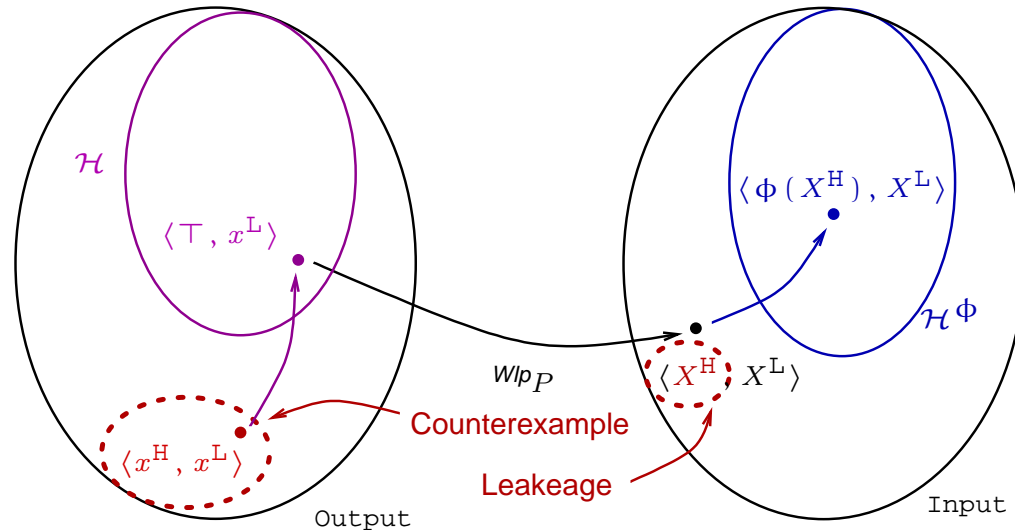
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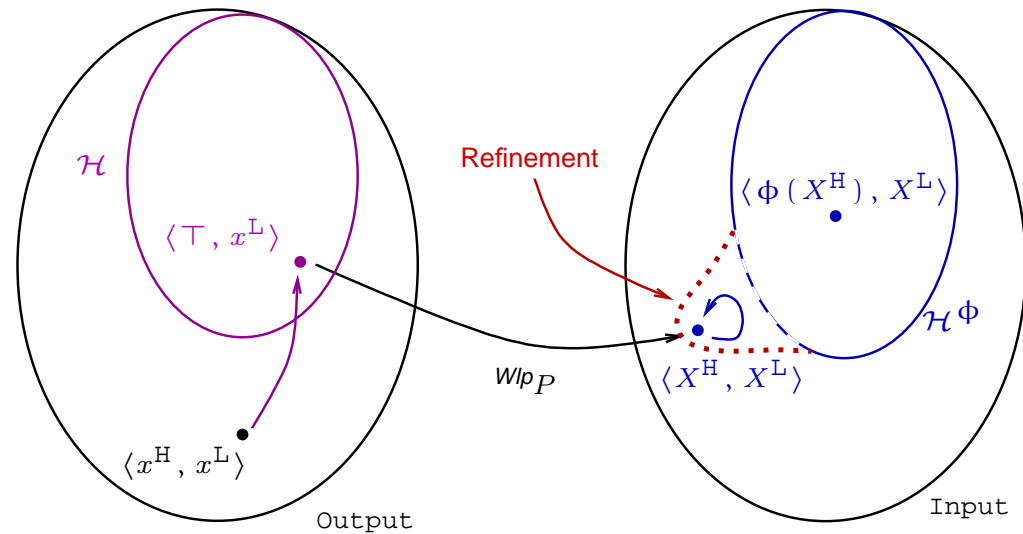
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Example: Oblivious Transfer Protocol

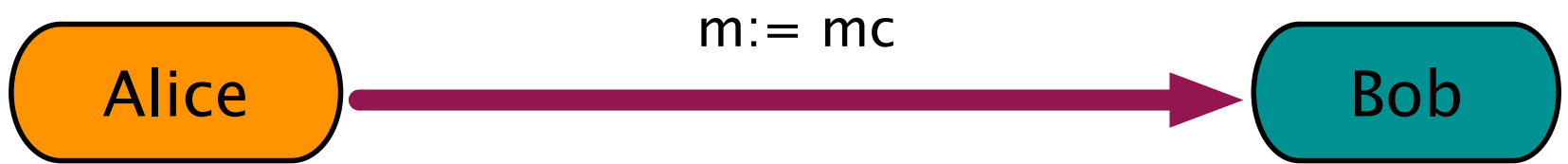
[C. Morgan]

Alice has two messages;
Bob knows their "names",
but not their contents.

$m_0, m_1: M$

Bob asks for one,
by name.

$c: \{0,1\}$



Alice sends the message...

...Bob receives it

PROBLEM: Alice is not to know which message Bob asked for
Bob is not to know the other message

Example: Oblivious Transfer Protocol

[C. Morgan]

Ted: Trusted party

	Alice	Bob
Hid:	$r; d; c \in \{0, 1\}; m$	$m_0; m_1; r_0; r_1$
Vis:	$m_0; m_1; r_0; r_1; f_0; f_1; e$	$c; m; d; r; f_0; f_1; e$

$$P \stackrel{\text{def}}{=} \left[\begin{array}{l} r_0, r_1 \in M; d \in \{0, 1\}; \\ r := r_d; \\ e := c \oplus d; \\ f_0, f_1 := m_0 \oplus r_e, m_1 \oplus r_{1 \oplus e}; \\ m := f_c \oplus r; \end{array} \right.$$

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Vis:	$m_0; m_1; r_0; r_1; f_0; f_1; e$	$c; m; d; r; f_0; f_1; e$

Bob's point of view: He has not to see $m_{1 \oplus c}$

$r_0, r_1 \in M; d \in \{0, 1\};$

$r := r_d;$

$\{(c = d, f_0 = m_0 \oplus r_0, f_1 = m_1 \oplus r_1) \vee (c \neq d, f_0 = m_0 \oplus r_1, f_1 = m_1 \oplus r_0)\}$

$e := c \oplus d;$

$\{f_0 = m_0 \oplus r_e, f_1 = m_1 \oplus r_{1 \oplus e}\}$

$f_0, f_1 := m_0 \oplus r_e, m_1 \oplus r_{1 \oplus e};$

$m := f_c \oplus r;$

$\{f_0; f_1; m\}$

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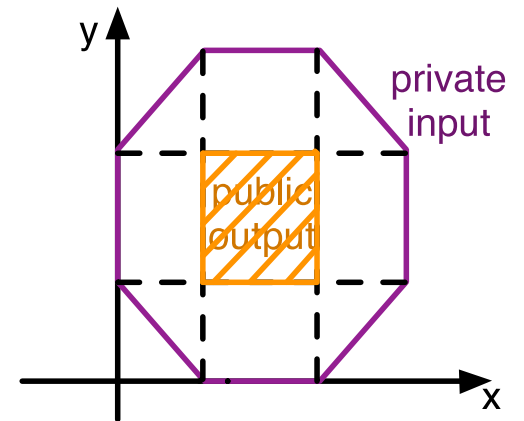
Soundness guarantees that Bob knows $m = m_c, f_c, r_d$

Wlp guarantees that Bob knows only $f_c = m_c \oplus r_d$ and $f_{1 \oplus c} = m_{1 \oplus c} \oplus r_{1 \oplus d}$

$f_{1 \oplus c}$ tells *almost nothing* of the secret $m_{1 \oplus c}$

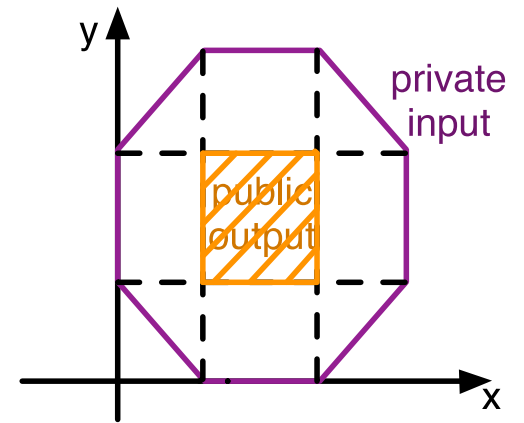
Declassified *Abstract* non-interference

$P \stackrel{\text{def}}{=} \left[\begin{array}{l} \text{if } (d \leq x + y \leq d + d_x + d_y \wedge -d_y \leq x - y \leq d_x) \text{ then} \\ \quad \text{if } (x \geq 0 \wedge x \leq d) \text{ then } x_L := d; \\ \quad \text{if } (x > d \wedge x \leq d_x) \text{ then } x_L := x; \\ \quad \text{if } (x > d_x \wedge x \leq d_x + d) \text{ then } x_L := d_x; \\ \quad \text{if } (y \geq 0 \wedge y \leq d) \text{ then } y_L := d; \\ \quad \text{if } (y > d \wedge y \leq d_y) \text{ then } y_L := y; \\ \quad \text{if } (y > d_y \wedge y \leq d_y + d) \text{ then } y_L := d_y; \end{array} \right.$



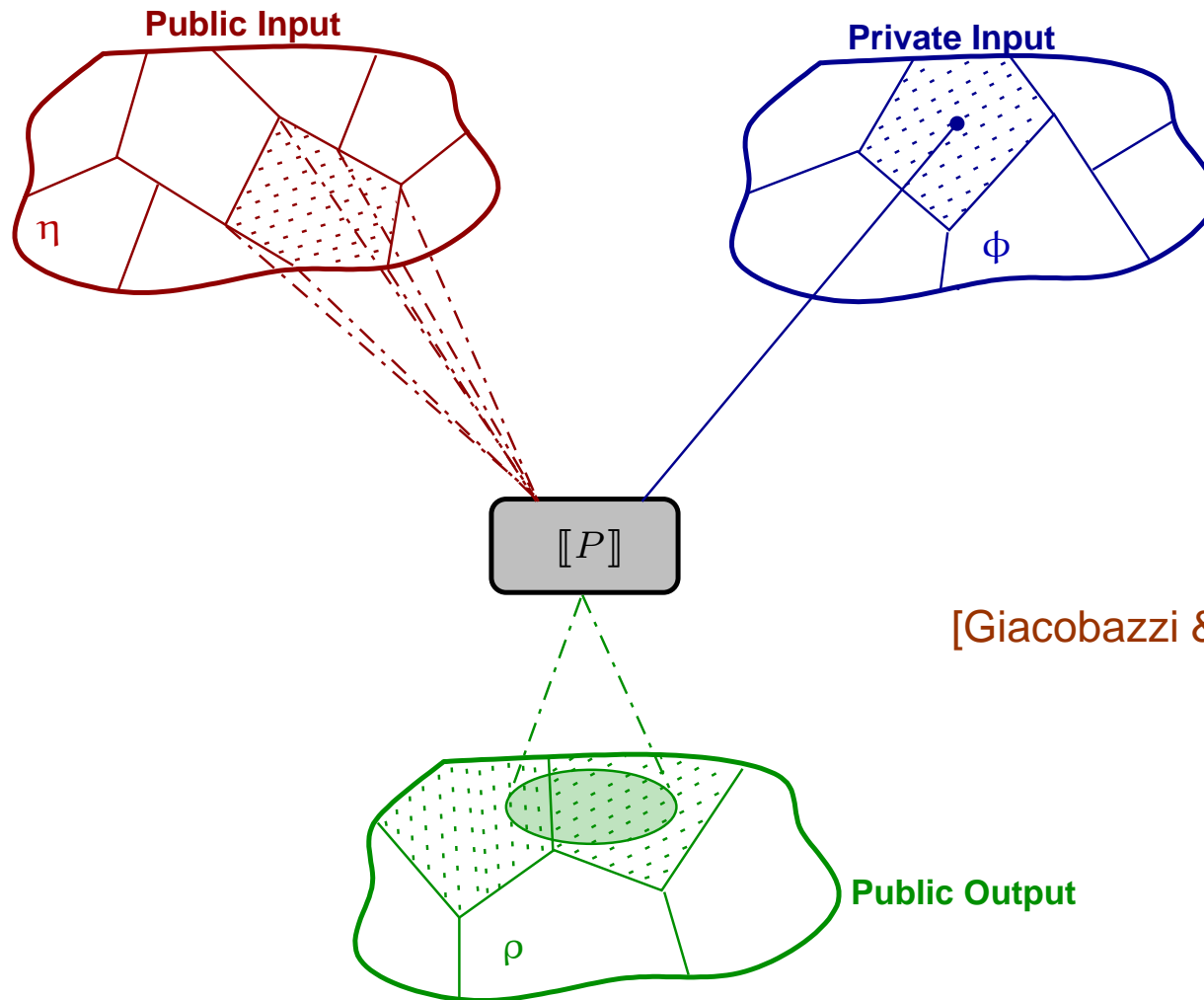
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$$\mathcal{H}^\Phi_\eta \circ Wlp_P \circ \mathcal{H}_\rho = Wlp_P \circ \mathcal{H}_\rho$$

Declassified *Abstract* non-interference

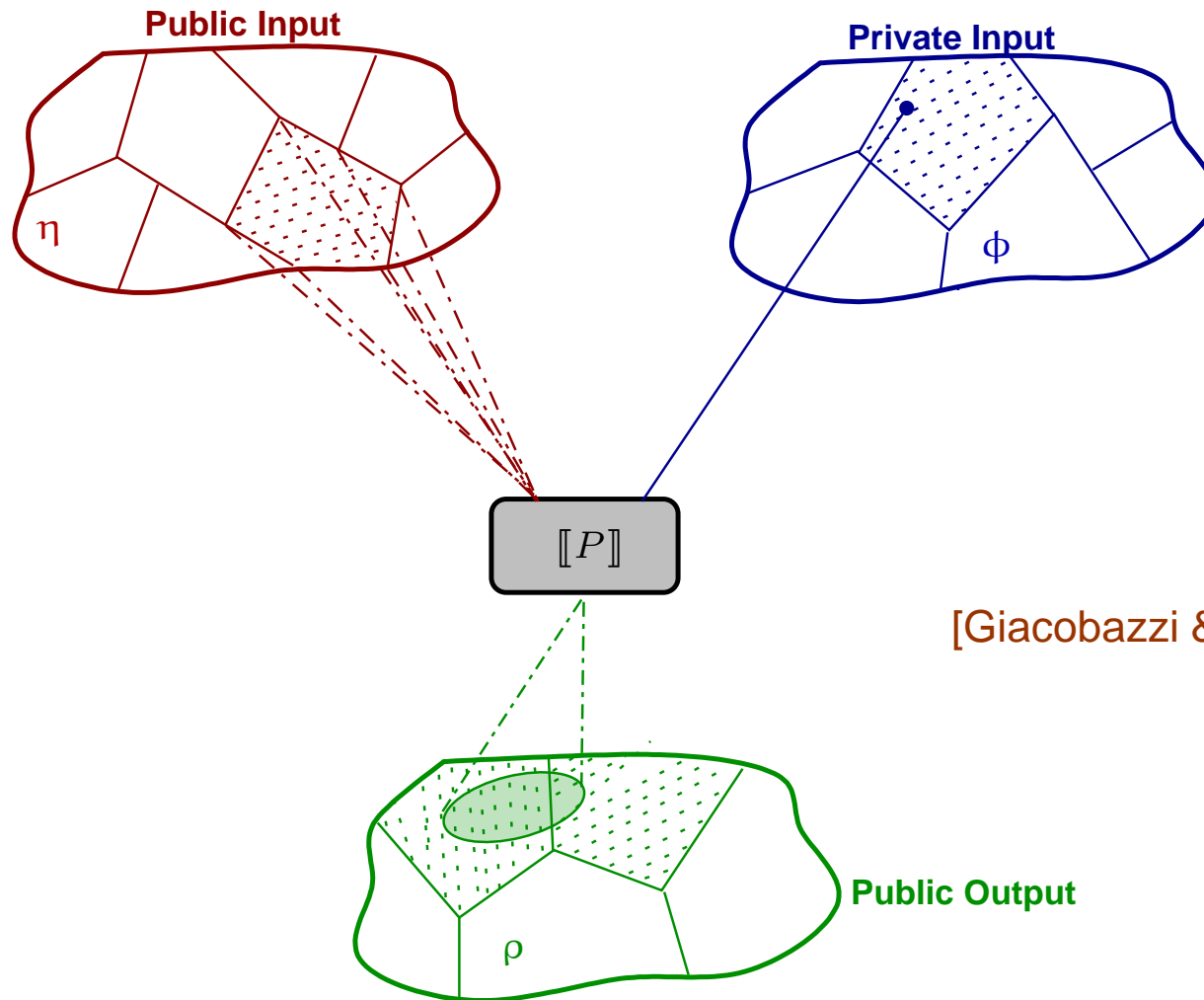


[Giacobazzi & Mastroeni '04]

$$\rho, \eta \in \text{Abs}(\wp(\mathbb{V}^L)), \phi \in \text{Abs}(\wp(\mathbb{V}^H)): (\eta)P(\phi \Rightarrow \rho):$$

$$\eta(l_1) = \eta(l_2) \text{ and } \phi(h_1) = \phi(h_2) \Rightarrow \rho([[P]](h_1, \eta(l_1))^L) = \rho([[P]](h_2, \eta(l_2))^L)$$

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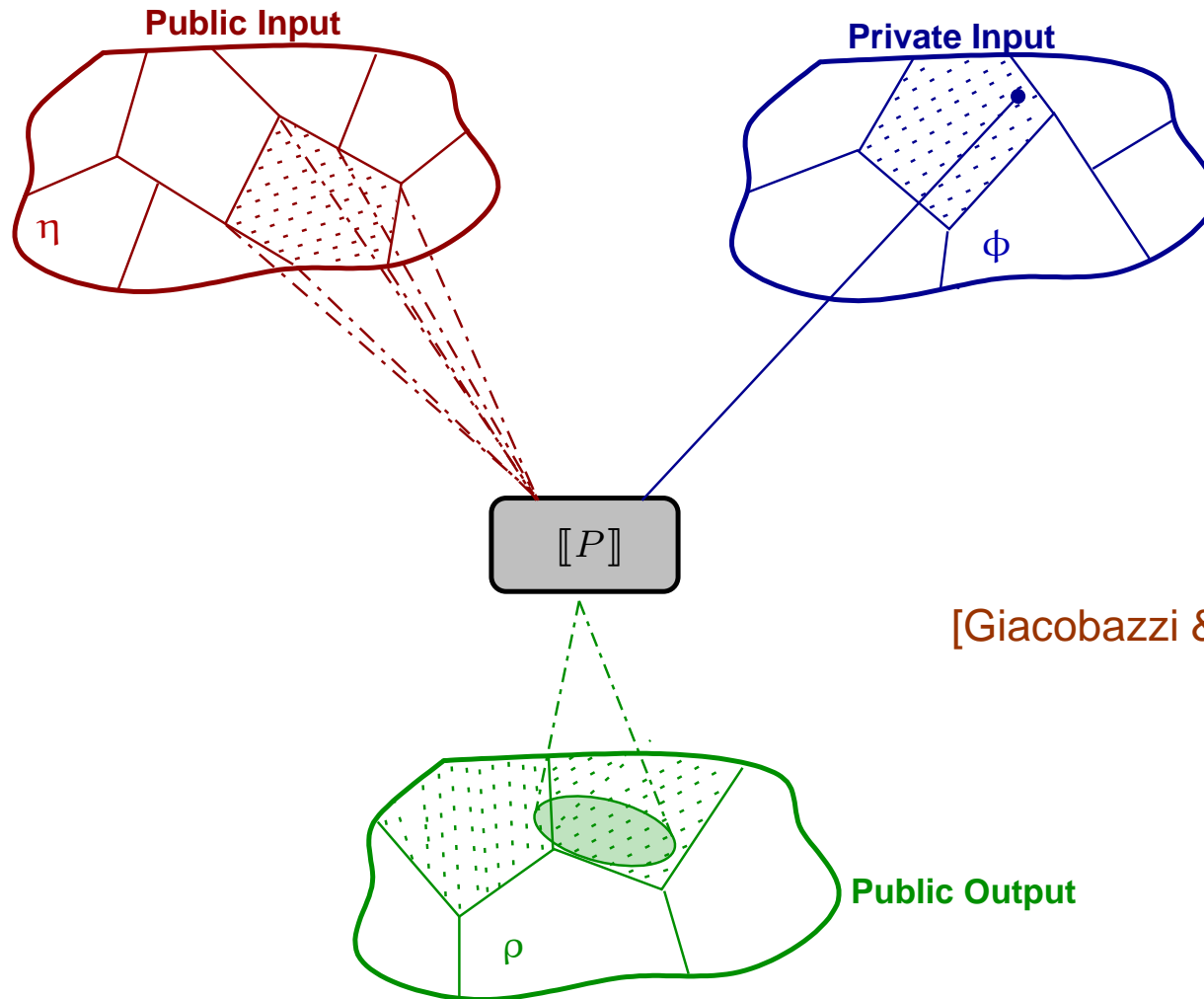


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Abstract Model checking DNI

- ⑥ **Robust declassification** transforms the attacker observational capability [Zdancewic & Myers '01]:

$$\forall \sigma, \sigma' \in \Sigma . \langle \sigma, \sigma' \rangle \in S[\approx] \Leftrightarrow Obs_{\sigma}(S, \approx) \equiv Obs_{\sigma'}(S, \approx)$$

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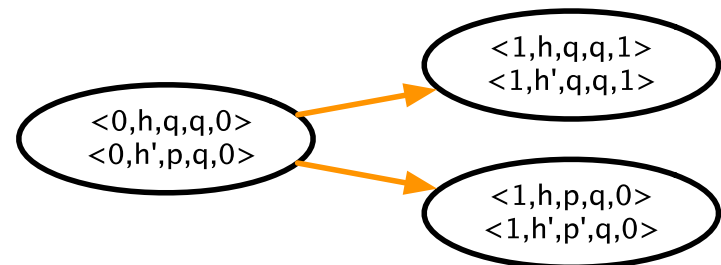
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The public variables are t, q, r , hence the partition induced by \mathcal{H} is:

$$\langle t, h, p, q, r \rangle \equiv \langle t', h', p', q', r' \rangle \text{ iff } t = t' \wedge q = q' \wedge r = r'$$

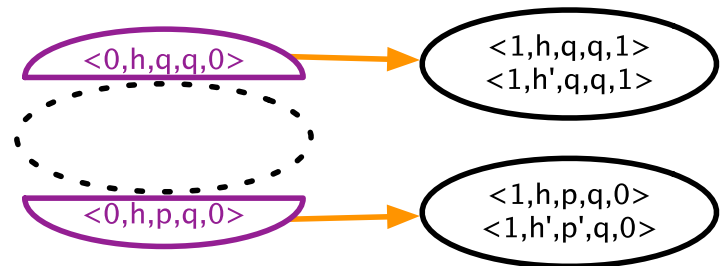
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Discussion

- ⑥ **What already exists:** *Several studies about declassification, derivation of counterexamples and refinements... nothing that combines all together*
 - ▣ Several approaches for modelling and checking declassification policies: PER model [Sabelfeld and Sands], dynamic logic [Darvas et al.], robust declassification [Zdancewic and Myers], Delimited release [Sabelfeld and Myers], Relaxed non-interference [Li and Zdancewic],...;
 - ▣ Derivation of counterexamples of secure information flows (without declassification) [Unno et al.];
 - ▣ Preservation of secrecy under refinement [Alur et al.];

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- ⑥ **What we have to do:** We are interested in...
 - ▣ ...extending our approach to more complex systems;
 - ▣ ...exploiting this connection for implementing our approach;

Abstract Interpretation

Consider the complete lattice $\langle C, \leq, \wedge, \vee, \perp, \top \rangle$, $A_i \in uco(C)$

Lattice of Abstract Domains \equiv Lattice uco

$$A \equiv \rho(C)$$

$$\langle uco(C), \sqsubseteq, \sqcap, \sqcup, \lambda x. \top, \lambda x. x \rangle$$

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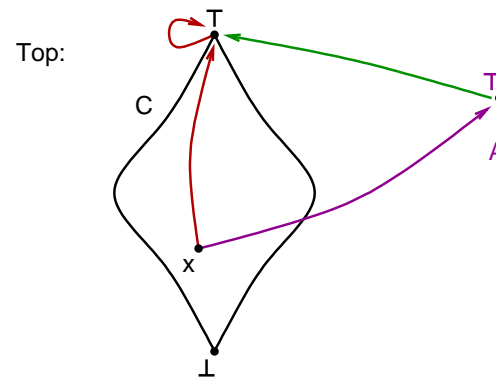
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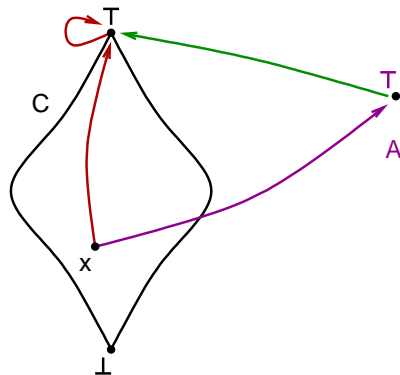
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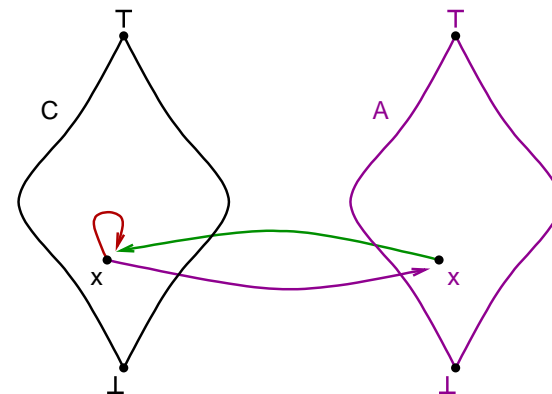
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Top:



Bottom:

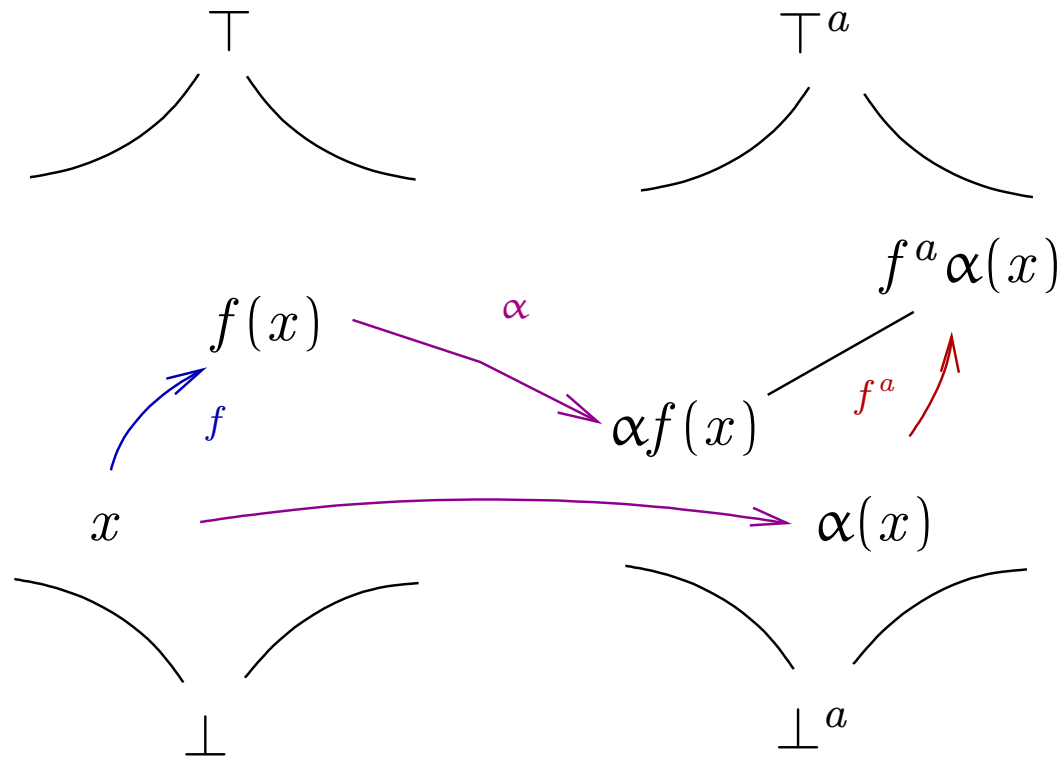


Abstract domain *backward* completeness

Let $\langle A, \alpha, \gamma, C \rangle$ a Galois insertion. [Cousot & Cousot '77,'79]

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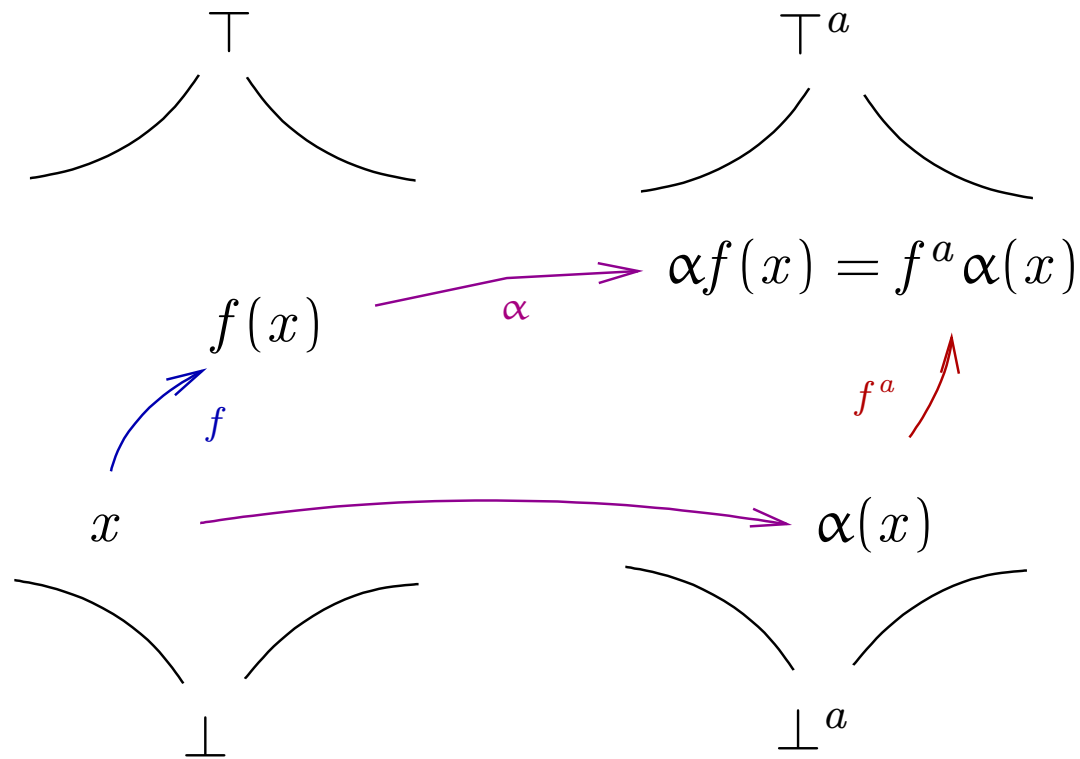
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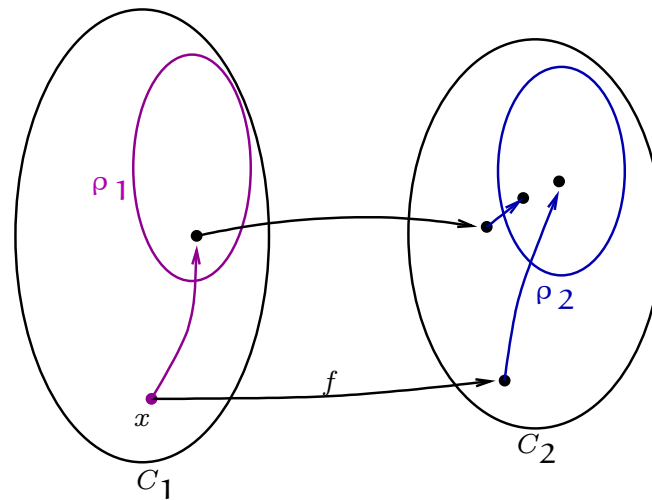
|||

$\rho f \rho = \rho f$



Making *backward* complete

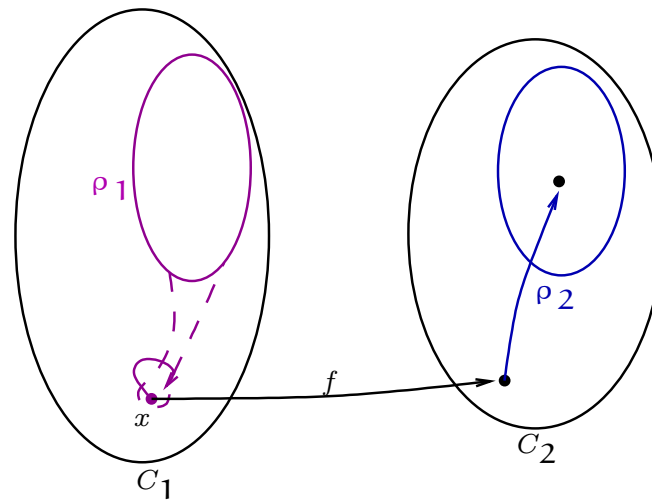
Giacobazzi et al. '00



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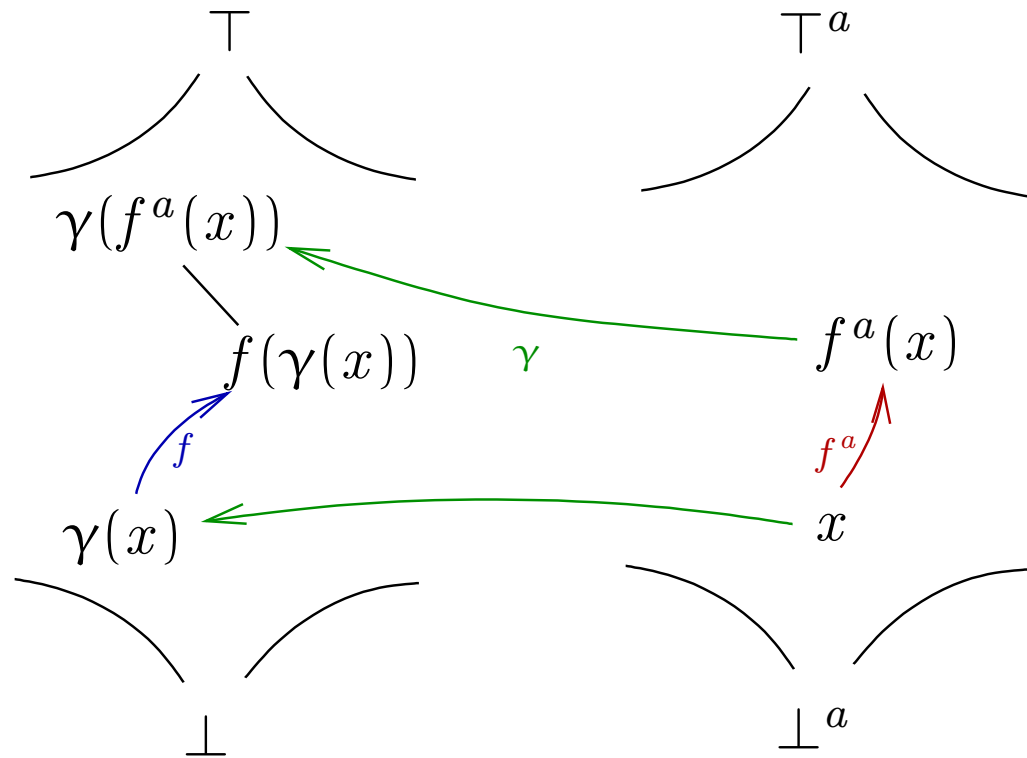
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