## Bioinformatics

## Scoring Matrices

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## Scoring Matrices

- Learning Objectives
- To explain the requirement for a scoring system reflecting possible biological relationships
- To describe the development of PAM scoring matrices
- To describe the development of BLOSUM scoring matrices


## Scoring Matrices

- Database search to identify homologous sequences based on similarity scores
- Ignore position of symbols when scoring
- Similarity scores are additive over positions on each sequence to enable DP
- Scores for each possible pairing, e.g. proteins composed of 20 amino acids, $20 \times 20$ scoring matrix


## Scoring Matrices

- Scoring matrix should reflect
- Degree of biological relationship between the amino-acids or nucleotides
- The probability that two AA's occur in homologous positions in sequences that share a common ancestor
- Or that one sequence is the ancestor of the other
- Scoring schemes based on physico-chemical properties also proposed


## Scoring Matrices

- Use of Identity
- Unequal AA's score zero, equal AA's score 1. Overall score can then be normalised by length of sequences to provide percentage identity
- Use of Genetic Code
- How many mutations required in NA's to transform one AA to another
- Phe (Codes UUU \& UUC) to Asn (AAU, AAC)
- Use of AA Classification
- Similarity based on properties such as charge, acidic/basic, hydrophobicity, etc


## Scoring Matrices

- Scoring matrices should be developed from experimental data
- Reflecting the kind of relationships occurring in nature
- Point Accepted Mutation (PAM) matrices
- Dayhoff (1978)
- Estimated substitution probabilities
- Using known mutational (substitution) histories


## Scoring Matrices

- Dayhoff employed 71 groups of near homologous sequences ( $>85 \%$ identity)
- For each group a phylogenetic tree constructed
- Mutations accepted by species are estimated
- New AA must have similar functional characteristics to one replaced
- Requires strong physico-chemical similarity
- Dependent on how critical position of AA is to protein
- Employs time intervals based on number of mutations per residue


## Scoring Matrices

Overall Dayhoff Procedure:-

- Divide set of sequences into groups of similar sequences multiple alignment for each group
- Construct phylogenetic tree for each group
- Define evolutionary model to explain evolution
- Construct substitution matrices
- The substitution matrix for an evolutionary time interval $t$ gives for each pair of AA $(a, b)$ an estimate for the probability of $a$ to mutate to $b$ in a time interval $t$.


## Scoring Matrices

- Evolutionary Model
- Assumptions : The probability of a mutation in one position of a sequence is only dependent on which $A A$ is in the position
- Independent of position and neighbour AA's
- Independent of previous mutations in the position
- No need to consider position of AA's in sequence
- Biological clock - rate of mutations constant over time
- Time of evolution measured by number of mutations observed in given number of AA's. 1-PAM = one accepted mutation per 100 residues


## Scoring Matrices

- Calculating Substitution Matrix - count number of accepted mutations


|  | A | C | D | G | H | I | K | L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A |  | 1 | 2 |  |  |  |  |  |
| C | 1 |  |  |  |  |  | 1 |  |
| D | 2 |  |  |  |  |  | 1 |  |
| G |  |  |  |  |  | 1 |  |  |
| H |  |  |  |  |  |  |  | 1 |
| I |  |  |  | 1 |  |  |  |  |
| K |  | 1 | 1 |  |  |  |  |  |
| L |  |  |  |  | 1 |  |  |  |

## Scoring Matrices

- Once all accepted mutations identified calculate
- The number of $a$ to $b$ or $b$ to $a$ mutations from table - denoted as $f_{a b}$
- The total number of mutations in which $a$ takes part - denoted as $f_{a}=\Sigma_{b \neq a} f_{a b}$
- The total number of mutations $f=\Sigma_{a} f_{a}$ (each mutation counted twice)
- Calculate relative occurrence of AA's
- $p_{a}$ where $\Sigma_{a} p_{a}=1$


## Scoring Matrices

- Calculate the relative mutability for each AA
- Measure of probability that $a$ will mutate in the evolutionary time being considered
- Mutability depends on $f_{a}$
- As $f_{a}$ increases so should mutability $m_{a}$; AA occurring in many mutations indicates high mutability
- As $p_{a}$ increases mutability should decrease ; many occurrences of AA indicate many mutations due to frequent occurrence of $A A$
- Mutability can be defined as $m_{a}=K f_{a} / p_{a}$ where $K$ is a constant


## Scoring Matrices

- Probability that an arbitrary mutation contains $a$
- $2 f_{a} / f$
- Probability that an arbitrary mutation is from $a$
$-f_{a} / f$
- For 100 AA's there are $100 p_{a}$ occurrences of $a$
- Probability to select $a 1 / 100 p_{a}$
- Probability of any of $a$ to mutate
- $m_{a}=\left(1 / 100 p_{a}\right) \times\left(f_{a} / f\right)$
- Probability that $a$ mutates in 1 PAM time unit defined by $m_{a}$


## Scoring Matrices

- Probability that $a$ mutates to $b$ given that $a$ mutates is $f_{a b} / f_{a}$
- Probability that $a$ mutates to $b$ in time $t=1$ PAM

$$
-M_{a b}=m_{a} f_{a b} / f_{a} \text { when } a \neq b
$$

```
X=0
C 12
S 0}
T -2 1 3
P - - 1 crecre Log-odds PAM 250 matrix
G -3 1 1
N -4 1
D -5 0
E -5 0
Q -5 -1 -1 0
H -3 -1 -1 0
R -4 0
K -5 0 0
M -5 -2 -1 -2 -2 -1 -3 -2 -3 -2 -2 -1 -2 0
I -2 -1 0 0-2 -1 -3 -2 -2 (-2 -2 -2 -2 -2 -2 2 5
L -6 -3 -4 -2 -3 -2 -4
V -2 -1 0}0
F -4 -3 -3 -5 -4 -5 -4 -6 -5 -5 -5 -2 -4 4
W 0 -3 -3 -5 -3 -5 -2 -4 -4 -4 0
Y [r - -2 
```


## Dayhoff mutation matrix (1978) - summary

- Point Accepted Mutation (PAM)
- Dayhof matrices derived from sequences $85 \%$ identical
- Evolutionary distance of 1 PAM = probability of 1 point mutation per 100 residues
- Likelihood (odds) ratio for residues $a$ and $b$ :

Probability $a-b$ is a mutation / probability $a-b$ is chance

- PAM matrices contain log-odds figures
val $>0$ : likely mutation
$\mathrm{val}=0 \quad$ : random mutation
vak $<0$ : unlikely mutation
- 250 PAM : similarity scores equivalent to $20 \%$ identity
- low PAM - good for finding short, strong local similarities high PAM = long weak similarities


## Scoring Matrices

- What about longer evolutionary times ?
- Consider two mutation periods 2-PAM
$-a$ is mutated to $b$ in first period and unchanged in second
- Probability is $M_{a b} M_{b b}$
$-a$ is unchanged in first period but mutated to $b$ in the second
- Probability is $M_{a a} M_{a b}$
- $a$ is mutated to $c$ in the first which is mutated to $b$ in the second
- Probability is $M_{a c} M_{c b}$
- Final probability for $a$ to be replaced with $b$

$$
-M^{2}{ }_{a b}=M_{a b} M_{b b}+M_{a a} M_{a b}+\Sigma_{\mathrm{c} \neq \mathrm{a}, b} M_{a c} M_{c b}=\Sigma_{\mathrm{c}} M_{a c} M_{c b}
$$

## Scoring Matrices

- Simple definition of matrix multiplication
- $M^{2}{ }_{a b}=\Sigma_{\mathrm{c}} M_{a c} M_{c b}$
- $M^{3}{ }_{a b}=\Sigma_{c} M^{2}{ }_{a c} M_{c b}$ etc
- Typically $M^{40} M^{120} M^{160} M^{250}$ are used in scoring
- Low values find short local alignments, High values find longer and weaker alignments
- Two AA's can be opposite in alignment not as a results of homology but by pure chance
- Need to use odds-ratio $\mathrm{O}_{\mathrm{ab}}=M_{a b} / P_{b}$ (Use of Log)
- $\mathrm{O}_{\mathrm{ab}}>1: b$ replaces $a$ more often in bologically related sequences than in random sequences where $b$ occurs with probability $P_{b}$
- $\mathrm{O}_{\mathrm{ab}}<1: b$ replaces $a$ less often in bologically related sequences than in random sequences where b occurs with probability $P_{b}$


## BLOSUM Scoring Matrices

- PAM matrices derived from sequences with at least $85 \%$ identity
- Alignments usually performed on sequences with less similarity
- Henikoff \& Henikoff (1992) develop scoring system based on more diverse sequences
- BLOSUM - BLOcks SUbstitution Matrix
- Blocks defined as ungapped regions of aligned AA's from related proteins
- Employed > 2000 blocks to derive scoring matrix


## BLOSUM Scoring Matrices

- Statistics of occurrence of AA pairs obtained
- As with PAM frequency of co-occurrence of AA pairs and individual AA's employed to derive Odds ratio
- BLOSUM matrices for different evolutionary distances
- Unlike PAM cannot derive direct from original matrix
- Scoring Matrices derived from Blocks with differing levels of identity


## BLOSUM Scoring Matrices

- Overall procedure to develop a BLOSUM X matrix
- Collect a set of multiple alignments
- Find the Blocks (no gaps)
- Group segments of Blocks with X\% identity
- Count the occurrence of all pairs of AA's
- Employ these counts to obtain odds ratio (log)
- Most common BLOSUM matrices are $45,62 \& 80$


## Scoring Matrices

- Differences between PAM \& BLOSUM
- PAM based on predictions of mutations when proteins diverge from common ancestor - explicit evolutionary model
- BLOSUM based on common regions (BLOCKS) in protein families
- BLOSUM better designed to find conserved domains
- BLOSUM - Much larger data set used than for the PAM matrix
- BLOSUM matrices with small percentage correspond to PAM with large evolutionary distances
- BLOSUM64 is roughly equivalent to PAM 120

