SIGMA-PURE-INJECTIVE OBJECTS, CHARACTERISATIONS AND CLASSIFICATIONS FROM REPRESENTATION THEORY

RAPHAEL BENNETT-TENNENHAUS

It is well-known and documented that both pure-injective (PI) and Sigma-pure-injective (SPI) modules may be characterised in various ways. Crawley-Boevey developed purity for locally finitely presented additive categories and characterised PI and SPI objects. Krause defined purity for compactly generated triangulated categories and characterised PI objects. Garkusha and Prest then built on the work of Krause and gave a many-sorted language for this situation. The aforementioned characterisations rely on pulling information from the functor category using different restrictions of the Yoneda functor.

In this talk, I will focus on SPI objects, and show how one may characterise SPI objects in the triangulated setting. Time permitting, I will then use these characterisations to describe SPI objects in the homotopy category of a gentle algebra. This talk is based on joint work with Crawley-Boevey published in the Journal of Algebra, work published in Fundamenta Mathematicae, and an article on arXiv 1911.07691.

A DEFINABLE APPROACH TO TENSOR TRIANGULAR GEOMETRY

ISAAC BIRD

The Balmer spectrum of a tensor triangulated category yields a classification of thick tensor ideals. The recently introduced homological spectrum enables a functorial approach to the same problem, and enables one to obtain the Balmer spectrum from the functor category. In a big tt-category, one can equip the usual Ziegler spectrum with a coarser topology which is compatible with the monoidal structure - the closed sets now correspond to tensor-closed definable subcategories rather than the usual ones. I will show how, from this tensor Ziegler spectrum, one can recover the homological spectrum, and therefore also the Balmer spectrum, from tensor triangular geometry. I shall also show how definable functors between triangulated categories can be used to obtain functoriality of the homological spectrum beyond the setting of geometric functors, thereby expanding the scope of results of Balmer. This talk is based on joint work with Jordan Williamson.

CLOSURE PROPERTIES OF ORTHOGONAL CLASSES ASSOCIATED TO COSILTING OBJECTS

SIMON BREAZ

I will present some results that use closure properties of left perpendicular classes to obtain characterizations for cosilting objects in various triangulated categories. In particular, we will see that a weak cogenerator in a triangulated category with produts is cosilting if and only if it belongs to its left perpendicular class, and this class is closed with respect to products. This result extends a similar theorem, known for pure-injective cogenerators in compactly generated categories. In the last part of the proof we will see that the pure-injective cosilting objects in compactly generated categories can be recognized by using the clousre of the associated left orthogonal class under pure subobjects.

DESCENT IN TENSOR TRIANGULAR GEOMETRY

NATALIA CASTELLANA

Given a tensor triangulated category (tt-category), one way to study it is by classifying its thick, smashing and localizing tensor ideals. Descent methods apply when one can reduce these problems to another tt-category via a tt-functor with good properties, e.g. base change with respect to a descendable commutative algebra. In this work we describe equalizer diagrams relating lattices of localizing and smashing ideals through base change, which yield to a coequalizer diagram for Balmer spectra. We apply these results to particular examples, e.g. faithful Galois extensions. This is joint work with T. Barthel, D. Heard, N. Naumann, L. Pol and B. Sanders.

FLAT COTORSION MODULES AND THE EXCHANGE PROPERTY

MANUEL CORTÉS IZURDIAGA

In a recent work, we have introduced the class of right strong exchange rings which contains the classes of local, left perfect, left continuous and left cotorsion rings, and we have proved that right strong exchange rings are semi-regular. And therefore, they are exchange rings.

On the other hand, flat cotorsion right modules have right cotorsion endomorphism rings which are, in particular, left strong exchange. This means that flat cotorsion modules satisfy the finite exchange property, as had been already noted by Guil Asensio and Herzog. Consequently, one may ask the natural related question of whether flat cotorsion right modules satisfy the generalized exchange property. We will give in this talk a positive answer to this question.

This talk is based on two papers, one with P. A. Guil Asensio and other with P. A. Guil Asensio and A. Srivastava.

GABRIEL-POPESCU THEOREM REVISITED

SEPTIMIU CRIVEI

Using a generalization of the Mitchell Lemma and one-sided exact categories, we extend to AB5 categories a series of classical results from the theory of Grothendieck categories, namely the Takeuchi Lemma, the Ulmer Theorem and the Gabriel-Popescu Theorem. The generating set of objects of a Grothendieck category from the classical Gabriel-Popescu Theorem is weakened to an arbitrary set $\mathcal{U}$ of objects of an AB5 category $\mathcal{A}$, and we show that the restriction of the Yoneda functor $T : \mathcal{A} \to (\mathcal{U}^{\mathrm{op}}, \mathrm{Ab})$ to a suitable coreflective full subcategory of $\mathcal{A}$ is fully faithful, and has a left adjoint which is conflation-exact with respect to the inflation-exact structure induced from $\mathcal{A}$. We give applications to tilting theory and its generalizations, equivalences of categories and some particular categories. This is mainly based on joint work with Constantin Năstăsescu.

SOME ASPECTS OF (STRONG) GENERATION IN MODULE CATEGORIES

SOUVIK DEY

Strong generation is a concept introduced for triangulated categories by Bondal-Van den Bergh and Rouquier. Iyengar-Takahashi introduced the concept of strong generation for abelian categories with enough projective objects. I will present highlights of various joint works with Ryo Takahashi, Pat Lank, and Anirban Bhaduri on passage of strong generation under certain morphisms of commutative Noetherian rings and for schemes and provide some upper bounds for generation time.

STRUCTURE OF SEMIARTINIAN RINGS

KATERINA FUKOVA

For (von Neumann) regular semiartinian rings with primitive factors artinian there is an invariant called dimension sequence (Theorem 2.1 in [1]) formed by slices of socle chain of the ring. The necessary conditions on this invariant was studied for example in [2]. We will focus on how much the dimension sequence determines the ring. In the case of commutative ring if we consider the dimension sequence with finitely many slices all of them being countable, the corresponding ring is (up to isomorphism) given by one construction from the ring of eventually constant sequences. We will also discuss the case for dimension sequence of infinitely many countable slices.

References

[1] P. Růžička, J. Trlifaj, J. Žemlička, *Criteria of Steadiness*, Marcel Dekker, Abelian Groups, Module Theory and Topology, 1998.

[2] J. Žemlička, *On Socle Chains of Semiartinian Rings with Primitive Factors Artinian*, Lobachevsii Journal of Mathematics, Volume 37, 2016. Pages 316-322.

NONCOMMUTATIVE TWIST EQUIVALENCE ARISING FROM A (PARTIALLY) MINIMAL PROJECTIVE RESOLUTION

MARINA GODINHO

Given an object X in a Frobenius exact category satisfying mild conditions, we construct a special projective resolution of the stable endomorphism algebra of X as a module over the endomorphism algebra of X. This resolution is minimal with respect to objects in the Frobenius category which are not projective. With this technology, we specify precisely when the suspension functor of the stable category applied to X has finite order. When this is the case, we show that the derived restriction of scalars functor (induced from quotient morphism between the endomorphism algebra of X and the stable endomorphism algebra of X) is spherical. As a result, we are able to construct the noncommutative twist equivalence, an interesting derived autoequivalence of the endomorphism algebra of X. This result has applications in geometry, and for very singular 3-folds we obtain new autoequivalences.

TENSOR-ORTHOGONAL PAIRS IN DERIVED CATEGORIES

MICHAL HRBEK

A co-t-structure can be seen as the derived category analog of the notion of a complete and hereditary cotorsion pair in the module category. We introduce the notion of a tensor-structure in the derived category setting as the analog of the module-theoretic notion of a hereditary Tor-pair. Tensor-structures generalize both the compactly generated t-structures and the Bousfield classes in a derived category. The lattice of tensor-structures embeds into the lattice of co-t-structures in the derived category of left or right modules.

In the derived category of a commutative noetherian ring, we provide a classification of tensor-structures generated by complexes of finite flat dimension in terms of sequences of subsets of the Zariski spectrum. This provides the "minimal" common generalization of the classification results of Neeman for localizing subcategories and Alonso-Jeremías-Saorín for compactly generated t-structures. This is a joint work with Dolors Herbera and Giovanna Le Gros, and the first part of a two-part series of talks, the second part will be delivered by Giovanna Le Gros.

TOR-PAIRS AND TENSOR-ORTHOGONAL PAIRS OVER COMMUTATIVE NOETHERIAN RINGS

GIOVANNA LE GROS

Over a commutative noetherian ring, collections of subsets of the spectrum classify families of objects in its derived category or module category. In the derived category, this includes the compactly generated t-structures by sp-filtrations due to Alonso-Jeremías-Saorín, and localising subcategories by subsets of the spectrum due to Neeman. Similarly, in the module category, finite sequences of specialisation closed subsets of the spectrum which do not contain the associated primes of the ring R, classify cotilting cotorsion pairs due to Angeleri Hügel-Pospíšil-Šťovíček-Trlifaj.

We consider an extension of the above results: On the derived category side, we introduce tensor-structures, and we show that the ones generated by bounded complexes of flat modules are in bijective correspondence with certain sequences of subsets of spectrum indexed by the integers. On the module category side, we consider hereditary Tor-pairs generated by modules of bounded flat dimension over commutative noetherian rings, and show that these are classified by sequences of subsets of the spectrum with a certain condition depending on the depth of the localisations at primes of the spectrum.

This talk is based on joint work with Dolors Herbera and Michal Hrbek. This is the first part of a two-part series of talks, the first part of this talk will be delivered by Michal Hrbek.

BOUNDED T-STRUCTURES AND THE FINITISTIC DIMENSION OF A TRIANGULATED CATEGORY

KABEER MANALI RAHUL

Neeman recently settled a conjecture by Antieau, Gepner and Heller on the existence of bounded t-structures on the derived category of perfect complexes. We prove a triangulated categorical generalisation of that theorem. In particular, we will show that under some finiteness hypothesis, the existence of a bounded t-structure implies that the singularity category, appropriately defined, vanishes. To achieve this, we also introduce the notion of finitistic dimension for triangulated categories. Finally, we also show that all t-structures on the completion under these hypotheses are equivalent. This proves that all bounded t-structures on the bounded derived category of a noetherian finite dimensional scheme are equivalent, generalising a result by Neeman. This is joint work with Rudradip Biswas, Hongxing Chen, Chris Parker, and Junhua Zheng.

A REPRESENTABILITY THEOREM FOR TRIANGULATED CATEGORIES

GEORGE CIPRIAN MODOI

In this talk we present a general Brown representability theorem for triangulated categories, inspired by similar results appearing in [2, Theorem 4.16] and [1, Theorem 4.2]. Our theorem generalises both above mentioned results, and it holds for triangulated R-linear categories, with R being a commutative, but non necessary noetherian, ring. The result we will present are a part of an ongoing project, in which we want to see how far we can generalize the results in [1], by omitting hypotheses as, for example, the noetherian assumption before.

References

[1] A. Neeman, Triangulated categories with a single compact generator and a Brown representability theorem, preprint arXiv:1804.02240[math.CT].

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MUTATION OF SIGNED QUIVERS AND PRESENTATIONS OF SIMPLE LIE ALGEBRAS

DAVIDE MORIGI

In this talk we introduce a signed variant of (valued) quivers and a mutation rule, that generalizes the classical Fomin-Zelevinsky mutation of quivers. We associate to any signed valued quiver a Lie-theoretic object that is a signed analogue of the Cartan counterpart, from which we can construct root systems and a Lie algebra via a "Serre-like" presentation.

We then restrict our attention to the Dynkin case. For root systems, we show some compatibility results with the appropriate concept of mutation. Using results from Barot-Rivera and Perez-Rivera, we also show that signed quivers in the same mutation class yield isomorphic Lie algebras.

This is based on a joint project with Joe Grant (arXiv number 2403.14595).

ON GENERALIZATIONS OF AN OLD THEOREM OF RICKARD'S

AMNON NEEMAN

In 1989 Rickard published a couple of papers on Morita equivalence of derived categories. We will begin with a quick review of his results.

The interest in this old result was revived in 2018 by Krause, who asked for improvements. Krause's question almost immediately led to a couple of results we will briefly review.

And then we will talk about very recent work, joint with Canonaco and Stellari, which goes much further in improving Rickard's theorem. This is work still in progress, in the sense that we are trying to sharpen and generalize the theorems that we can already prove.

COMMUTATIVE HEARTS: CONTINUITY OF MUTATION

SERGIO PAVON

Hearts of compactly generated t-structures in the derived category of a commutative noetherian ring are related by a (right) mutation operation, which is realised as an HRS-tilt at a hereditary torsion pair of finite type. This operation induces a natural bijection between the Gabriel spectra of the two hearts. Using the special properties of residue fields, which correspond to the points of the Gabriel spectra, we show that right mutation induces an open map, with respect to their full support topologies. This is ongoing joint work with Michal Hrbek and Jorge Vitória.

ABSTRACT REPRESENTATION THEORY VIA COHERENT AUSLANDER-REITEN DIAGRAMS

ÁLVARO SÁNCHEZ

In this talk we explain a method to study representations of quivers over arbitrary stable ∞-categories in terms of Auslander-Reiten diagrams.

Our techniques allow us to internally visualize (a significant piece of) the Auslander-Reiten quiver of the derived category of a hereditary finite-dimensional algebra inside much more general (∞-)categories of representations, such as representations over arbitrary rings, schemes, dg algebras, or ring spectra. This is provided by an equivalence with a certain *mesh subcategory* of representations of the repetitive quiver, which we build inductively using abstract reflection functors.

As an application we obtain that the automorphism group of the non-regular components of the Auslander-Reiten quiver acts on the ∞-category of representations over any stable ∞-category. When specialized to representations of trees over a field, this action gives an isomorphism with the derived Picard group as shown by Miyachi and Yekutieli.

THE DERIVATOR ASSOCIATED TO A DG-CATEGORY

CHIARA SAVA

This talk is based on a joint work in progress with Francesco Genovese. In triangulated categories not all homotopy limits and colimits are functorial; for this reason we work with enhancements which are higher categorical structures whose homotopy categories are triangulated. Among the most important enhancements there are derivators, ∞-categories and differential graded categories (dg-categories). All these structures can be related: indeed, we can define the ∞-category associated to a dg-category and, analogously, the derivator associated to an ∞-category. In this talk, by developing a new theory of homotopy limits and colimits in dg-categories, we close a gap in the existing literature and provide a direct construction of a derivator associated to a (pretriangulated) dg-category. As a consequence, we will be able to associate a derivator to the dg-enhancement of the stable category of Gorenstein projective modules and, given suitable ring R and small category I, we also prove that the derivator associated to the dg-category of totally acyclic complexes of projective modules, evaluated in I, is equivalent to the category of totally acyclic complexes of projective RI-modules.

EXACT STRUCTURES AND PURITY

KEVIN SCHLEGEL

The notion of a purity category and a Ziegler spectrum is, in its most general form, defined for a locally finitely presented category $\mathcal{A}$. Given an exact structure $\mathcal{E}$ on the full subcategory of finitely presented objects in $\mathcal{A}$, we define a relative purity category $\mathcal{P}$ such that $\mathcal{A}$ is a full subcategory of $\mathcal{P}$ closed under extensions. This induces an exact structure on $\mathcal{A}$ with enough injective objects and we show that the indecomposable injective objects form a closed set of the Ziegler spectrum of $\mathcal{A}$. Moreover, we specify to the case that $\mathcal{A}$ is a module category over an Artin algebra. In this case, the closed set of the Ziegler spectrum obtained from the exact structure gives rise to an fp-idempotent ideal. This is used to prove generalized Auslander-Reiten formulas for the Ext-groups relative to the exact structure.

PURITY, EXACT DIRECTED COLIMITS AND T-STRUCTURES

JAN STOVICEK

Jensen and Lenzing characterized pure-injective modules by a condition that naturally generalizes to any additive category with products. This in principle allows one to define pure-injective objects in any such category. I will explain that this naive idea has interesting consequences: In joint work with Positselski, we characterized exactness of directed colimits in an abelian category with products and an injective cogenerator via pure-injectivity of the cogenerator. In joint work with Saorín, we used this criterion to study when hearts of t-structures have exact directed colimits.

APPROXIMATIONS AND VOPĚNKA'S PRINCIPLES

JAN TRLIFAJ

The approximation classes of modules that arise as components of cotorsion pairs are tied up by Salce's duality. We will consider general approximation classes of modules and investigate possibilities of dualization in dependence on closure properties of these classes. While some proofs are easily dualized, other dualizations require large cardinal principles, and some fail in ZFC, with counterexamples provided by classes of $\aleph_1$-projective modules over non-perfect rings. For example, we will show that Vopěnka's Principle implies that each covering class of modules closed under homomorphic images is of the form $\mathrm{Gen}(M)$ for a module M, and that the latter property restricted to classes generated by $\aleph_1$-free abelian groups implies Weak Vopěnka's Principle (joint work with Asmae Ben Yassine).

A study of Sylvester rank functions via functor categories

Simone Virili*

May 2024

Abstract

Given a ring R, a **Sylvester module rank function** $\rho\colon \text{mod-}R \to \mathbb{R}_{\geqslant 0}$ on the category of finitely presented right R-modules is an isomorphism-invariant function which is additive on coproducts, subadditive on right-exact sequences, monotone on quotients, and normalized by $\rho(R) = 1$. Classically, they have been studied by P. Malcomson, A.H. Schofield and, in a slightly different context, by W. Crawley-Boevey while, more recently, they played an important role in A. Jaikin-Zapirain's approach to the Atiyah and to the Lück approximation conjectures.

A **length function** on a Grothendieck category $\mathcal{G}$ is an isomorphism-invariant function $L\colon \mathcal{G} \to \mathbb{R}_{\geqslant 0} \cup \{\infty\}$ which is additive on short exact sequences, and "continuous" on direct unions of subobjects. For R a ring, a length function L on Mod-R is said to be **normalized** if $L(R) = 1$. Any normalized length function $L\colon \text{Mod-}R \to \mathbb{R}_{\geqslant 0} \cup \{\infty\}$, restricts to a rank function on mod-R. In fact, extending an argument of Crawley-Boevey, I have recently proved that this gives a bijective correspondence between normalized length functions and a family of (quite special) rank functions called **additive**. It is natural to conclude from such a result that the theory of length functions is just a first approximation to the finer theory of rank functions.

Going in the opposite direction, H. Li has recently developed a theory of **bivariant rank functions**, obtaining as a natural application some new results on rank functions related to ring epimorphisms, and new (much simpler) proofs for several deep results of Schofield.

In this talk I will start by observing that any rank function ρ on mod-R extends to a length function on the functor category $\mathcal{G}(R) := [R\text{-mod}, \text{Ab}]$ (viewing each $P \in \text{mod-}R$ as the functor $H_P := P \otimes_R -\colon R\text{-mod} \to \text{Ab}$). In fact, this gives a bijection between rank functions on mod-R and, suitably normalized, length functions on $\mathcal{G}(R)$. As a consequence, we can now affirm that

The theory of length functions (on locally finitely presented Grothendieck categories) is not restrictive at all: it contains the theory of rank functions as a special case.

This also suggests that the strategy, commonly used in the model theory of modules, of studying pure-injective modules as injective objects in the functor category also applies in this context. Exploiting this connection I will extend and give new, simplified, proofs of several results of Schofield, Li, and Jaikin-Zapirain.

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COMMUTATIVE HEARTS: THE GABRIEL SPECTRUM

JORGE VITÓRIA

A notable fact of hearts of compactly generated nondegenerate t-structures in the derived category of a commutative noetherian ring R is that they contain suitable shifts of all residue fields. In this talk, we show that if $\mathrm{Mod}(R)$ is a Gabriel category, then so is every such heart. Moreover, we use these residue fields to show that the Gabriel spectrum of such hearts is a T_0 Alexandrov topological space. This is ongoing joint work with Michal Hrbek and Sergio Pavon.

PROPOSAL FOR A LECTURE; ON THE RING THEORY OF DIFFERENTIAL GRADED RINGS

ALEXANDER ZIMMERMANN

Differential graded algebras proved to be highly useful in many different areas, such as algebraic geometry, algebraic topology, and others. Though defined in 1954 by Cartan, only few discussions were done on the pure ring theory of this class of algebras. Orlov [3,4] studied in two papers mainly geometrically motivated properties of again mainly finite dimensional differential graded algebras. Goodbody [1] proved a version of Nakayama's lemma for this kind of rings, using Orlov's approach. DG-semisimplicity was studied independently by Aldrich and Garcia-Rozas. We give an alternative systematic discussion, defining dg-simple in contrast to dg-semisimple differential graded algebras, different concepts of differential graded Jacobson radicals, differential graded prime ideals, differential graded nil radicals. We show that Ore localisation at homogeneous regular elements in differential graded rings satisfying the Ore condition has a natural differential graded structure extending the initial differential graded structure. We extend Goldie's theorem on localisation of semiprime rings with finite Goldie dimension to the differential graded situation. Goodearl and Stafford [2] showed that for graded rings the hypothesis of the ring to be a graded-prime ring is necessary for a graded Goldie theorem, and likewise we also need this kind of hypothesis.

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A FUNCTORIAL APPROACH TO RANK FUNCTIONS ON TRIANGULATED CATEGORIES

ALEXANDRA ZVONAREVA

A rank function on a small triangulated category $\mathcal{C}$ is an assignment of a non-negative real number to any object of $\mathcal{C}$, which satisfies certain natural axioms. Rank functions were recently introduced by Chuang and Lazarev. We study rank functions on $\mathcal{C}$ via its abelianisation mod$\mathcal{C}$ and prove that every rank function on $\mathcal{C}$ can be interpreted as an additive function on mod$\mathcal{C}$. As a consequence, every integral rank function has a unique decomposition into irreducible ones. Furthermore, integral rank functions are related to a number of important concepts in the functor category Mod$\mathcal{C}$. Based on joint work with Teresa Conde, Mikhail Gorsky, and Frederik Marks.