Motion Planning in Dynamic Environments Using the Relative Velocity Paradigm

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Abstract

This paper presents a simple and efficient approach to the computation of avoidance maneuvers among moving obstacles. The method is discussed for the case of a single maneuvering object avoiding several obstacles moving on known linear trajectories. The original dynamic problem is transformed into several static problems using the relative velocity between the maneuvering object and each obstacle. The static problems are then converted into a single problem by means of a vector transformation, then the set of velocity vectors guaranteeing the avoidance of all the obstacles is computed. Within this set, the best maneuver for the particular problem can be selected. The geometric background of this approach is developed for both 2-D and 3-D cases and the method is applied to an example of a 3-D avoidance maneuver.

1 Introduction

We consider the problem of computing an approximate avoidance maneuver for an object moving in a time-varying environment with both fixed and moving obstacles. The object for which the maneuver is computed is called the *object* and the other elements in the environment are referred to as *obstacles*. Both *object* and *obstacles* are modeled by *circles* in the plane and *spheres* in the space. We assume a deterministic environment in which fixed obstacles are at known positions and moving obstacles have known rectilinear trajectories with constant speeds. Given this simplified model, we wish to determine whether the *object* will collide with any of the *obstacles* before a given time. In case of a collision, a trajectory correction is computed that avoids the forecast collisions. The trajectory correction may consist of several maneuvers in which the *object*'s velocity vector may change in direction and/or modulus.

This approach transforms each moving *obstacle* into a fixed one by considering **the relative velocity** of the *object* with respect to that *obstacle*. We first map the *object* and the *obstacles* to the the *object's Configuration Space* and then, using the relative velocity we add the sets of feasible velocities and build the **velocity obstacles** for the *object*. The tip of the *object*'s velocity vector must be outside the velocity obstacle to avoid the collision. This construction corresponds to taking a *snapshot* of the object *State Space* at a given time. Simple vector operations allow then to determine the set of *one-step* maneuvers that, from the current position of the object, will avoid the obstacles colliding with the object before a specified time. By iterating this elementary step, the modified trajectory from start to finish can be computed.

This representation allows to generate the graph in the state space between the object start and goal positions made of a sequence of one-step maneuvers. This graph, consisting of straight line segments with constant speed, generalizes the visibility graph [7, 8] to the case of moving obstacles. In the computation of the avoidance maneuvers, the dynamic constraints of the *object* are not considered directly, but they are replaced by constant bounds on the velocity and the turning angle. The correct, state dependent, actuator limits will be accounted for by an optimization algorithm that uses the avoidance maneuvers as its nominal trajectory [4]. In this paper we introduce the basic concepts of the relative velocity paradigm and their application to the computation of a single avoidance maneuver in both planar and spatial cases.

The need for studying the problem of automatic planning in dynamic environments arises from many real situations. Complex manufacturing tasks require the coordination of multiple robots and moving carriers. Similarly, the control of traffic in air, sea, and on land is reaching the limits of human controllers and can benefit from some automatic maneuvering capability. The problem of planning in a dynamic environment has been first studied in [10] where the authors analyzed the *asteroid avoidance* problem. A *space-time* approach is applied in [3] to the case of two moving robots. In [6] the planning problem is decomposed into two phases: avoidance of static obstacles, and velocity scheduling for the avoidance of moving obstacles. In [5] a *collision front* is used to represent the locus of the collision

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points between two objects and to compute a time minimal path. Relative velocity planning has been studied in [2] for the case of collision avoidance among aircraft.

2 The Relative Velocity Paradigm

The approach presented in this paper uses **relative velocity** to detect possible collisions among moving objects and to compute a maneuver satisfying a set of collision conditions. We are concerned only with collision avoidance and not with hitting moving targets. A geometric construction is used to determine whether there are collisions between the *object* and the *obstacles* and to compute a new velocity vector for the *object*.

Defining the velocities with respect to a common inertial frame allows to represent this dynamic environment in the *Velocity Space* defined as the *Tangent Bundle* $T\mathbf{R}^n$ to the Euclidean Space $\mathbf{R}^n$ ($n = 2, 3$) [1]. In these cases, $T\mathbf{R}^n$ can be effectively represented on $\mathbf{R}^n$ itself to visualize position and velocity at each point of a trajectory, as shown in figure 1 for two objects on the plane. The four dimensional state space of objects **A** and **B** in figure 1 is visualized by attaching the velocity vector to their centers. The motion of an object in the velocity space is then defined as

$$\mathbf{p}(t) = (\mathbf{x}_p(t), \mathbf{v}_p(t))$$

where $\mathbf{x}_p(t)$ are the point's coordinates and $\mathbf{v}_p(t)$ is its velocity vector. In the following discussion, we will assume that the time is frozen at t_0 and that the *object*'s velocity is constant to t_0^- and may change at $t_0{}^+$ to avoid a collision.

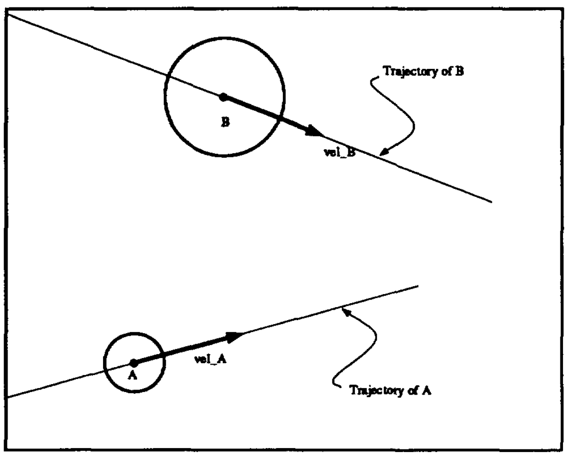

Figure 1: Scenario for the Planar case

We denote the *object* with $\mathbf{obj}_0$ and the *obstacles* with $\mathbf{obs}_i$. Using the concept of *Configuration Space* obstacle for $\mathbf{obj}_0$ due to $\mathbf{obs}_i$ [9], in this simple environment we can reduce $\mathbf{obj}_0$ to a point and grow $\mathbf{obs}_i$ by $\mathbf{obj}_0$'s radius.

For every pair $\mathbf{obj}_0$ and $\mathbf{obs}_i$, we define the **relative velocity** $\mathbf{v}_{0,i}$ of $\mathbf{obj}_0$ with respect to $\mathbf{obs}_i$

$$\mathbf{v}_{0,i} = \mathbf{v}_0 - \mathbf{v}_i \quad i = 1, \ldots, m \tag{1}$$

where m is the number of obstacles. The relative velocity transforms the single dynamic planning problem into m static problems. In fact, given $\mathbf{obj}_0$ at position $\mathbf{x}_0$ with constant velocity $\mathbf{v}_0$, the computation of a maneuver that avoids obstacle $\mathbf{obs}_i$ at position $\mathbf{x}_i(t_0)$ with velocity $\mathbf{v}_i$, is equivalent to the computation of the avoidance of the static obstacle $\mathbf{obs}_i$ by $\mathbf{obj}_0$ moving at the relative velocity $\mathbf{v}_{0,i}$.

If we define the **relative trajectory** $trj_{0,i}$ of $\mathbf{obj}_0$ with respect to $\mathbf{obs}_i$ as:

$$trj_{0,i} = \{(\mathbf{x}, \dot{\mathbf{x}}) \mid (\dot{\mathbf{x}}(t_0) = \mathbf{v}_{0,i}, \mathbf{x}(t_0) = \mathbf{x}_0\} \tag{2}$$

then there will be a collision between $\mathbf{obj}_0$ and $\mathbf{obs}_i$ if $\mathbf{v}_{0,i}$ does not change and if

$$trj_{0,i} \cap \mathbf{obs}_i \neq \emptyset \tag{3}$$

The set of velocities $\mathbf{v}_{0,i}$ for which (3) is satisfied defines the *Relative Collision Cone* $\mathbf{CC}_{0,i}$:

$$\mathbf{CC}_{0,i} = \{trj_{0,i} \mid trj_{0,i} \cap \mathbf{obs}_i \neq \emptyset\} \tag{4}$$

In figure 2 the relative collision cone $\mathbf{CC}_{A,B}$ is constructed for the case of two disks **A** and **B**. The cone is formed by the planar sector with apex in $\hat{\mathbf{A}}$ and limited by the two tangents from $\hat{\mathbf{A}}$ to $\hat{\mathbf{B}}$. Any relative velocity of **A**, $\mathbf{v}_{A,B}$, that remains within $\mathbf{CC}_{A,B}$ is guaranteed to cause a collision between **A** and **B**.

Since the underlying space is the velocity space, the relative collision cone represents both relative trajectories and relative velocities causing a collision. It defines *indirectly* the set of colliding $\mathbf{v}_A$. This set can be specified directly by defining the *Absolute Collision Cone* $\mathbf{CC}_0$ that identifies the set of absolute velocities $\mathbf{v}_0$ causing a collision between $\mathbf{obj}_0$ and $\mathbf{obs}_i$. The absolute collision cone is obtained by translating the relative collision cone of the vector $\mathbf{v}_{obs_i}$. Then $\mathbf{obj}_0$ will collide with $\mathbf{obs}_i$ when the tip of $\mathbf{v}_{obj_0}$ is inside $\mathbf{CC}_0$. Figure 3 shows both the relative (light grey) and the absolute (dark grey) collision cones for the planar case of disks **A** and **B**. There will be a collision between **A** and **B** unless $\mathbf{v}_A$ is changed, since the tip of vector $\mathbf{v}_A$ is inside the absolute collision cone $\mathbf{CC}_A$.

The avoidance maneuvers of $\mathbf{obj}_0$ consist of the absolute velocity $\mathbf{v}_0$ whose tip is outside the absolute collision cone $\mathbf{CC}_0(t_0)$, that is:

$$\begin{aligned} \mathbf{v}_0 &\in \overline{\mathbf{CC}_0(t_0)} \\ \mathbf{CC}_0(t_0) &= \{\mathbf{v}_0 \mid trj_{0,i} \in \mathbf{CC}_{0,i}(t_0)\} \end{aligned} \tag{5}$$

The absolute collision cone $\mathbf{CC}_0(t_0)$ can be thought of as the **Velocity Obstacle** $\mathbf{VO}_i$ for $\mathbf{obj}_0$ due to $\mathbf{obs}_i$. The velocity obstacle can be viewed as an extension of the concept of configuration space obstacle to the velocity space in that the trajectory of $\mathbf{obj}_0$ is collision free if the following two conditions hold:

1. $\mathbf{obj}_0$ is not within a configuration obstacle $\mathbf{CO}_i$, and

2. the tip of $\mathbf{v}_{obj_0}$ is not within a velocity obstacle $\mathbf{VO}_i$.

The absolute collision cone permits the computation of avoidance maneuvers of group of moving obstacles since each cone represents a set of colliding velocities for $\mathbf{obj}_0$. The set union of the absolute collision cones due to the obstacles in the environment is the **Multiple Velocity Obstacle MVO** for $\mathbf{obj}_0$ due to all the $\mathbf{obj}_i$'s.

In the following the collision cones are computed for two and three dimensions.

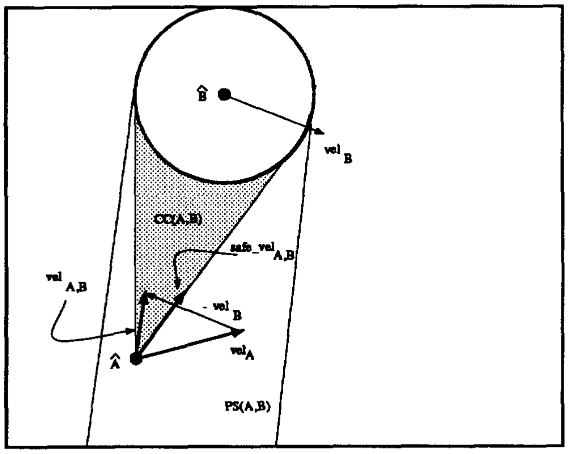

Figure 2: Geometry of the Planar Maneuver

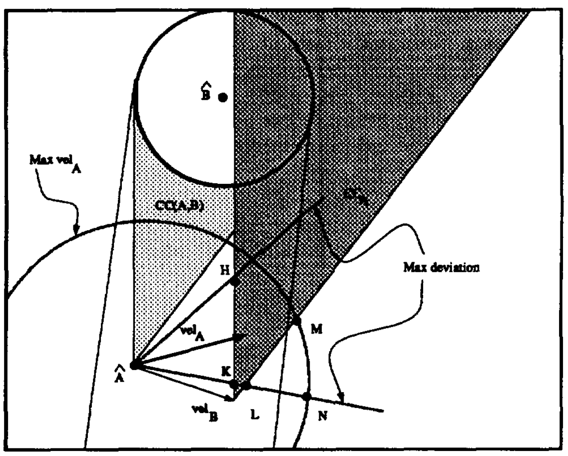

Figure 3: Construction of the Collision Sector CS_A

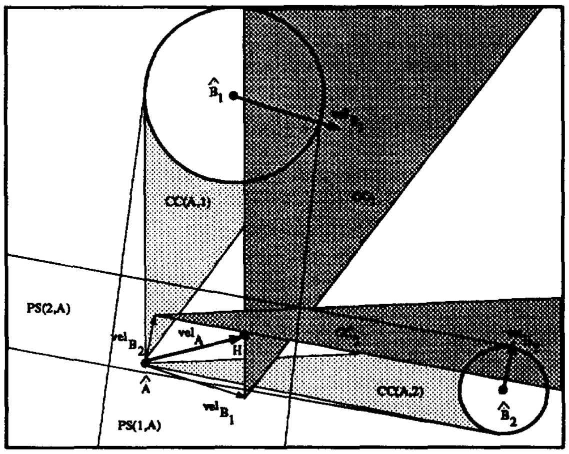

Figure 4: Avoidance of Multiple Obstacles

2.1 Planar Avoidance of Moving Disks

The simplest scenario for a planar avoidance maneuver consists of two disks **A** and **B** moving with arbitrary constant speeds on straight line trajectories as shown in figure 1. Disk **A** is the *object* and disk **B** is the *obstacle*.

We compute the *configuration space obstacle* of disk **A**, consisting of $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ of figure 2, and we determine whether there exists a potential collision between the object and the obstacles, only in this case we compute a trajectory correction. For this purpose, it is convenient to consider $\hat{\mathbf{A}}$ stationary and $\hat{\mathbf{B}}$ moving at the relative velocity $\mathbf{v}_{B,A}$:

$$\mathbf{v}_{B,A} = (\mathbf{v}_B - \mathbf{v}_A)$$

The area swept by $\hat{\mathbf{B}}$ consists of the planar stripe **PS**, as shown in figure 2:

$$\mathbf{PS}_{B,A} = \{\mathbf{x} \mid \mathbf{x} \in trj_{B,A}\}$$

A collision may occur between **A** and **B** if both keep the same velocities and if the point corresponding to $\hat{\mathbf{A}}$ is within the stripe **PS**:

$$\begin{aligned}&(\mathbf{B}(t) \cap \mathbf{A}(t) \neq \emptyset) \ \text{ iff} \\ &\quad (\mathbf{v}_A(t) = \mathbf{v}_A(t_0))\ \&\ (\mathbf{v}_B(t) = \mathbf{v}_B(t_0)) \\ &\quad \&\ (\hat{\mathbf{A}} \in \mathbf{PS}_{B,A})\end{aligned} \tag{6}$$

where $\mathbf{v}_{A,0}$ and $\mathbf{v}_{B,0}$ are the velocities at the time t_0 when the avoidance maneuver is computed.

The avoidance maneuver is computed using the velocity obstacle of **A** found by translating the relative collision cone $\mathbf{CC}_{A,B}$ by vector $\mathbf{v}_B$ to the absolute collision cone $\mathbf{CC}_A$. The relative collision cone is limited by the two tangents from $\hat{\mathbf{A}}$ to $\hat{\mathbf{B}}$.

A collision avoiding maneuver consists of a velocity $\mathbf{v}_A^*$ whose tip is outside the absolute collision cone $\mathbf{CC}_A$ and therefore, the corresponding relative velocity $\mathbf{v}_{A,B}^*$ will not satisfy the collision test of equation (3). We summarize this result in the following:

> *All absolute velocities of* $\mathbf{A}$, $\mathbf{v}_A(t_0^+)$, *guaranteeing the avoidance of* $\mathbf{B}$ *are included in the complement of the absolute collision cone* $\mathbf{CC}_A(t_0)$:

$$(\mathbf{B}(t) \cap \mathbf{A}(t) \neq \emptyset) \text{ iff } (\mathbf{v}_A(t_0^+) \in \overline{CC_A(t_0)})$$

Constraints on $\mathbf{v}_A$ can be easily added to the geometric construction in figure 3 and can further limit the set of non-colliding $\mathbf{v}_A$. Examples of such constraints in the velocity space are *maximum velocity* and *maximum turning angle* limits. In figure 3 the limit on the change in velocity of $\mathbf{A}$ is given by a circle centered at $\hat{\mathbf{A}}$. The maximum turning angle is the limit on the change in direction of $\mathbf{v}_A$ given by lines $\hat{\mathbf{A}}\mathbf{H}$ and $\hat{\mathbf{A}}\mathbf{K}$. The addition of the two limits reduces the safe velocity set to two disjoint subsets. The first subset consists of the velocities whose tip is within the triangle $\hat{\mathbf{A}}\mathbf{HK}$ and the second subset consists of the velocities whose tip is in the circular segment $\mathbf{LMN}$.

For the case of multiple obstacles, the absolute collision cones can be combined into a single velocity obstacle $\mathbf{MVO}$, and the safe velocity set is given by all the velocities whose tip is outside the union of all the absolute collision cones. The set of safe velocities is then defined as:

$$\{\mathbf{v}_A \in \overline{\mathbf{MVO}}\} \tag{7}$$

$$\mathbf{MVO} = \cup_{i=1}^{m} \mathbf{CC}_i \tag{8}$$

where $\mathbf{B}_i$ are the m obstacles.

Figure 4 shows obstacles $\mathbf{B}_1$ and $\mathbf{B}_2$, their relative collision cones $\mathbf{CC}_{A,1}$ and $\mathbf{CC}_{A,2}$, and their absolute collision cones $\mathbf{CC}_1$ and $\mathbf{CC}_2$. The absolute velocity represented by segment $\hat{\mathbf{A}}\mathbf{H}$ corresponds to a maneuver whose trajectory will be tangent to both obstacles $\mathbf{B}_1$ and $\mathbf{B}_2$.

2.2 Spatial Avoidance of Moving Spheres

In this section we extend the previous discussion to the case of collision avoidance among spheres. The basic scenario consists of two spheres $\mathbf{A}$ and $\mathbf{B}$ moving at constant speeds on straight line trajectories as shown in figure 5. We are interested in finding an avoidance maneuver for sphere $\mathbf{A}$.

The configuration space obstacle for $\mathbf{A}$ due to $\mathbf{B}$ is given by $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$. In general, avoidance maneuvers can be computed for any plane belonging to the bundle of planes intersecting at $\hat{\mathbf{A}}$. However, the visualization of the avoidance geometry is simplified if we consider only the maneuvers on the XY plane. In the following, the plane containing the avoidance maneuver is called the *maneuver plane*.

The collision test can be written as in equation (3):

$$(\mathbf{B}(t) \cap \mathbf{A}(t) \neq \emptyset) \text{ iff } \tag{9}$$

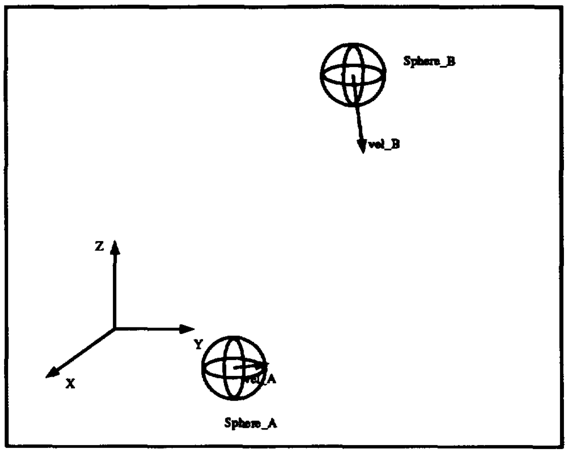

Figure 5: The Spatial Avoidance

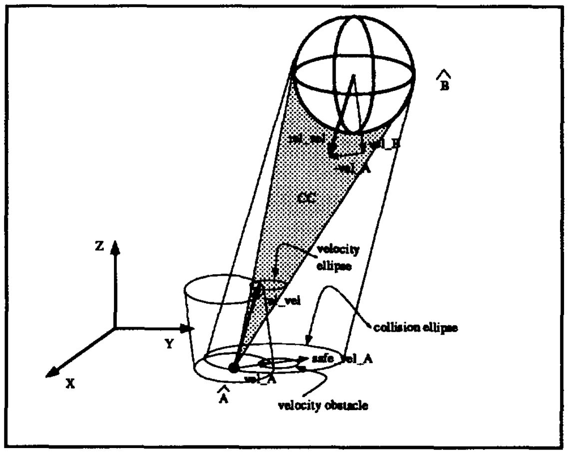

Figure 6: The Avoidance Maneuvers

$$(\mathbf{v}_A(t) = \mathbf{v}_A(t_0)) \;\&\; (\mathbf{v}_B(t) = \mathbf{v}_B(t_0)) \;\&\; (\hat{\mathbf{A}} \in \mathbf{SS}_{B,A})$$

where $SS_{B,A}$ is the volume swept by $\hat{\mathbf{B}}$ and consists of a cylinder with direction parallel to $\mathbf{v}_{B,A}$ and tangent to $\hat{\mathbf{B}}$ (figure 6).

The test can be simplified by considering the intersection of the cylinder with any plane of the bundle centered at $\hat{\mathbf{A}}$, XY in this case. In general, the intersection is a *collision ellipse* defined by:

$$\rho^2 = \frac{max\_axis^2 \; min\_axis^2}{max\_axis^2 \sin^2\theta + min\_axis^2 \cos^2\theta}$$

$$min\_axis = radius$$

$$max\_axis = \frac{radius}{\cos(tilt)} \tag{10}$$

where $0 \leq \theta \leq 2\pi$ measures the angle between the major axis of the ellipse and the reference x-axis, $\vec{\rho}^2$ is the square of the distance of a point of the ellipse from the center, *tilt* is the angle between the relative velocity and the plane, and *radius* is the radius obstacle.

The condition $(\hat{\mathbf{A}} \in \mathbf{SS}_{B,A})$ of equation 10 is equivalent to saying that $\hat{\mathbf{A}}$ is internal to the collision ellipse, that is:

$$\|\vec{AF}_1 + \vec{AF}_2\| \leq min\_axis + max\_axis \tag{11}$$

where F_1 and F_2 are the two foci of the ellipse, and $\vec{AF}_i$ are the vectors from $\hat{\mathbf{A}}$ to each focus as shown in figure 6. Here too, the collision is avoided if

$$\mathbf{v}_{A,B} \notin CC$$

where CC is the relative collision cone with apex in $\hat{\mathbf{A}}$ and tangent to $\hat{\mathbf{B}}$. The surface of the cone is the boundary between safe and collision velocities.

When the avoidance maneuver is limited to the maneuver plane, the velocity obstacle becomes a subset of the absolute collision cone and may be represented by a closed set on the maneuver plane. In this case, the velocity obstacle is smaller, which simplifies the selection of the avoidance velocity. Also in this case, we take advantage of the fact that the velocity space $T\mathbf{R}^3$ is represented on $\mathbf{R}^3$ itself and that point coordinates and velocity vectors can be represented in the same picture. The intersection of the relative collision cone with a plane parallel to the maneuver plane at a distance $-\mathbf{v}_B$ identifies the set of relative velocities $\mathbf{v}_{A,B} \in CC$ that can be formed by keeping $\mathbf{v}_B$ constant and varying $\mathbf{v}_A$. This set is the *velocity ellipse* of figure 6.

The *velocity obstacle* **VO** for **A** due to **B** relative to the chosen maneuver plane is computed by projecting the velocity ellipse onto the maneuver plane XY in the direction of $\mathbf{v}_B$ as shown in figure 6. The velocity obstacle permits to define the avoidance velocities on the maneuver plane as:

$$\mathbf{v}_A = \{\mathbf{v}_A \in \overline{\mathbf{VO}}\} \tag{12}$$

Figure also 6 shows two types of avoidance maneuver that can be easily computed from the velocity obstacle, with a constant direction or a constant speed. The first case is given by the intersections of the original path of **A** with **VO**: the two interseciton points are the lower and upper boundaries on the speed guaranteeing an avoidance without changing direction. The avoidance maneuvers at constant speed are given by the intersections of **VO** with a circle centered at $\hat{\mathbf{A}}$ and radius equal to $|\mathbf{v}_A|$. All other velocity vectors whose tip is outside the velocity obstacle **VO** correspond to maneuvers in the XY plane that require a change in both magnitude and direction.

In the case of several obstacles, the avoidance maneuver can be computed from the union of the single velocity obstacles, by defining the *multiple velocity obstacle* **MVO**:

$$\mathbf{MVO} = \cup_{i=1}^{m} \mathbf{VO}_i \tag{13}$$

and by chosing an avoidance velocity satisfying equation (8).

3 An Example of Avoidance Maneuver

In this example we demonstrate the use of relative velocity to select an avoidance maneuver in the 3D environment shown in figure 7. The object for which to plan the maneuver is *sphere0* and the moving obstacles are *sphere1-4*. The velocities of each object are shown as line segments positioned at the center of each object. They were chosen so that all objects are on a collision course with *sphere0*. If these velocities remain unchanged, then *sphere0* will collide with all the obstacles, as indicated in figure 8 by the darkened circles. Note that figures 8, 9, are orthogonal projections of the spheres on the XY plane. The objective then is to compute an avoidance maneuver consisting of a single change of direction of the velocity $\mathbf{v}_0$ of *sphere0* at constant speed in the XY plane.

The test for collision is carried out by computing the collision ellipses for each obstacle. To select the desired $\mathbf{v}_0 \in \overline{\mathbf{MVO}}$ we need to compute the geometric shape of **MVO**. Then the maneuver requiring the least change in direction corresponds to one of the intersections of the circle of radius $|\mathbf{v}_A|$ with **MVO**. The trajectory resulting from the change velocity of *sphere0* is shown in of figure 9 where it does not collide with any of the obstacles.

4 Conclusion

A new method has been presented for computing avoidance maneuvers of moving obstacles in two and three dimensional spaces. We converted the dynamic planning problem into a static problem by using the relative velocity of the object with respect to each obstacle. We then introduced the concept of *absolute collision cone* which allows to plan the avoidance of multiple obstacles moving at constant speeds. The absolute collision cone represents geometrically the collision region in the velocity space and can be tought of as a velocity obstacle. An avoidance maneuver is then planned simply by selecting a velocity vector that points outside of the obstacle collision cone. This representation allows for considering constraints related to system dynamics such as limits on the velocity and on its deviation angle. These constraints are easily superimposed on the absolute collision cone to limit the set of avoidance maneuvers.

This method can be used to develop a graph in the state space of the moving object from which a complete trajectory can be computed. Such a trajectory can be used as the initial guess to a dynamic optimization that takes into account the full system dynamics. The geometric nature of this approach makes it attractive for interactive applications, such as motion planning of multiple mobile robots and air traffic control.

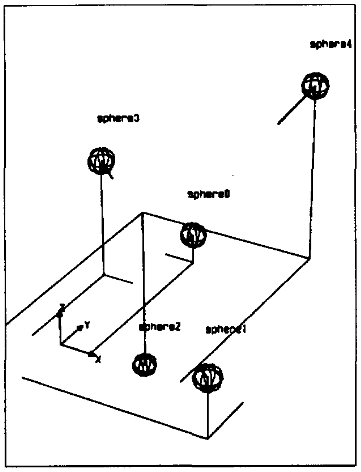

Figure 7: Example Perspective View

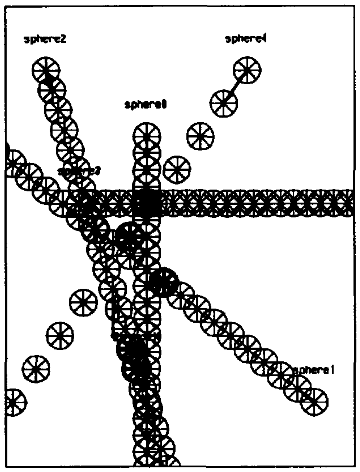

Figure 8: Original Scenario Simulation

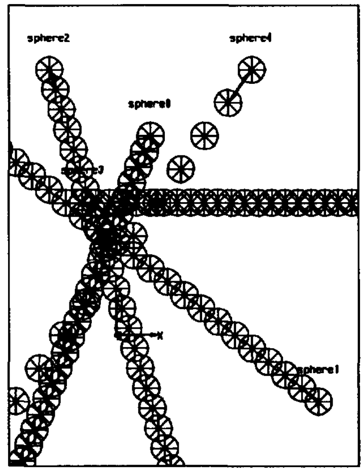

Figure 9: Avoidance Maneuver

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