Linear Methods for Regression: Subset Selection

Statistical learning – Part II

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Motivation

Possible **problems of Least Squares Estimation** (LSE):

- **Prediction accuracy**:
  - Low bias, large variance
  - Can sometimes be improved by shrinking. Sacrifice a bit of bias but reduce variance

- **Interpretation:**
  - Identification of a small subset of variables with the strongest effect

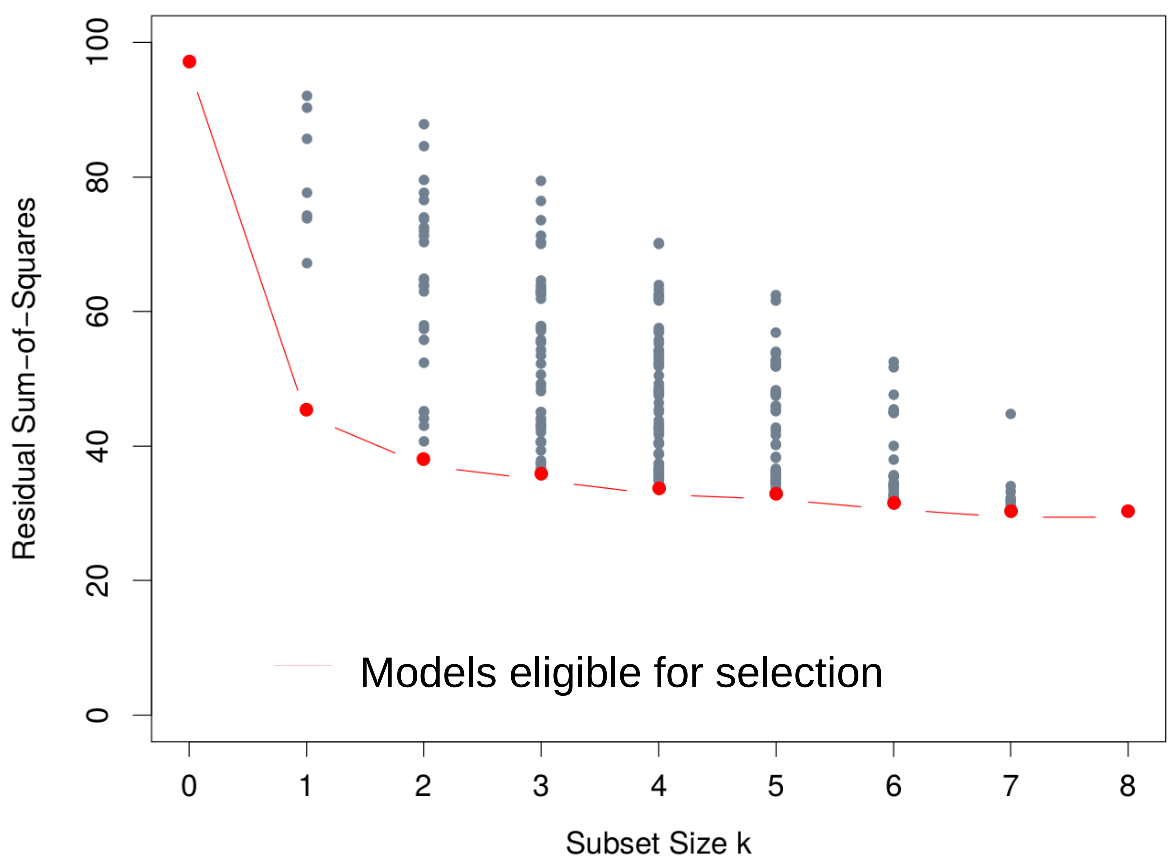
Solution: **Model selection**

- Here we describe different **strategies** to **variable subset selection** with **linear regression.**

- In next lectures **shrinkage** and **dimension-reduction** approaches for controlling variance.

Best-subset selection

- It finds for each **k={0,1,2,…,p}** the subset of size *k* that gives **smaller Residual Sum of Squares (RSS).**
- **Leaps and bounds** procedure (Furnival and Wilson, 1974): feasible for p as large as 30 or 40.
- RSSs of **all subset models** for the prostate cancer example:



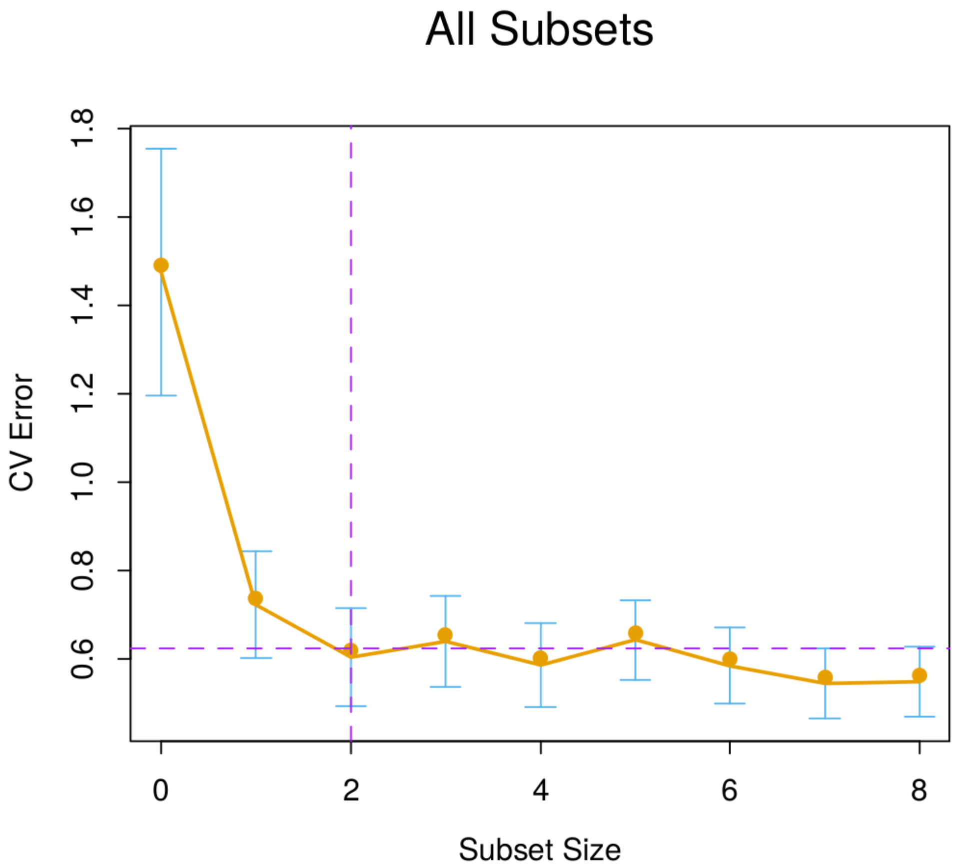
- Best subset of size 2 need not include the variables in the best subset of size 1
- **Best-subset curve** is necessary **decreasing**. It cannot be used to select the subset size k

General idea of best-subset selection

Use the training data to produce a **sequence of models** varying in **complexity** and indexed by a single parameter.

- **Cross-validation** and the **AIC** criterion (presented in next lectures) can be used to estimate the **best parameter k**.

Term	LS	Best Subset
Intercept	2.465	2.477
lcavol	0.680	0.740
lweight	0.263	0.316
age	−0.141	
lbph	0.210	
svi	0.305	
lcp	−0.288	
gleason	−0.021	
pgg45	0.267	
Test Error	0.521	0.492
Std Error	0.179	0.143



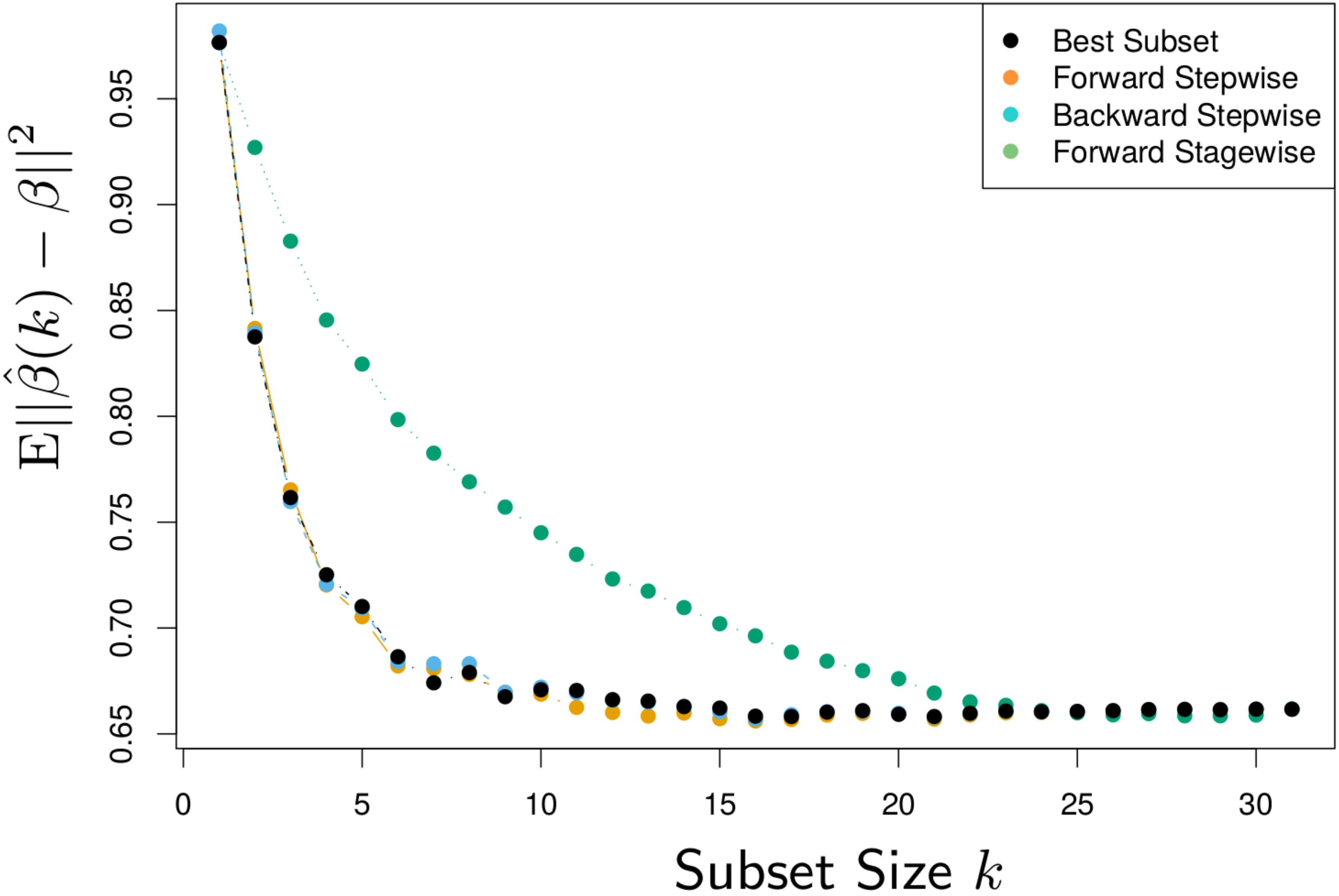
Forward-Stepwise Selection

- Search all possible subsets is **infeasible for large p**, hence we seek a **good path** through them.
- **Forward-stepwise selection:**
  - starts with the **intercept**
  - sequentially **adds** into the model the predictor that **most improve** the fit (e.g., RSS)
  - produces a sequence of models indexed by k, the subset size
  - is a **greedy algorithm**, producing a **nested sequence of models**
  - is **suboptimal** compared to best-subset selection
  - is applicable with **large p**
  - has **lower variance** but perhaps **higher bias** than best-subset selection

Backward-Stepwise Selection

- **Backward-stepwise selection:**
  - starts with the **full model**
  - sequentially **deletes** the predictor that has the **least impact** on the fit (e.g., RSS)
  - the **candidate** for dropping is the variable with the **smallest Z-score**
  - can only be used when N > p
  - produces a sequence of models indexed by k, the subset size
  - is a **greedy algorithm**, producing a **nested sequence of models**
  - is **suboptimal** compared to best-subset selection
  - is applicable with **large p**
  - has **lower variance** but perhaps **higher bias**

Comparison



On the prostate cancer example, best-subset, forward and backward selection all gave exactly the same sequence of terms.

Hybrid stepwise selection strategies

- Hybrid stepwise-selection strategies consider **both forward and backward moves** at each step, and **select the “best” of the two**.

- The R function called *step* uses the Akaike (**AIC**) criterion for weighting the choices, i.e., at each step an add or drop is performed that minimizes the AIC score.

- Notice that **standard errors** of coefficients in non-full models are not valid since they do not account for the search process.

  - **Bootstrap** techniques (presented in next lectures) can be used to solve this problem

Exercise: Prediction on the prostate cancer dataset

See text of Exercise 3

References

[Hastie 2009] Trevor Hastie, Robert Tibshirani, Jerome Friedman. The Elements of Statistical Learning: Data Mining, Inference, and Prediction (second edition). Springer. 2009.