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Registration of Multiple Acoustic Range Views for Underwater Scene Reconstruction

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This paper proposes a technique for the three-dimensional reconstruction of an underwater environment from multiple acoustic range views acquired by a remotely operated vehicle. The problem is made challenging by the very noisy nature of the data, the low resolution, and the narrow field of view. Our main contribution is a new global registration technique to distribute registration errors evenly across all views. Our approach does not use data points after the first pairwise registration, for it works only on the transformations. Therefore, it is fast and occupies only a small amount of memory. Experimental results suggest the global registration technique is effective in equalizing the error. Moreover, we introduce a statistically sound thresholding (the X84 rejection rule) to improve ICP robustness against noise and nonoverlapping data.

Key Words: multiple views; registration; ICP; 3D reconstruction; acoustic imaging.

1. INTRODUCTION

In this paper we address the problem of the registration of many 3D point sets, coming from an acoustic range sensor. Typically, the term *registration* is used for the geometric alignment of a pair or more 3D data point sets, while the term *fusion* is used when one wants to get a single surface representation from registered 3D data sets.

Our data come from a high frequency acoustic camera, called Echoscope [1], with a typical resolution of 3 cm at 500 KHz. Speckle noise is typically present due to the coherent nature of the acoustic signals. The final goal is to provide a 3D scene model to the human operator(s) of an underwater Remotely Operated Vehicle (ROV), in order to facilitate the navigation and the understanding of the surrounding environment.

The registration of two point sets is usually performed by the Iterative Closest Point (ICP) procedure [2, 3]. ICP assumes that one point set is a subset of the other; when this assumption does not hold, false matches are created that negatively influence the convergence of the

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ICP to the solution. In order to overcome this problem, many variants to ICP have been proposed, including the search of closest points in the direction of the local surface normal [3], the use of thresholds to limit the maximum distance between points [4], disallowing matching on the surface boundaries [5], and the use of robust regression [6, 7]. In this paper we use the X84 outlier rejection rule [8] to discard false correspondences. This is an improvement over [4], because there are no free parameters and because it achieves a larger basin of attraction.

A widely used approach to the registration of many views is to sequentially apply pairwise registration until all the views are combined. Chen and Medioni [3], for instance, proposed an incremental approach in which two views are registered and merged, building a *metaview*. The next view is then registered and merged with the metaview and the process is repeated for all the views. A similar approach was taken also by Masuda [9]. Jin *et al.* [10] proposed to incrementally build a surface model onto which new views can be registered and already registered views can be adjusted.

These schemes do not compute the optimal solution, because of the accumulation of registration errors, as pointed out by [11] and [12]. They do not use all the available information. Multiview registration, instead, must exploit the information present in the unused overlapping view pairs, distributing the alignment error evenly between every pairwise registration. Bergevin *et al.* [12] registered multiple range images simultaneously, using an extended ICP algorithm. They converted the sequential registration relationship into a star-shaped relationship and then imposed the *well-balanced network* constraint. A network of range views is well balanced when the registration error is similar for all transformation matrices and the transformation matrix between any two views is uniquely defined regardless of the path chosen to link the views. Pulli [13] proposed to use the pairwise alignments as constraints that the multiview step enforces while evenly diffusing the pairwise registration errors. In such a way, computational time is reduced as well as memory storage. He introduces the concept of a *virtual mate* to enforce the pairwise alignments as constraints. Eggert *et al.* [14] use a force-based optimization in which incremental pose adjustments are computed simultaneously for all point sets, resulting in a globally optimal set of transformations. In [15], couples of range images are incrementally registered together with a final registration between the first and the last view, by using the inverse calibration procedure of the range-finder.

Some works focus on computing the global registration given the correspondences (i.e., the N-view point set registration problem). In [16], a force-based optimization approach is proposed. Assuming the points' correspondences among the data sets are known, interconnections using springs between corresponding points is simulated. Pennec [17] introduces an iterative algorithm based on the concept of *mean shape*. Benjemaa and Schmitt [11] use a quaternion approach similar to [18]. These techniques have been compared in [19], and the result is that, not considering speed, Pennec's method is the best one, whereas [11] is the fastest. In a recent work, Williams and Bennamoun [20] proposed a new technique in which rotations are first computed iteratively, and then translations are obtained as the solution of a linear system. The method has been integrated into a generalized multiview ICP.

All the multiview alignment methods need to keep data of all—or at least some—views in memory at the same time, reducing drastically performance, especially when aligning large data sets. Our global registration approach differs from all the others because we enforce the constraints arising from the pairwise registration directly on the transformation matrices, without the need to go over data points again, after the initial pairwise registration between all the overlapping views. The idea comes from [21] where it was applied to the construction of planar mosaics from video (2D) images. Here we propose to extend the technique to the registration of multiple 3D point sets. In our case we end up with a nonlinear system of equations (because of the parameterization of the rotations) that we solve with the Gauss–Newton algorithm. In the context of medical imaging Roche *et al.* [22] proposed a similar method for the registration of 3D ultrasound images and magnetic resonance images. This technique differs from ours in the formulation of the objective function and in the representation of rotations. Following [11, 18, 23] we used quaternions to represent rotations, because of their well-known good properties [24]. In the field of 3D registration, the closest work to our is [13], because both are based on the simultaneous satisfaction of constraints provided by the pairwise registration, and neither relies on the solution of the N-view point set registration problem. Our work differs in the formulation of the constraints which do not involve data points.

We would like to stress that none of the works on multiple views registration present in the literature deal with the particular kind of 3D data we are using. In fact, (i) the resolution is never better than some centimeters, unlike classical range data; and (ii) the motion of the sensor is quite unstable and cannot be controlled with precision, so acquired images from a fixed position may be different due to speckle noise and sensor floating.

2. ROBUST PAIRWISE REGISTRATION

Pairwise registration was addressed using the classical ICP algorithm [2] to which we added an outlier rejection rule (called X84) in order to cater to nonoverlapping areas between views.

2.1. Two View Point Set Registration

Let us suppose that we have two sets of 3D points, V^i and V^j, which correspond to a single shape but are expressed in different reference frames. The registration consists in finding a 3D transformation which, when applied to V^j, minimizes the distance between the two point sets. In general, point correspondences are unknown.

For each point $\mathbf{y}_i$ from the set V^j, there exists at least one point on the surface of V^i that is closer to $\mathbf{y}_i$ than all the other points in V^i. This is the *closest point*, $\mathbf{x}_i$. The basic idea behind the ICP algorithm is that, under certain conditions, closest points are a reasonable approximation to the true point correspondences. The ICP algorithm can be summarized as follows:

(1) For each point in V^j, compute the closest point in V^i;

(2) With the correspondence from step 1, compute the incremental transformation $(\mathbf{R}^{i,j}, \mathbf{t}^{i,j})$;

(3) Apply the incremental transformation from step 2 to the set V^j;

(4) If the change in total mean square error is less than a threshold, terminate. Else goto step 1.

Besl and McKay [2] proved that this algorithm is guaranteed to converge monotonically to a local minimum of the mean square error. As for step 2, efficient, noniterative solutions to this problem (known as the *point set registration problem*) were compared in [25], and the one based on singular value decomposition was found to be the best in terms of accuracy and stability.

ICP can give very accurate results when one set is a subset of the other, but results deteriorate with outliers, created by nonoverlapping areas between views. In this case, the overlapping surface portions must start very close to each other to ensure convergence, making the initial position a critical parameter.

Modifications to the original ICP have been proposed to achieve accurate registration of partially overlapping point sets [4–7]. We implemented a variation similar to the one proposed by Zhang [4] using outlier diagnostics to limit the maximum allowable distance between closest points.

2.2. Robust Outlier Rejection

As pointed out by Zhang, the distribution of the residuals for two fully overlapping sets approximates a Gaussian, when the registration is good. The nonoverlapped points skew this distribution: they are *outliers*. Therefore, good correspondences can be discriminated by applying outlier diagnostics on the distribution of closest point distances ϵ. To this end, we employ a simple but effective model-free rejection rule, called X84 [8], which uses robust estimates for location and scale (i.e., the spread of the distribution) to set a rejection threshold. The median is a robust location estimator, and the Median Absolute Deviation (MAD), defined as

$$\mathrm{MAD} = \operatorname*{med}_i \left\{ \left| \epsilon_i - \operatorname*{med}_j \epsilon_j \right| \right\}, \tag{1}$$

is a robust estimator of the scale. The X84 rule prescribes to reject values that are more than k MADs away from the median. Under the hypothesis of Gaussian distribution, a value of $k = 5.2$ is adequate in practice, as the resulting threshold contains more than 99.9% of the distribution.

The X84 rejection rule has a breakdown point of 50%: any majority of the data can overrule any minority. The computational cost of X84 is dominated by the cost of the median, which is $O(N)$, where N is the size of the data point set. The most costly procedure inside ICP is the establishment of point correspondence, which costs $O(N \log N)$. Therefore X84 does not increase the asymptotic complexity of ICP.

3. MULTIVIEW REGISTRATION

Assume that there are M overlapping point sets (or views) $V^1 \dots V^M$, each taken from a different viewpoint. The objective is to find the best rigid transformations $\mathbf{G}^1 \dots \mathbf{G}^M$ to apply to each set, bringing them into a common reference frame where they are seamlessly aligned. Let $\mathbf{G}^{i,j}$ be the rigid transformation matrix (in homogeneous coordinates) that registers view j onto view i, i.e.,

$$V^i = \mathbf{G}^{i,j} V^j, \tag{2}$$

where the equality holds only for the overlapping portions of the two points sets V^i and $\mathbf{G}^{i,j} V^j$. If we choose (arbitrarily) view k as the reference one, then the unknown rigid transformations $\mathbf{G}^1 \dots \mathbf{G}^M$ are respectively $\mathbf{G}^{k,1} \dots \mathbf{G}^{k,M}$. As customary, we will take $k = 1$.

These rigid transformations are not independent of each other, being linked by a composition relationship. We can therefore estimate the alignment $\mathbf{G}^j$ of image V^j on the reference view (defined by the image V^1), by first registering V^j onto any view V^i and then using $\mathbf{G}^i$ to map the result into the space of V^1:

$$\mathbf{G}^j = \mathbf{G}^i \mathbf{G}^{i,j}. \tag{3}$$

This relationship can be used to compute $\mathbf{G}^i$ when all the matrices $\mathbf{G}^{i-1,i} \ldots \mathbf{G}^{1,2}$ are known, by simply chaining them:

$$\mathbf{G}^i = \prod_{j=2}^{i} \mathbf{G}^{j-1,j}. \tag{4}$$

The global registration matrix $\mathbf{G}^i$ will map V^i into the space of V^1 (the reference view).

As is well known, the combination of pairwise registration does not yield the optimal result. For example, if $\mathbf{G}^{k,i}$ and $\mathbf{G}^{i,j}$ are optimal in the sense that they minimize the mean square error distance between the respective sets, then $\mathbf{G}^{k,j}$ computed by composition does not necessarily minimize the mean square error between views V^j and V^k. Moreover, small registration errors accumulate so that views near the end of a sequence have a large cumulative error.

3.1. *Global Transformations Adjustment*

In order to improve the quality of global registration, let us suppose we have locally registered all spatially overlapping view pairs, in addition to those that are adjacent in the image sequence. As the ROV moves back and forth, we can obtain good alignment also between distant views in the temporal sequence.

The aim of the proposed method is to optimize the information coming from every pairwise registration, obtained by the alignment of all the overlapped range images. The innovative contribution consists in obtaining a global registration by introducing algebraic constraints on the *transformations*, instead of data points.

We first perform pairwise registration between every view and each of its overlapping views, thereby computing the $\mathbf{G}^{i,j}$ matrices whenever it is possible. By considering many equations as (3), we can build a system of equations in which the $\mathbf{G}^{i,j}$ are known quantities obtained by pairwise image registration, and the matrices $\mathbf{G}^{1,i} = \mathbf{G}^i$ $(2 \leq i \leq M)$ are the sought unknowns. By decomposing the homogeneous transformation matrices into rotation and translation, as $\mathbf{G} = [\begin{smallmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{smallmatrix}]$, Eq. (3) becomes:

$$\begin{cases} \mathbf{R}^j = \mathbf{R}^i \mathbf{R}^{i,j} \\ \mathbf{t}^j = \mathbf{R}^i \mathbf{t}^{i,j} + \mathbf{t}^i, \end{cases} \tag{5}$$

where $\mathbf{R}$ is a rotation matrix and $\mathbf{t}$ is a translation vector. Although this system of equations is essentially linear, a number of problems arise when formulating solutions that account for the nonlinear constraints on the components of $\mathbf{R}$. In order to respect these constraints, the rotation matrices must be suitably parameterized, ending up with a system of nonlinear equations.

This nonlinear least squares problem can be cast as the minimization of the following objective function

$$\min \sum_{i,j} ((\text{angle}(\mathbf{R}^i \mathbf{R}^{i,j} (\mathbf{R}^j)^\top)/\sigma_\alpha)^2 + (\|\mathbf{R}^i \mathbf{t}^{i,j} + \mathbf{t}^i - \mathbf{t}^j\|/\sigma_t)^2), \tag{6}$$

where angle$(\cdot)$ is an operator that takes a rotation matrix and returns the angle of rotation,[2] and σ_α and σ_t are normalization factors. Starting from the global registration obtained by chaining pairwise transformation (Eq. (4)), a least squares solution is iteratively sought, using a standard Gauss–Newton algorithm.

The estimated transformations $\mathbf{G}^2 \ldots \mathbf{G}^M$ are influenced by all the measured pairwise transformations, and the registration error is distributed over all the estimated transformations. In this sense, the final registration graph is very close to a well-balanced graph as defined in [12]. As the objective function includes only the matrix components, the complexity of the proposed algorithm is independent on the number of points involved and depends only on the number of available pairwise registrations.

3.2. *Dealing with Rotations*

One of the most convenient ways to represent rotations are quaternions. They have a number of mathematical properties that make them particularly well suited to the requirements of an iterative gradient-based search for rotation and translation [24]. Rotations are represented by *unit* quaternions. Instead of requiring the quaternion $\mathbf{q} = [u, v, w, s]$ to be a unit vector, following [24], we enforce the constraint that the rotation matrix is orthonormal by dividing the matrix by the squared length of the quaternion

$$\mathbf{R}(\mathbf{q}) = \frac{1}{\mathbf{q} \cdot \bar{\mathbf{q}}} \mathbf{R}_u(\mathbf{q}), \tag{7}$$

where $\mathbf{R}_u(\mathbf{q})$ is the rotation matrix associated to the quaternion. This constraint is necessary in general to ensure the gradient accurately reflects the differential properties of a change in the quaternion parameters.

3.3. *Summary of the Algorithm*

Step 1. For each overlapping view pair, compute the pairwise registration matrix $\mathbf{G}^{i,j}$, using ICP + X84. Accept $\mathbf{G}^{i,j}$ only if the final registration error is below a certain threshold;

Step 2. Compute a starting guess for each $\mathbf{G}^i$ by chaining pairwise transformations as in Eq. (4);

Step 3. Solve the system of equations defined in Eq. (5) with Gauss–Newton.[3] At each step enforce orthogonality of the rotation matrix with Eq. (7);

Step 4. Apply the transformations $\mathbf{G}^i$ to the view V^i, $i = 2, \ldots, M$.

Registered sets of points must be fused in order to get a single 3D model. Surface reconstruction from multiple range images can be addressed as the problem of surface reconstruction from a set of unorganized 3D points, disregarding the original 2.5D nature of the

[2] Any (nonzero) rotation in 3D space has a unique representation as a rotation angle about an (oriented) axis.

[3] We used the MATLAB `lsqnonlin` function, which implements a quasi-Newton method with a mixed quadratic and cubic line search procedure, with numerical Jacobian.

data. We used the algorithm by Hoppe *et al.* [26], for which a public domain implementation exists.

4. RESULTS DESCRIPTION

In synthetic experiments we simulated the movement of an underwater ROV around the external part of an offshore rig using the OpenGL library to generate synthetic range images. Given a 3D model of part of the rig, range images were obtained by moving the (virtual) camera and extracting the *z-buffer* for each view. In order to asses the final registration, we made the last view to coincide with the first one.

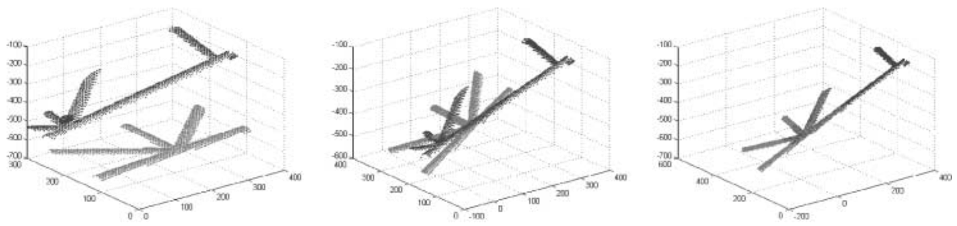
In Fig. 1, we show an example of two point sets that Zhang's ICP fails to align. Instead, our ICP algorithm with X84 rejection rule recovers the correct rigid transformation.

As for the global registration, in order to evaluate the performance of our technique, we computed the *registration error* of a view as the mean square distance between its points and their closest points in the mosaic composed by all the already registered views (outliers were discarded according to the X84 rule). The improvement over the chained pairwise alignment is shown as a histogram depicting, for each view, the difference between the registration errors of the two techniques (a positive value means an improvement of our method).

Experiment 1 consists of a synthetic sequence of 29 range images. The benefit brought by the global registration can be appraised in Figs. 2a and 2b. The histogram in Fig. 2c shows that the global registration improves especially near the end of the sequence (as expected).

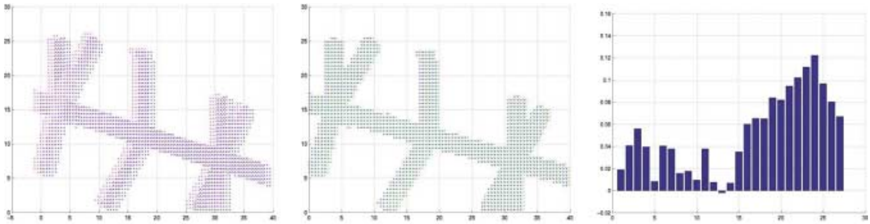
In Experiment 2, we generated a sequence composed by 37 range images. We wanted to test the performance of the global registration algorithm in the presence of an incorrect pairwise registration (view 35). In this case the chaining of pairwise transformations inevitably propagates the error. In our global multiview registration, thanks to the information coming from the other pairwise transformations linking (indirectly) views 35 and 34, the correct registration is achieved, and the error is distributed over the whole sequence. Figure 3c shows the improvement obtained by the optimal global registration, which is concentrated on view 35, as expected. The benefit brought by the global registration is also clearly visible in Figs. 3a, and 3b and also in Fig. 4 where the reconstructed surfaces are shown for both techniques.

In Experiments 3 and 4 we added Gaussian white noise with different standard deviation ($\sigma = 0.02$ and $\sigma = 0.045$, respectively) to the synthetic images of Experiment 1. The relative histograms are shown in Fig. 5.



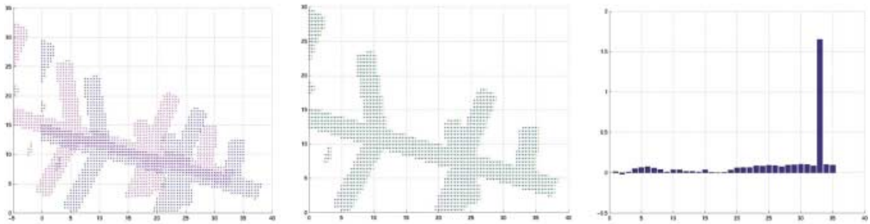
(a) (b) (c)

FIG. 1. In (a) the two point sets are in the start position, (b) shows the result of Zhang's ICP algorithm, and (c) shows the result of ICP + X84.



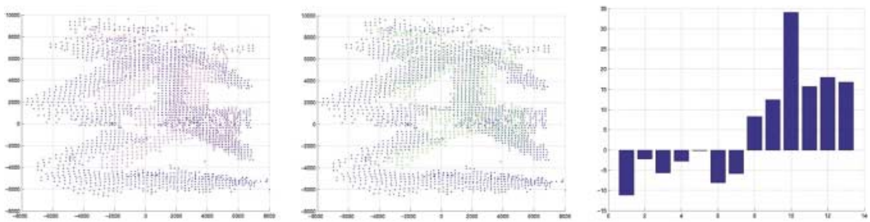
(a) (b) (c)

FIG. 2. Experiment 1. Alignment between view 1 and view 29 for chained pairwise registration (a) and global registration (b). Histogram of the differences of the registration error (c). A positive value corresponds to an improvement.



(a) (b) (c)

FIG. 3. Experiment 2. Alignment between view 1 and view 37 for chained pairwise registration (a) and global registration (b). Histogram of the differences of the registration errors (c).



(a) (b) (c)

FIG. 6. Experiment 5. Alignment between first view and last view for chained pairwise registration (a) and global registration (b). Histogram of the differences of the registration errors (c).

TABLE 1
Average registration errors

Experiment	Chained pairwise registration	Global registration	% difference
1	0.24095	0.19258	20.1
2	0.28960	0.19630	32.2
3	0.36328	0.32936	9.3
4	0.50290	0.47200	6.1
5	15.47955	15.01574	3.0

Note. The synthetic and real images are not the same scale.

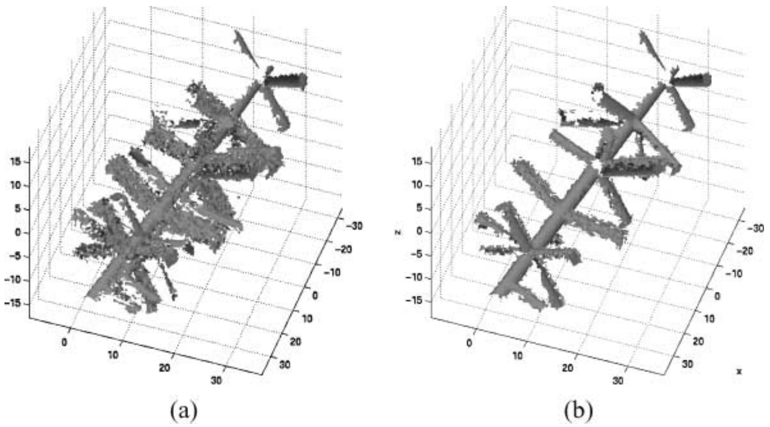


FIG. 4. Surface reconstruction using Hoppe and DeRose algorithm. Chained pairwise registration (a) and global multiview registration (b).

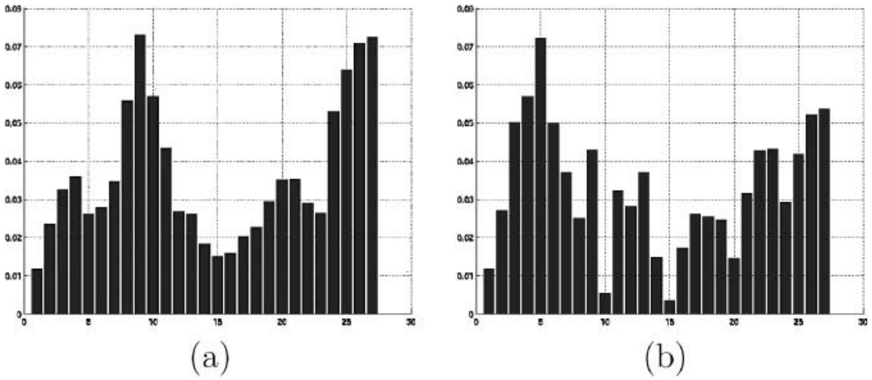


FIG. 5. Histogram of the differences between the registration errors for the chained pairwise registration and the global registration, in Experiments 3(a) and 4(b).

TABLE 2
Misalignment between the last and the first view (cm)

Experiment	Chained pairwise registration	Global registration	% difference
1	1.9584	0.1340	93.2
2	29.8662	0.1362	99.5
3	4.0279	2.1979	45.4
4	13.8420	11.1094	19.7

Real acoustic data were acquired by an underwater ROV using the Echoscope camera [1], which outputs a 64×64 range image. The noise corrupts the acoustic signals and decreases the reliability of the estimated 3D measures. Resolution depends on the frequency of the acoustic signal (it is about 3 cm at 500 KHz): the higher the frequency, the higher the resolution, and the narrower the field of view.

In Experiment 5 we used a sequence of 15 real acoustic range images that are partial views of a tubular structure. Figures 6a and 6b show the overlay of the first and last views for both pairwise registration and global multiview registration. Even if the images are rather noisy and quite difficult to understand, it can still be noticed that our technique yields a better alignment. The histogram shown in Fig. 6c confirms this improvement. The light worsening at the beginning of the sequence is compensated by the good improvement near the end. A more accurate evaluation is not possible in the real case because true correspondences (as in synthetic experiments) are not known.

Tables 1 and 2 summarize the numerical results obtained in all the experiments. Table 1 reports the average (over the views) registration errors for both algorithms. In Table 2 a more meaningful evaluation is obtained by calculating the registration error (misalignment) between the first and the last view (which should coincide), knowing the correct point correspondences.

Our global multiview registration algorithm always improves over pairwise registration. When the noise level was increased in the experiments on synthetic data, our algorithm continued to perform better. Improvements were also seen in the experiment involving real data. The typical computing time in these experiments was 25 s for each pairwise registration and about 3 min for the subsequent global optimization. The code was written in MATLAB and run on a PII 350 MHz PC with Linux.

5. CONCLUSIONS

In this paper we propose a technique for 3D object reconstruction from multiple acoustic range views, acquired by an underwater acoustic sensor. As data are noisy and of low resolution, and the field of view is narrow, we want to provide the human operator(s) with a synthetic 3D model of the scene, in order to facilitate the navigation and the understanding of the surrounding environment. To this end, we address the problem of registering many 3D views, starting from pairwise registration between all the overlapping views.

Our contribution is twofold. First we modified Zhang's ICP by introducing the X84 rejection rule, which does not depend on user-specified thresholds and is more effective in achieving a larger convergence basin. Moreover, we proposed a new global multiview registration technique to distribute registration errors evenly across all views. Our approach differs from all the others because we enforce the constraints arising from the pairwise registration directly on the transformations, and we do not rely on the solution of the N-view point set registration problem. The complexity of our technique does not depend on the number of points involved, but only on the number of views.

The drawback is that the error is only spread among the views, but does not get reduced significantly. Consequently, this technique is well suited for all the applications where speed can be traded for accuracy.

Future work will be aimed at automatically detecting the degree of overlap between views and introducing a weight for each term of Eq. (6), depending on the amount of overlap. Moreover, we are starting to convert the software in C++ and to make some optimization to the ICP implementation. At the end of the process we expect a speed-up of a factor 20 when running on a state-of-the-art computer.

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