Interpolation for McCain-Turner Causal Theories

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Abstract

We give a modal presentation of the McCain and Turner's "causal theories"; we show how to formalise, in this framework, Foo and Zhang's interpolation argument [Zhang and Foo, 2002].

1 Introduction

Consider McCain and Turner's theory of "causal reasoning" [McCain and Turner, 1997]; this starts from a collection of *causal laws* (written $\phi \rhd \psi$), and defines a logical consequence relation as follows:

1. Suppose that we are given a set of causal laws: call it $\mathbb{T}$, formulated in some language $\mathfrak{L}$. Let M be a model of $\mathfrak{L}$. Given a model M of our language, define a theory as follows:
$$\mathbb{T}^M \stackrel{\text{def}}{=} \{\psi | \text{ for some } \phi \in \mathfrak{L}, \phi \rhd \psi \text{ and } M \vDash \phi\} \quad (1)$$
2. Now we say that M is *causally explained* (according to $\mathbb{T}$) if it is the only model of $\mathbb{T}^M$.
3. Finally, we say that $\phi \in \mathfrak{L}$ is a *consequence* of a causal theory $\mathbb{T}$ if ϕ is true in every $\mathbb{T}$-causally explained model.

With a particular choice of causal theory $\mathbb{T}$, this gives – it seems – a consequence relation appropriate for causal reasoning of the usual sort. It has, furthermore, good mathematical properties: as I argue [2002b; 2002a], this consequence relation is independent of the vocabulary that it is formulated in (a feature not shared by circumscription-based approaches).

Interesting though it is, this system has some disadvantages. It is defined in terms of models: models, however, are large, computationally unwieldy objects. Furthermore, although these systems *seem* to work, they are not metatheoretically transparent: on the formal level, it is hard (and extremely bureaucratic) to prove their correctness, whereas, on the informal level, they do not provide very much insight into why they work.

Consequently, we [2002b] have defined a modal system which can be used to reformulate these McCain-Turner theories. It is given by a sequent calculus: the non-modal rules are given in Table 1, whereas the modal operator is given by the rules in Table 2. The introduction and elimination rules are analogous to Lifschitz's predicate completion reformulation of McCain-Turner [Lifschitz, 1998]; however, our rules are more general. Furthermore, these rules have a richer metatheory than predicate completion: this (and particularly the cut elimination result) will also be useful to us.

Remark 1. It is easy to verify that we could just as well have used left and right rules where the $Q_1, \ldots, Q_k$ (and the corresponding sets of Ps) range over *minimal* sets such that
$$Q_1, \ldots, Q_k \vdash X.$$
In effect, the rules define $\Box X$ as a large disjunction of conjunctions like $P_1 \wedge \ldots \wedge P_k$; allowing non-minimal entailmennts means allowing more disjuncts, each of which entails one of the non-minimal disjuncts. Such an enlargement replaces the disjunction with a logically equivalent one.

However, although the system *with* the restriction to minimal sets may well be important for applications (it is certainly more computationally tractable), it is considerably clumsier to work with: we will, therefore, use the non-minimal system.

This system is (as the notation implies) a modal logic:

Proposition 1. *$\Box$ is a **K** modality.*

Proof. [White, 2002b] □

Notice that □L is, in general, infinitary (and one can invent examples where it is undecidable). However, in standard applications of the system, we can show that it remains quite tractable. For this, however, we need the following metatheoretical results.

Proposition 2. *The system given by Tables 1 and 2 has cut elimination: that is, given any proof of a sequent $\Gamma \vdash \Delta$, there is a proof of the same sequent without using the multicut rule.*

Proof. Given in [White, 2002b]; the proof uses the methods of Schroeder-Heister [1992]. □

Now from cut elimination, we can (as is standard) conclude that our system is consistent. We can also derive more interesting results. Note first that, if $\phi \rhd \psi$ is a causal law, $\phi \vdash \Box\psi$ is trivially a theorem of our system. Using cut elimination, we can prove a sort of converse. We suppose that we start off with a non-modal language $\mathfrak{L}$, in which the original McCain-Turner theory is formulated: if $\mathfrak{L}_\Box$ is $\mathfrak{L}$ extended by our modal operator $\Box$, then we have:

$$\frac{}{A \vdash A}\ \text{Ax} \qquad \frac{}{\bot\vdash}\ L\bot$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\ LW \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta}\ RW$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\ LC \qquad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta}\ RC$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ \neg L \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ \neg R$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ \wedge L \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ \wedge R$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ \vee L \qquad \frac{\Gamma \vdash A, B\Delta}{\Gamma \vdash A \vee B, \Delta}\ \vee R$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ \to L \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ \to R$$

$$\frac{\Gamma, A[x/t] \vdash \Delta}{\Gamma, \forall x A \vdash \Delta}\ \forall L \qquad \frac{\Gamma \vdash A[x/y], \Delta}{\Gamma \vdash \forall x A, \Delta}\ \forall R^{a}$$

$$\frac{\Gamma, A[x/y] \vdash \Delta}{\Gamma, \exists x A \vdash \Delta}\ \exists L^{b} \qquad \frac{\Gamma \vdash A[x/t], \Delta}{\Gamma \vdash \exists x A, \Delta}\ \exists R$$

$$\frac{\Gamma \vdash X^m, \Delta \quad \Gamma', X^n \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ \text{multicut}^{c}$$

[a] y not free in Γ or Δ, and either $y = x$ or y not free in A
[b] y not free in Γ or Δ, and either $y = x$ or y not free in A
[c] where X^n stands for n occurrences of X; $m, n > 0$

Table 1: The Non-Modal Rules

$$\frac{\Gamma \vdash P_1 \wedge \ldots \wedge P_n, \Delta \quad Q_1, \ldots, Q_n \vdash X}{\Gamma \vdash \Box X, \Delta}\ \Box R^{a}$$

$$\frac{\{\Gamma, P_{i_1}, \ldots, P_{i_k} \vdash \Delta, \quad Q_{i_1}, \ldots, Q_{i_k} \vdash X\}_{i=1,\ldots,n}}{\Gamma, \Box X \vdash \Delta}\ \Box L^{b}$$

[a] where, for all i, $P_i \rhd Q_i$

[b] where, for each i, we have $P_{i_1} \rhd Q_{i_1}, \ldots P_{i_k} \rhd Q_{i_k}$, and where the $\{P_{i_j}\}$ and $\{Q_{i_j}\}$, for $i = 1, \ldots n$, are the only such sets of Ps and Qs that there are.

Table 2: The Basic Modal Rules

Proposition 3. *Let M be a model of $\mathfrak{L}$, and let $P \in \mathfrak{L}$. Then $\mathbb{T}^M \vdash P$ iff there is a valid entailment $\Gamma \vdash \Box P$, with Γ a set of propositions of $\mathfrak{L}$ true in M.*

Proof. Only if is clear: we can simply use the proof of $\mathbb{T}^M \vdash P$ to get a proof of $\Gamma \vdash \Box P$. The other direction needs cut elimination: we take a cut free proof of $\Gamma \vdash \Box P$ and, by induction on the complexity of the proof tree, find a set of causal laws with their bodies true in M and their heads entailing P. $\Box$

From this follows:

Proposition 4. *The canonical model of our modal logic is given as follows: the worlds are all the models of our non-modal language $\mathfrak{L}$, whereas the accessibility relation $\mathcal{R}$ is given by*

$$M\mathcal{R}M' \quad \textit{iff} \quad M' \textit{ is a model of } \mathbb{T}^M. \tag{2}$$

Proof. We first prove that, in any model M of $\mathfrak{L}_\Box$, the truth values of modal propositions are given by the truth-values of the non-modal propositions: thus, models of $\mathfrak{L}_\Box$ are given by models of $\mathfrak{L}$. By cut elimination, $\mathfrak{L}_\Box$ is a conservative extension of $\mathfrak{L}$, so each model of $\mathfrak{L}$ can be extended to a model of $\mathfrak{L}_\Box$. Now the worlds of the canonical model are precisely the models of $\mathfrak{L}_\Box$, which are the models of $\mathfrak{L}$; the fact that the accessibility relation is given by (2) is standard. $\Box$

Our modal system thus tells us everything we want to know about the causal theory: the worlds of its canonical model give us the models of the original language, whereas the accessibility relation on the canonical model gives us the deductive closures of the sets $\mathbb{T}^M$. From this we can work out the McCain-Turner entailment relation: more formally, we have

Proposition 5. *Let $P \in \mathfrak{L}$. Then P is causally entailed by $\mathbb{T}$ iff we have*

$$\Gamma, \Gamma' \vdash P$$

where Γ is a set of propositions of the form $\Box A \vdash A$, and Γ' is a set of propositions of the form $A \vdash \Box A$, and where the modal operator is defined by $\mathbb{T}$.

Proof. By (2), together with standard results [van Benthem, 1984], the causally explained worlds are precisely those in which $\Box A \to A$ and $A \to \Box A$ hold, for all A. The result follows. $\Box$

Finally, note a further consequence of cut elimination: proof search for entailments of the form $\Gamma \vdash \Box\Delta$, where Γ and Δ are sets of non-modal propositions, is monotonic in Γ, Δ and the elements of $\mathbb{T}$, and is also generally quite tractable. Nonmonotonicity only arises when we have to deal with applications of the left rule for $\Box$. This is an illustration of a rather more general theme: that, if we can find logical systems with robust mathematical properties (cut elimination, for example, or interpolation), then, even though our reasoning may be nonmonotonic, it can, still, be quite tractable [Zhang and Foo, 2002].

1. $f_0 \rhd f_0$ and $\neg f_0 \rhd \neg f_0$, for any fluent f at time 0;
2. $a_t \rhd a_t$ and $\neg a_t \Rightarrow \neg a_t$, for any action a at any time t;
3. $f_{t-1} \wedge f_t \rhd f_t$ and $\neg f_{t-1} \wedge \neg f_t \rhd \neg f_t$, for any fluent f and any time t;
4. $f_{t-1} \wedge a_{t-1} \rhd g_t$, for any time t, where f is the precondition and g is the postcondition of action a.
5. $\neg P \rhd \bot$, for any domain constraint P.

Table 3: McCain and Turner's Laws

Example 1 (Necessitation). As an example, we show that the rule of necessitation is admissible: that is, that, if we have a proof Π of $\Gamma \vdash A$, we can also prove $\Box\Gamma \vdash \Box A$ (where, as usual, $\Box\Gamma = \{\Box\gamma | \gamma \in \Gamma\}$ for a set of formulae Γ).

We first apply $\Box$L to all of the $\Box\gamma$:

$$\frac{\left\{\phi_{1_1}, \ldots, \phi_{1_{k_1}}, \ldots, \phi_{n_1}, \ldots, \phi_{n_{k_n}} \vdash \Box A\right\}}{\Box\Gamma \vdash \Box A}$$

where $\Gamma = \{\gamma_1, \ldots, \gamma_n\}$, where the entailments in the brackets are given by all of the sets of bodies of causal rules ϕ_i where the corresponding heads satisfy

$$\psi_{i_1}, \ldots, \psi_{i_{k_i}} \vdash \gamma_i \tag{3}$$

for each i.

Consider one of the entailments in brackets. From the entailments (3), together with our proof of $\Gamma \vdash A$, we can derive a proof of

$$\psi_{1_1}, \ldots, \psi_{1_{k_1}}, \ldots, \psi_{n_1}, \ldots, \psi_{n_{k_n}} \vdash A;$$

we can use this proof as a side condition for an application of $\Box$R, and thus prove the required entailment.

2 The Original Causal Laws

McCain and Turner's original laws are given in Table 3; here we suppose that we have sets of fluents $\{f_i\}$ (where f_i stands for the fluent f at time t, and of action symbols a_j, and also that the domain constraints (if any) are given by sentences P.

Example 2 (Using the Constraints). Suppose that we have to prove an entailment of the form $\Gamma \vdash \Box A, \Delta$. We can get the constraints onto the left hand side like this:

$$\cfrac{\cfrac{\cfrac{\cfrac{\vdots}{\Gamma, C \vdash \Box A, \Delta}}{\Gamma \vdash \neg C, \Box A, \Delta}\,\neg\text{R}}{\Gamma \vdash \Box A, \Box A, \Delta}\,\Box\text{R}}{\Gamma \vdash \Box A, \Delta}\,\text{RC}$$

where the application of $\Box$R is justified by the entailment $\bot \vdash A$.

2.1 The Meaning of These Rules

McCain and Turner's system is often described as a system of causal reasoning: I would like to argue, however, that we can interpret it in a rather more general sense, and that, so interpreted, it can be seen as a continuation of a well-established tradition.

The idea of questions and answers is quite appropriate here. According to Hintikka [1976; 1972], and Harrah [1975] a question can be regarded as denoting its set of possible answers (out of which an appropriate answer selects one). For example, in Harrah's system our $\Box P$ would be called the "assertive core" of the question, whereas his *indicated replies* would, in our system, be combinations of rule bodies $\phi_, \ldots, \phi_k$ such that $\phi_1, \ldots, \phi_k \vdash \Box P$. Here we have two rules for $\Box$, left rules and right rules; when we apply a left rule to the necessitation of a given fluent, we get the set of possible answers to a question. When we apply a right rule, we have to select an answer from the set of appropriate ones. The duality of left rules and right rules, then, corresponds to a duality of questions and answers.

Now, *in the case of McCain and Turner's original examples*, these questions and answers come from the domain of causal explanation: the operators give us answers to the questions that arise in the process of constructing a causally explained narrative. However, there is no need to limit this system to merely *causal* questions, or causal explanations. In fact, Parsons and Jennings ([Parsons and Jennings, 1996]; see also [Parsons *et al.*, 1998]) have described a consequence relation, $\vdash_{\text{ACR}}$, which is intended to capture the practice of argumentation from a given set of basic arguments. Their system turns out (see [White, 2003]) to be a special case of ours: to each proof of theirs

$$\Delta \vdash_{\text{ACR}} (p, A)$$

– which says that the argument to p from premises A is valid, given the basic arguments in Δ – we can associate a proof in our system of

$$A \vdash \Box_{\Delta} p,$$

where $\Box_{\Delta}$ is the modality obtained by taking the basic arguments in Δ as "causal" axioms (although, of course, they need not be causal, and, in Parsons and Jenning's case, they are not causal).

3 The Zhang-Foo Interpolation Argument

Zhang and Foo argue that reasoning about the frame problem can be made considerably more tractable if one uses the meta-hypothesis that "local queries require only local frame axioms" [2002, p. 359]. Making sense of this hypothesis depends, of course, on being able to give a sense to "local": Zhang and Foo interpret it using the idea of a sublanguage, so that *local* reasoning would be reasoning would be reasoning which could be carried out in some appropriate sublanguage. So, as they write,

> … engineers can localise their language so that it involves only the relevant components yet is sufficient for specifying the system and expressing possible future queries … [A]nswering a query in the local language might only require local frame axioms. Therefore the number of frame axioms will mainly depend on the size of the local language. [2002, p. 358]

Substantiating this hypothesis involves metatheoretical work: what we need to show is some sort of *interpolation* property [Troelstra and Schwichtenberg, 1996, Section 4.2] for the logic concerned. Zhang and Foo [2002, pp. 365ff.] establish such a property for their logic (a variant of dynamic logic); here we establish it for ours.

We first lay down some assumptions and some notation. Suppose that the relation symbols of our language $\mathfrak{L}$ can be partitioned into two disjoint sets $\mathcal{R}'$ and $\mathcal{R}''$, and that the constants can likewise be partitioned into two disjoint sets $\mathcal{C}'$ and $\mathcal{C}''$; let $\mathfrak{L}'$ and $\mathfrak{L}''$ be the corresponding sublanguages of $\mathfrak{L}$, and, similarly, let $\mathfrak{L}'_{\Box}$ and $\mathfrak{L}''_{\Box}$ be the corresponding sublanguages of $\mathfrak{L}_{\Box}$.

Definition 1. C is an *implicit constraint* if $\neg C \vdash \Box \bot$.

Note that an implicit constraint must be true in every causally explained model: if it were to be false in a model, then $\Box \bot$ would be true in that model, and so $\bot$ would be true in that model, which is a contradiction.

Proposition 6. *Suppose that the causal rules are such that, if $\phi \rhd \psi$, then either $\phi, \psi \in \mathfrak{L}'$ or $\phi, \psi \in \mathfrak{L}''$. Define a modal operator $\Box'$ by rules restricted to $\mathfrak{L}'$: the right rule, for example, will be*

$$\frac{\Gamma \vdash \phi'_1 \wedge \ldots \wedge \phi'_k, \Delta}{\Gamma \vdash \Box' X}$$

whenever $\phi'_i \rhd \psi'_i$ for all i, $\psi'_1, \ldots, \psi'_k \vdash X$, and where $\phi'_i, \psi'_i \in \mathfrak{L}$ for all i. The left rule is similar. Similarly, define a modal operator $\Box''$ by rules restricted to $\mathfrak{L}''$. Then, if $X' \in \mathfrak{L}'$, $\Box X \cong C'' \rightarrow \Box' X'$, where C'' is a conjunction of domain constraints in $\mathfrak{L}''$. Correspondingly, if $X'' \in \mathfrak{L}''$, $\Box X'' \cong \Box'' X''$.

Proof. For any X, $\Box' X \vdash \Box X$: we take the sequent $\Box' X \vdash \Box X$ and apply $\Box X$, which gives us a set of sequents of the form

$$\phi'_1, \ldots, \phi'_k \vdash \Box X$$

where $\psi'_1, \ldots, \psi' k \vdash X$: but now we can simply apply $\Box R$.

The converse is not so immediate. We have to prove $C'', \Box X' \vdash \Box' X'$, and so we have to prove sequents of the form

$$C'', \phi_1, \ldots, \phi_k \vdash \Box' X',$$

where $X' \in \mathfrak{L}'$, but where now the ϕs are no longer restricted to lie in $\mathfrak{L}'$. We know that $\psi_1, \ldots, \psi_k \vdash X$: re-order the ψs if necessary so that we have

$$\overbrace{\psi'_1, \ldots, \psi'_j}^{\mathfrak{L}'}, \overbrace{\psi''_{j+1}, \ldots, \psi''_k}^{\mathfrak{L}''} \vdash X$$

By interpolation for $\mathfrak{L}$, there is a proposition $F \in \mathfrak{L}' \cap \mathfrak{L}''$ such that

$$\begin{aligned} \psi''_{j+1}, \ldots, \psi''_k &\vdash F \\ \psi'_1, \ldots, \psi'_j, F &\vdash X \end{aligned}$$

Now, by assumption, $\mathfrak{L}' \cap \mathfrak{L}'' = \{\top, \bot\}$, there are two cases: if $F = \top$, then we already have $\psi'_1, \ldots, \psi'_j \vdash X$, and we can apply $\Box' R$.

If $F = \bot$, then $\psi''_{j+1}, \ldots, \psi''_k \vdash \bot$, and so $\phi''_{j+1} \wedge \ldots \wedge \phi''_k$ is the negation of an implicit constraint in $\mathfrak{L}''$: we simply add it to C'', and we can prove the desired sequent. ☐

We now specialise to a particular class of theories: in these theories, there will be temporally indexed proposition and action symbols, and the "causal rules" will be of the form given in Table 3: that is, both heads and bodies will be conjunctions of fluents and actions. Call such a theory a *fluent-based* theory.

Lemma 1. *Let $\mathbb{T}$ be a fluent-based theory: then a model M is causally explained iff, for every conjunction P of fluents and actions, $M \vDash P \leftrightarrow \Box P$.*

Proof. Any proposition in $\mathfrak{L}$ can be put into disjunctive normal form. So, suppose that $P \cong P_1 \vee \ldots \vee P_k$, where the P_is are conjunctions of fluents and action symbols. To work out $\Box P$, we have to consider proofs of

$$\psi_1, \ldots, \psi_l \vdash P_1 \vee \ldots \vee P_k$$

By cut elimination for $\mathfrak{L}$, and the form of the ψ_i, every such proof comes from a proof of

$$\psi_1, \ldots, \psi_l \vdash P_j$$

for a suitable j. Consequently, *for fluent-based theories*, we have

$$\Box P \cong \Box P_1 \vee \ldots \vee \Box P_k,$$

and this shows that, for such theories, propositions in the modal language can be put into "disjunctive normal form"; the disjuncts here will be conjunctions of

1. conjunctions of fluents and action symbols, and
2. modalisations of fluents and action symbols.

The result follows. □

Consequently, we have

Corollary 1. *Suppose that $\mathbb{T}$ is a fluent-based theory, and that the fluents and action symbols can each be partitioned into two subsets, $\mathfrak{L}'$ and $\mathfrak{L}''$, such that the causal rules and the constraints respect the partition (i.e., for each rule $\phi \rhd \psi$, either $\phi, \psi \in \mathfrak{L}'$ or $\phi, \psi \in \mathfrak{L}''$, and the constraints are a conjunction of propositions each of which involves only propositions in $\mathfrak{L}'$ or $\mathfrak{L}''$. Then a model is causally explained (with respect to $\Box$ iff its restrictions to $\mathfrak{L}'$ and $\mathfrak{L}''$ are causally explained (with respect to $\Box'$ and $\Box''$).*

Proof. Since $\mathfrak{L}' \subseteq \mathfrak{L}$ and $\mathfrak{L}'' \subseteq \mathfrak{L}$, for every $P' \in \mathfrak{L}'$, there is a conjunction C'' of $\mathfrak{L}''$-constraints such that $(C'' \to \Box' P') \cong \Box P'$ (and similarly for $P'' \in \mathfrak{L}''$).

Suppose now that M is causally explained: by the above argument, for every $P' \in \mathfrak{L}'$ there is a conjunction C'' of implicit constraints such that $M \vDash P' \leftrightarrow (C'' \to \Box' P')$. However, C'' is a constraint and M is causally explained, so $M \vDash C''$; consequently, $M \vDash P' \leftrightarrow \Box' P'$. P' is arbitrary, so M' is causally explained.

Conversely, suppose that M' and M'' are causally explained. By Lemma 1, it is enough to prove $M \vDash P \leftrightarrow \Box P$ for every conjunction P of fluents and action symbols. However, each such is a conjunction $P' \wedge P''$ of fluents and action symbols from $\mathfrak{L}'$ and $\mathfrak{L}''$; so, $\Box P \cong (C'' \to \Box' P') \wedge (C' \to \Box'' P'')$, and we argue as above. □

Thus, every causally explained model of $\mathbb{T}$ is a pair of causally explained models, one of $\mathbb{T}'$ and one of $\mathbb{T}''$: and we have a local reasoning result very similar to Foo and Zhang's.

Results like these are not only useful for the tractability of causal reasoning: they also play an important role in the metatheory. Consider a causal theory in McCain and Turner's form: how do we verify its correctness? Suppose, for simplicity, that we have a scenario with only one action occurrence: partition the vocabulary into two parts, one of which ($\mathfrak{L}'$) contains the action, together with its preconditions and postconditions, whereas the other ($\mathfrak{L}''$) contains all the other primitives. It is elementary to verify that the only model of $\mathfrak{L}''$ in which no actions occur is the one in which nothing changes: consequently, we only have to verify correctness for $\mathfrak{L}'$, which is a much simpler affair.

4 Conclusion

Makinson [2003, pp. 10f.] has argued that nonmonotonic consequence relations are not closed under uniform substitution, and that they do not need to be. This is, I would argue, too hasty: the inclusions $\mathfrak{L}', \mathfrak{L}'' \subseteq \mathfrak{L}$ of this paper can be viewed as substitutions, and the main results can be viewed as showing that the consequence relation is, in fact, closed under the substitutions in question. The truth of the matter is surely this: although the consequence relation is not closed under substitution *in general*, there are certain substitutions that it *is* closed under. Furthermore, knowing which substitutions are possible tells us a good deal about the structure of the logic – indeed, if Zhang and Foo are correct, this knowledge can be regarded as the key to a tractable grasp of the logic in question.

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