Homework Assignment 4

Due **Tuesday 17-02-04**

1. Let α be Ackermann's function. Prove Lemma 2(ii):

$$y < n \Rightarrow \alpha(m, y) < \alpha(m, n), \qquad \text{for all } m,\, n \text{ and } y.$$

(*Hint:* Use main induction on n and secondary induction on m. You may use Lemma 2(i) and Lemma 3.)

We say that a number a is *congruent to b modulo m* (written $a \equiv b \ (mod\ m)$) if and only if $a - b = md$ for some $d \in \mathbf{Z}$. (Thus, if $a = md_0 + r_0$ and $b = md_1 + r_1$ with $0 \leq r_0, r_1 < m$, then $r_0 = r_1$.)

2. Prove the *Chinese Remainder Theorem: Let m_1, ..., m_r be any pairwise coprime integers, then the congruences*

$$x \equiv a_i \pmod{m_i} \qquad (i = 1, \ldots, r)$$

have a common solution, which is unique mod m, where $m = m_1 \cdot \ldots \cdot m_r$.

Moreover, writing $M_i = m/m_i$, we can obtain a solution in the form $x = \Sigma_{i \leq r} M_i x_i$, where x_i satisfies $M_i x_i \equiv a_i \pmod{m_i}$.

Hint: Read section 2.3 of Cohn's book and write exactly the part you need to prove theorem 5 (nothing less, nothing more).