WHEN IS THE HEART OF A t-STRUCTURE A GROTHENDIECK CATEGORY?

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JOINT WORK WITH MANUEL SAORÍN AND JAN ŠŤOVÍČEK

ABSTRACT. Let $\mathcal{D}$ be a triangulated category endowed with a t-structure $\mathfrak{t} = (\mathcal{U}, \Sigma\mathcal{V})$ and denote by $\mathcal{H} := \mathcal{U} \cap \Sigma\mathcal{V}$ its heart. In this seminar I will report on some recent results, obtained in collaboration with Manuel Saorín and Jan Šťovíček, partially answering the following well-known question:

Under what conditions on $\mathcal{D}$ and $\mathfrak{t}$
can we say that $\mathcal{H}$ is a Grothendieck category?

We will concentrate on the case when $\mathcal{D}$ is the base of a stable derivator. In this generality we will see that, under very natural hypotheses on $\mathfrak{t}$, direct limits in $\mathcal{H}$ are exact. Furthermore, when $\mathcal{D} = \mathrm{ho}(\mathcal{G})$ is the homotopy category of a suitable model structure on a Grothendieck category $\mathcal{G}$, $\mathcal{H}$ has also a set of generators. This last case includes derived categories of Grothendieck categories and of small dg categories.