Anatomy of a Stranger: A (Barely) non-Noetherian Ring

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In ongoing work with William Heinzer and Christel Rotthaus over the past twenty years we have been applying a construction technique for obtaining some- times baffling, sometimes badly behaved, sometimes Noetherian, sometimes non- Noetherian integral domains. This technique of intersecting fields with power series rings goes back to Akizuki in the 1930s and Nagata in the 1950s, and since then has also been employed by Nishimuri, Heitmann, Ogoma, the authors and others.

In particular we show how to obtain a three-dimensional near-Noetherian unique factorization domain B that is tantalizingly close to being Noetherian but is not quite-because exactly one prime ideal has height two and it is the only nonfinitely generated prime ideal of B. The unique maximal ideal of B is 2-generated. We also mention more mysterious generalizations to higher dimensions.