Extensions of star operations and Jaffard families

The concept of extension of a star operation of finite type on D to a localization of D was introduced by Houston, Mimouni and Park to study integral domains where $\mathrm{Star}(D)$, the set of star operations on D, has cardinality 2. The aim of this talk is to show how this idea can be generalized to star operations that are not necessarily of finite type and to flat overring of D, and how it can be used, in some cases, to decompose $\mathrm{Star}(D)$ as a product of $\mathrm{Star}(T)$, as T ranges in a family of overrings of D. In particular, this decomposition is possible if T varies in a Jaffard family of D, in which case the properties of $\star \in \mathrm{Star}(D)$ can be studied through the properties of its extensions $\star_T \in \mathrm{Star}(T)$. As a further application, it will be shown that, if D is a Prüfer domain with finite spectrum, the set $\mathrm{Star}(D)$ can be determined by joining some geometrial data (the homeomorphically irreducible tree associated to $\mathrm{Spec}(D)$) and some algebraic data (the primes P such that PD_P is principal).