A SURVEY OF HOPF ALGEBRAS OF SMALL DIMENSION

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Abstract. The problem posed by Kaplansky [K] of classification of Hopf algebras of a given finite dimension over the complex numbers is a nontrivial one, even for some small dimensions. It has been shown [Z],[Ng], [Ng1], [HN] that for p a prime, Hopf algebras of dimension p, p^2,$2p$ and $2p^2$ are either semisimple, pointed nonsemisimple, or the dual is pointed. It has been conjectured that this is the case also for dimension pq and p^n with p, q primes. At this time, the smallest dimension where the classification is incomplete is 24. The classification for dimension 27 has only recently been completed [BG] and in general, although [G], [BG] give many partial results, the classification for dimension p^3, p an odd prime, is still open.

This talk will give a little survey of what is known for some small dimensions and present some techniques based on [F] using the coradical filtration for dealing with these questions.

References

[BG] M. Beattie and G. A. García, Techniques for classifying Hopf algebras and applications to dimension p^3, arXiv:1108.6037v1 [math.QA]

[F] D. Fukuda, Structure of coradical filtration and its application to Hopf algebras of dimension pq, *Glasg. Math. J.* **50** (2008), no. 2, 183–190.

[G] G. A. García, On Hopf algebras of dimension p^3, *Tsukuba J. Math.* **29** (2005), no. 1, 259–284.

[HN] M. Hilgemann and S.-H. Ng, Hopf algebras of dimension $2p^2$, *J. London Math. Soc.*(2) **80** (2009), 295-310.

[K] I. Kaplansky, Bialgebras. Lecture Notes in Mathematics. University of Chicago, Chicago (1975)

[Ng] S-H. Ng, Non-semisimple Hopf algebras of dimension p^2, *J. Algebra* **255**, no. 1 (2002), 182–197.

[Ng1] ———— Hopf algebras of dimension $2p$, *Proc. Am. Math. Soc.* **133**(8), 2237-2242.

[Z] Y. Zhu, Hopf algebras of prime dimension, *Int. Math. Res. Not.* **1** (1994), 53–59.

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