Minicorso

Free Ideal Rings
Dolors Herbera – Universitat Autònoma de Barcelona

- martedì 12/01/2027, 10:00-12:30
- mercoledì 13/01/2027, 10:00-12:30
- giovedì 14/01/2027, 10:00-12:30

Sala Riunioni - Ca' Vignal 2

Irreducible representations of free algebras
Francesca Mantese – Università di Verona

mercoledì 20/01/2027 10:00-12:30
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Free Ideal Rings

Dolors Herbera

A Free Ideal Ring R (fir, for short) is an associative ring whose one-sided ideals are free as R-modules of unique rank. If we impose the property just for the finitely generated ideals, then R is said to be a semifir. Commutative principal ideal domains are firs, while commutative Bézout domains are semifirs.

Firs and semifirs are classes of rings that were introduced by Cohn in the 1960's with the aim of developing noncommutative ring theory along the lines of the classical theory of commutative principal ideal domains and Bézout domains. The free associative algebra over a field and the group algebra of a free group are among the most important motivating examples of firs.

In this short course we will explain the foundations of the theory of firs and semifirs. A basic reference is [1] and, especially for the course, the last chapter of the more accessible text [2].

The course is structured as follows:

1. Some non-commutative domains embeddable in division rings: skew-polynomial rings, tensor algebras and free algebras.

2. A non-commutative version of the euclidean algorithm: the weak algorithm.

3. Firs and semifirs.

4. Some classes of modules over firs and semifirs.

In **1,** we will introduce some of the objects of interest and briefly discuss some aspects of their localisation theory, which in the non-commutative setting has some surprises.

For further information on skew-polynomial rings, some useful additional references are [4] and [3]. Reference [4] also provides a nice introduction to the construction of Malcev-Neumann power series rings, useful to embed a free algebra and the group algebra over the free group into a division ring.

Domains with Euclidean division algorithm are principal ideal domains. In **2.** we will discuss a non-commutative extension of the division algorithm called the weak algorithm, and some of its variants. As a main result, we shall show that a free algebra over an arbitrary set X has the weak algorithm.

In **3.**, we will introduce firs, semifirs and some of their variants, like n-firs, and study their basic properties. The main result we shall prove is that a ring with weak algorithm is a fir. Consequently, the free associative algebra is a fir.

The content of part **4.** is somewhat tentative and may be adjusted depending on time and audience. The idea is to study some particular classes of modules over a free algebra, that seem to be closely related to the theory of representation of finite-dimensional algebras. This part will be based on chapter 5 of [1].

References

[1] P. M. Cohn, Free Rings and their Relations (Second Edition). London Mathematical Society Monographs **19**, 1985.

[2] P. M. Cohn, An Introduction to Ring Theory. Springer Undergraduate Mathematics Series, 2000.

[3] K. R. Goodearl and R. B. Warfield, Jr. An Introduction to Noncommutative Noetherian Rings. London Mathematical Society Student Texts **16**, Cambridge University Press, 2004.

[4] T. Y. Lam, A first course in noncommutative rings, 2nd ed., Graduate Texts in Mathematics **131**, Springer, New York, 2001.