4th Dolomites Workshop on Constructive Approximation and Applications

dedicated to

Annie Cuyt

on the occasion of her 60th birthday

Alba di Canazei (Italy), 8-13 September 2016

timetable and abstracts

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Arrival on Sept. 7; two shuttles from the airports of Venice and Verona on Sept. 7 around 16.00; no organized shuttle on Sept. 8. The welcome reception is on Sept. 8 at 18.00, the social dinner is on Sept. 12; the schedule of the other social activities is tentative. Return shuttles to be booked onsite by individuals or groups.

Plenary speakers

Annie Cuyt @60: A Life in Approximation

J.A.C. Weideman

We revisit a few milestones in the remarkable career of Annie Cuyt.

Beyond B-splines: generalized B-splines, exponential B-splines and pseudo-splines with emphasis on their refinement properties

Costanza Conti
University of Florence, Italy

Abstract

It is well known that B-splines are a powerful basis for polynomial splines with, beside other nice properties, minimal support with respect to a given degree and smoothness. Any spline function of given degree can be expressed as a linear combination of B-splines of that degree. Starting from a set of control points the latter is in fact the way curve and surface splines are constructed in computer-aided design and computer graphics. B-splines, in particular cardinal B-splines -i.e. B-splines with uniform knots,- find application also in other contexts than design like approximation theory, curve/surface fitting, numerical differentiation and integration, signal and image processing. Due to their refinability property cardinal B-splines are also suitable for multiresolution, multilevel and subdivision approaches which play an important role in numerical analysis. In spite of their celebrity, polynomial B-splines present several drawbacks. They have low approximation order and are not able to exactly reproduce geometries like conic sections which are important in design, biomedical imaging or isogeometric analysis. Also, they do not approximate well causal exponentials that play a fundamental role, for example, in classical system theory. This is why, in the last two decades several generalization of B-splines have been proposed, the most popular being Non Uniform Rational B-splineS (NURBS) that have received an increasing attention in the geometric modeling community, in particular. While NURBS are actually able to exactly reproduce a huge variety of geometries, transcendental curves like helix or cycloid are still excluded and modeling of manifolds with arbitrary topology is conceptually very complicated and extremely expensive. Moreover, NURBS require additional parameters or weights which do not have an evident geometric meaning and whose selection is often unclear. Last but not least, their rational nature is unpleasant with respect to differentiation and integration. To overcome the drawbacks of NURBS, generalized B-splines became, recently, an attractive alternative to the rational model. While classical B-splines are piecewise functions with sections in the space of algebraic polynomials, generalized B-splines are piecewise functions with sections in more general spaces. With a suitable selection of such spaces generalized B-splines allow exact representation of polynomial curves, conic sections, helices and other profiles. They possess all fundamental properties of polynomial B-splines and behave completely similar to B-splines with respect to differentiation and integration. Cardinal exponential B-splines are a crucial instance of such a class of basis suitable for multiresolution, multilevel and subdivision approaches. An other interesting generalization of polynomial B-splines recently emerged is given by pseudo-splines and, more in general by exponential pseudo-splines. Exponential pseudo-splines are a rich family of basis functions meeting various demands for balancing approximation power, regularity, support size, interpolation, reproduction capability and refinability. Their refinability properties combined with high approximation order make them useful, for example, to construct tight wavelet frames to be used in multiresultion analysis approaches in signal and image processing.

The talk will start with a review of polynomial B-splines with special emphasis on their refinement properties and on the corresponding subdivision algorithms for cardinal B-splines. We will then define and discuss exponential B-splines and exponential pseudo-splines in the uniform case yet by the help of a subdivision perspective.

Error Estimates and Convergence Rates for Filtered Back Projection

Armin Iske*

Computerized tomography allows us to reconstruct a bivariate function from Radon samples. The reconstruction is based on the *filtered back projection* (FBP) formula, which gives an analytical inversion of the Radon transform. However, the FBP formula is numerically unstable and suitable low-pass filters with a compactly supported window function and finite bandwidth are employed to make the reconstruction by FBP less sensitive to noise.

The objective of this talk is to analyse the intrinsic FBP reconstruction error which is incurred by the use of a low-pass filter. To this end, we prove L^2-error estimates on Sobolev spaces of fractional order. The obtained error bounds are affine-linear with respect to the distance between the filter's window function and the constant function 1 in the L^∞-norm. With assuming more regularity of the window function, we refine the error estimates to prove convergence for the FBP reconstruction in the L^2-norm as the filter's bandwidth goes to infinity. Further, we determine asymptotic convergence rates in terms of the bandwidth of the low-pass filter and the smoothness of the target function.

The talk is based on joint work with Matthias Beckmann (Hamburg).

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Computationally efficient radial basis function partition of unity methods

Elisabeth Larsson* Victor Shcherbakov* Alfa Heryudono‡

June 8, 2016

Radial basis function (RBF) methods provide a number of properties that make them attractive for solving partial differential equations, such as meshlessness, high order or spectral convergence rates for smooth problems, and ease of implementation. However, the global RBF methods that work very well in one and two space dimensions become increasingly computationally expensive for higher dimensions due to the dense linear systems that must be solved. To reduce computational cost, but still retain the advantages of global RBF approximation, localized RBF methods have been introduced. There are two main directions: Stencil-based approximations and partition of unity methods. In this talk, the focus is on RBF partition of unity methods (RBF-PUM). In RBF-PUM, the computational domain is covered by overlapping patches. Then a local RBF approximation is constructed in each patch, and finally the local approximations are blended into a global approximation using compactly supported partition of unity weight functions. This leads to sparse linear systems and a significantly lower computational cost than for the global method. We have explored different formulations of RBF-PUM both theoretically and computationally, and we provide insights into the convergence properties of the method, we explain how to formulate the method such that it remains robust for large scale problems, and we also show how to reduce the computational cost even further by particular choices of node distributions.

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Randomness and Potential Theory

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Abstract

Motivated by the search for "good" nodes in a compact set K in the complex plane for use in polynomial interpolation, i.e., arrays $\{z_{nj}\}_{n=1,2,...;\ j=0,...,n} \subset K$ so that the norms Λ_n of the projection operators $\mathcal{L}_n : C(K) \to \mathcal{P}_n \subset C(K)$ from $f \in C(K)$ to the Lagrange interpolating polynomial $\mathcal{L}_n(f) \in \mathcal{P}_n$ (polynomials of degree $\leq n$) with nodes at $z_{n0}, ..., z_{nn}$ satisfy $\Lambda_n^{1/n} \to 1$, we discuss two types of probabilistic questions whose answers are given by similar potential theoretic information. Introducing a measure μ on K, the reproducing kernel K_n for $\mathcal{P}_n$ in $L^2(\mu)$ plays a key role in the first type of question: *what are generic properties of random arrays?*

The reproducing kernel K_n reoccurs in the theory of random polynomials in $\mathbb{C}$. We will first discuss the classical setting of Kac-Hammersley: writing $p_n(z) = \sum_{j=0}^n a_j z^j = a_n \prod_{j=1}^n (z - \zeta_j) \in \mathcal{P}_n$ where the coefficients $a_0, ..., a_n$ are i.i.d. complex Gaussian random variables (appropriately normalized), one considers the normalized zero measure $\mu_n := \frac{1}{n} \sum_{j=1}^n \delta_{\zeta_j}$. More precisely, one is interested in the *asymptotic behavior of* $\{\mu_n\}_{n=1,...}$ *for sequences* $\{p_n\}_{n=1,...}$ *of random polynomials.*

Both random arrays and zeros of random polynomials have generalizations in a variety of other settings. Our talk will emphasize the basic situation, working on compact sets $K \subset \mathbb{C}$. A partial list of existing references for generalizations will be provided and briefly discussed.

Sparse approximation by modified Prony method

Gerlind Plonka[1], Vlada Pototskaia[1]

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Abstract

The classical Prony method works with exactly sampled data of the exponential sum

$$h(x) := \sum_{j=1}^{M} c_j \, \mathrm{e}^{f_j x}, \quad x \geq 0, \tag{1}$$

in the case of known order M. Following an idea of G.R. de Prony from 1795, we can recover all parameters c_j, f_j of the exponential sum (1), if sampled data $h(k)$, $k = 0, \ldots, 2M-1$ are given, where $z_j := \mathrm{e}^{f_j}$ are distinct values in $\mathbb{D} := \{z \in \mathbb{C} : 0 < |z| \leq 1\}$. If the number of terms M is unknown, we can use numerical methods, like the ESPRIT method, to evaluate M by considering the numerical rank of a suitable Hankel matrix that is build by the equidistant function values $h(k)$, see [2].

Recently, we extended the Prony-method to a reconstruction technique for sparse expansions of eigenfunctions of suitable linear operators, see [1]. This more general approach provides us with a tool to unify all Prony-like methods on the one hand and establishes a much broader field of applications of the method on the other hand. In particular, it can be shown that all well-known Prony-like reconstruction methods for exponentials and polynomials known so far, can be seen as special cases of this approach. The new insight into Prony-like methods enables us to derive also new reconstruction algorithms for orthogonal polynomial expansions and for expansions in finite-dimensional settings.

However, in many applications for sparse approximation, the function to be recovered is only approximatively of the form (1), and we may want to approximate h by an exponential sum with an a priory fixed number M of exponentials. The open questions are now:
Does the Prony method provide a good approximation of h if the number of terms M is underestimated? Can the error caused by the Prony approximation be exactly quantified in a suitable norm? Do we have to modify the Prony method in order to achieve better estimates? Can such a modified Prony method for sparse approximation be generalized to sparse expansions of eigenfunctions of linear operators?

References

[1] T. Peter and G. Plonka, *A generalized Prony method for reconstruction of sparse sums of eigenfunctions of linear operators.* Inverse Problems **29** (2013), 025001.

[2] G. Plonka and M. Tasche, *Prony methods for recovery of structured functions.* GAMM-Mitt. **37**(2) (2014), 239-258.

Low rank approximation of functions in polar and spherical geometries

Grady B. Wright
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A collection of algorithms for computing with functions defined on the unit disk or the surface of the unit two-sphere is presented. Central to these algorithms is a new scheme for approximating functions to essentially machine precision that combines a structure-preserving iterative variant of Gaussian elimination together with the double Fourier sphere method. The scheme produces low rank approximations of functions on the disk and sphere, ameliorates oversampling issues near the origin of the disk and poles of the sphere, converges geometrically for sufficiently analytic functions, and allows for stable differentiation. The low rank representation makes operations such as function evaluation, differentiation, and integration particularly efficient. A demonstration of the algorithms using the new Diskfun and Spherefun features of Chebfun will also be given. This is joint work with Prof. Alex Townsend and Heather Wilber (both at Cornell University).

A) Approximation theory in imaging science

Organizers: Wolfgang Erb and Andreas Weinmann

Sampling in Grassmannian spaces

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The Grassmannian space, as an example of a Riemannian manifold and a homogeneous space, shares many properties of the sphere but is slightly more involved. We shall study the approximation of functions on the Grassmannian from finitely many samples.

Learning co-sparse representations for applications in image processing

Martin Kiechle

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Recently, the co-sparse analysis model has gained much attention as an important alternative to synthesis sparse modeling in particular for image processing applications. In this talk, we consider learning co-sparse representations from data by optimizing a non-linear sparsifying objective. The presented framework allows for finding a solution to the problem efficiently by employing a geometric gradient method on a product of spheres structure. It is shown how such representations can be used effectively to regularize various inverse problems in image processing. Furthermore, extensions of the model to joint and structured co-sparse representations are considered that enable additional applications such as bi-modality image super-resolution and registration.

Counterexamples to the frame set conjecture for Hermite functions

Jakob Lemvig

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Frame set problems in Gabor analysis ask the question for which sampling and modulation rates the corresponding time-frequency shifts of a generating window allow for stable reproducing formulas of L^2-functions. In this talk we study the frame set for Hermite functions of order $4n+2$ and $4n+3$, $n = 0, 1, \ldots$. We show that the so-called frame set conjecture for these generators is false. Our arguments are based on properties of the Zak transform.

Structured sparse recovery from Fourier measurements using α-molecules

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Abstract

In this talk we discuss the non-linear approximation of piecewise smooth signals by localized systems such as wavelets and shearlets, or more generally, α-molecules. More precisely, we discuss recovery guarantees of sparse signals that are obtained by ℓ^1-minimization. We thereby, consider the ℓ^1-minimization problem in its analysis formulation using a multiscale transform that is associated to a localized system such as α-molecules. Furthermore, the multilevel structure is incorporated into the reconstruction problem by using a multilevel sampling scheme. One particular focus is the so-called *balancing property* that can be used to guarantee stable reconstructions, in particular for the case if α-shearlets are used as an approximating system and the subsampled Fourier operator as a sampling operator.

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APPROXIMATION TO THE FUNCTIONS EXPRESSED BY CONFORMAL MAPPINGS

Burçİn Oktay Yönet

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It‘s well known that conformal mappings play an important role in many applied disciplines especially applied mathematics, aerodynamics, thermodynamics, imaging sciences. The basic strategy in solving problems in this fields is to solve the problem on a simplier domain and to move the result to the main domain with the help of conformal mapping. Because of their importance and the difficulties of finding their expressions, the problem of approximation to conformal mappings by some functions whose properties are known is a special subject in approximation theory.

In this talk, we present our study on approximation problems to conformal mappings and the functions expressed with the help of conformal mappings by special polynomials on some complex domains, and give new results on approximation error according to the geometric properties of the domains.

ON APPROXIMATION PROPERTIES OF GENERALIZED (KANTOROVICH-TYPE) SAMPLING OPERATORS

OLGA ORLOVA, GERT TAMBERG

The theory of generalized sampling operators was developed at RWTH Aachen by P. L. Butzer and his students in the late 1970s and has been extended thoroughly since then. A generalized sampling operator generated by a kernel function $\varphi \in L^1(\mathbb{R})$ (which is constructed via the FCT of some window function) is defined for $f \in C(\mathbb{R})$ by

$$S_W^{\varphi} f(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W})\varphi(Wt-k) \quad (t \in \mathbb{R};\ W>0). \tag{1}$$

In [1] the authors introduced the Kantorovich version of operator (1) by replacing the exact value $f(k/W)$ with the Steklov mean of f on the interval $[k/W,(k+1)/W]$. We generalize the notion of Kantorovich-type sampling operator given in [1] by replacing the Steklov mean with its more general analogue, the Fejér-type singular integral and get for $f \in L^p(\mathbb{R})$ $(1 \leqslant p \leqslant \infty)$ the operator $(t \in \mathbb{R};\ W>0;\ n \in \mathbb{N})$

$$S_{W,n}^{\chi;\varphi} f(t) := \sum_{k=-\infty}^{\infty} \left(\int\limits_{-\infty}^{\infty} f(u) nW\chi(nW(\frac{k}{W}-u))\,du \right) \varphi(Wt-k). \tag{2}$$

The operators are well-defined when the following conditions are satisfied: $\varphi, \chi \in L^1(\mathbb{R})$, $\int\limits_{-\infty}^{\infty} \chi(u)du = 1$, $\sum_{k\in\mathbb{Z}} \varphi(u-k) = 1$, and $\sum_{k=-\infty}^{\infty} |\varphi(u-k)| < \infty$ $(u \in \mathbb{R})$.

By means of the operator (2) we are able to reconstruct functions (signals) which are not necessarily continuous. Moreover, our generalization allows us to take the measurement error into account. Our main goal is to estimate the rate of approximation by the above operators via high-order modulus of smoothness. We obtain these estimates for kernels with some certain properties.

We also consider the application of multivariate generalized sampling operators in digital image processing (more specifically, in resolution enhancement challenge). We demonstrate the superiority of some kernels φ over the commonly used bicubic kernel.

References

[1] C. Bardaro, P. L. Butzer, R. L. Stens, and G. Vinti, “Kantorovich-type generalized sampling series in the setting of Orlicz spaces,” *Sampling Theory in Signal and Image Processing*, vol. 6, pp. 29–52, 2007.

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A multivariate generalization of Pronys method

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Prony's method is a prototypical eigenvalue analysis based method for the reconstruction of a finitely supported complex measure on the unit circle from its moments up to a certain degree. In other words, it solves the super-resolution problem of deconvolving a signal into its generating spike train and a bandlimited convolution kernel from finitely many input data. Here, we want to present a generalization of this method to the multivariate case and provide simple conditions under which the problem admits a unique solution.

COMPRESSED MOTION SENSING FOR TOMOGRAPHIC PARTICLE IMAGE VELOCIMETRY

STEFANIA PETRA

ABSTRACT. In previous work [PS14, PSS13] we analyzed representative ill-posed scenarios of Tomographic Particle Image Velocimetry (Tomo PIV) [ESWvO07] with a focus on conditions for unique volume reconstruction. Based on sparse random seedings of a region of interest with small particles, the corresponding systems of linear projection equations were probabilistically analyzed in order to determine: (i) the ability of unique reconstruction in terms of the imaging geometry and the critical sparsity parameter, and (ii) sharpness of the transition to non-unique reconstruction with ghost particles when choosing the sparsity parameter improperly. We showed that the sparsity parameter directly relates to the seeding density used for Tomo PIV that is chosen empirically to date. Our results provide a basic mathematical characterization of the Tomo PIV volume reconstruction problem that is an essential prerequisite for any algorithm used to actually compute the reconstruction. Moreover, we have connected the sparse volume function reconstruction problem from few tomographic projections to major developments in compressed sensing (CS) and found out that the predicted critical seeding lies below the theoretical optimal threshold in CS.

In more recent work [DPS16] we complement the standard tomographic sensor, based on few projections, by additional measurements of moving objects at two subsequent points in time. Denoting by A the Tomo PIV sensor that corresponds to few projections synchronously recorded with few cameras only, the standard approach is to reconstruct an image pair (u, u_t) from $Au \approx b$, $Au_t \approx b_t$ and then - in a subsequent step - to estimate the unknown flow transport mapping $T_t(u) = u_t$ by cross-correlating (u, u_t).

Our approach is to use the available information at time step t, to consider the projections b_t as additional measurements together with b and to *jointly estimate* the images and the transformation parameters from the available multi-view measurements. Thus, we solve

$$\min_{T_t, u \geq 0} \|Au - b\|^2 + \|AT_t(u) - b_t\|^2$$

and regard $AT_t(\cdot)$ as an *additional* sensor. From the CS viewpoint this raises the key question if and how much the recovery performance of the complemented sensor

$$A_T := \begin{pmatrix} A \\ AT_t(\cdot) \end{pmatrix}, \quad A_T\, u = \begin{pmatrix} b \\ b_t \end{pmatrix}$$

improves, under the assumption that T_t is known. We call compressed sensing in connection with the correspondence information $u_t = T_t(u)$ *compressed motion sensing*. We evaluate both theoretically and numerically the recovery performance of A vs. A_T and show that our approach enables highly compressed sensing in dynamic imaging scenarios of practical relevance.

REFERENCES

[DPS16] R. Dalitz, S. Petra, and C. Schnörr, *Compressed Motion Sensing*, 2016, Technical report will be made publicly available on a preprint server.

[ESWvO07] G. Elsinga, F. Scarano, B. Wieneke, and B. van Oudheusden, *Tomographic particle image velocimetry*, Exp. Fluids **41** (2007), 933–947.

[PS14] S. Petra and C. Schnörr, *Average Case Recovery Analysis of Tomographic Compressive Sensing*, Linear Algebra Appl. **441** (2014), 168–198.

[PSS13] S. Petra, C. Schnörr, and A. Schröder, *Critical Parameter Values and Reconstruction Properties of Discrete Tomography: Application to Experimental Fluid Dynamics*, Fundam Inform. **125** (2013), 285–312.

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An algorithmic framework for Mumford-Shah regularization of inverse problems in imaging

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The Mumford-Shah model is a very powerful variational approach for edge preserving regularization of image reconstruction processes. However, it is algorithmically challenging because one has to deal with a non-smooth and non-convex functional. We propose a new efficient algorithmic framework for Mumford-Shah regularization of inverse problems in imaging. It is based on a splitting into specific subproblems that can be solved exactly. We derive fast solvers for the subproblems which are key for an efficient overall algorithm. Our method neither requires a priori knowledge on the gray or color levels nor on the shape of the discontinuity set. We demonstrate the wide applicability of the method for different modalities. In particular, we consider the reconstruction from Radon data, inpainting, and deconvolution. Our method can be easily adapted to many further imaging setups. The relevant condition is that the proximal mapping of the data fidelity can be evaluated within reasonable time. In other words, it can be used whenever classical Tikhonov regularization is possible. This is joint work with Kilian Hohm and Andreas Weinmann.

On approximation properties of generalized sampling operators

Gert Tamberg
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A natural application of sampling operators is imaging. We can represent an discrete 2D image f as a continuous function using sampling series

$$(Sf)(x,y) := \sum_{j,k} f(j,k)s_1(x-j)s_2(y-k). \tag{1}$$

Many image resizing (resampling) algorithms use such type of representation.

The generalized sampling operator is given by ($t \in \mathbb{R}$; $w > 0$)

$$(S_w f)(t) := \sum_{k=-\infty}^{\infty} f\Big(\frac{k}{w}\Big)s(wt-k). \tag{2}$$

In this talk we study an even band-limited kernel s, defined as Fourier cosine transform of an even window function $\lambda \in C_{[-1,1]}$, $\lambda(0) = 1$, $\lambda(u) = 0$ ($|u| \geqslant 1$).

We say that $f \in BV(\mathbb{R})$, the space of functions of bounded variation on $\mathbb{R}$, if $f \in BV[a,b]$ for every finite $[a,b] \subset \mathbb{R}$ and the total variation

$$V_{\mathbb{R}}[f] := \lim_{n\to\infty} V_{[-n,n]}[f]$$

is finite. If $f \in AC(\mathbb{R})$, the space of absolutely continuous functions, then we have $AC(\mathbb{R}) \subset BV(\mathbb{R})$ and $V_{\mathbb{R}}[f] = \|f'\|_1$.

We will estimate the order of approximation of the sampling operator (2) for functions f belonging in the space of absolutely continuous functions $AC(\mathbb{R}) \subset BV(\mathbb{R})$ in terms of modulus of smoothness.

Exploiting hidden sparsity for image reconstruction from nonlinear measurements

Andreas Tillmann

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In recent years, the concept of sparsity has been successfully exploited in various signal and image reconstruction tasks. In particular, in the context of compressed sensing, it was shown that sparse or compressible signals (i.e., those with few relevant components) can be efficiently and reliably recovered from few linear measurements. Furthermore, since a basis or dictionary with respect to which a signal is sparsely representable is generally not known a priori, several machine learning methods were developed to automatically train suitable dictionaries and associated representation coefficients directly from the measurement data.

In this talk, we discuss novel variants of such sparsity-based learning tasks. The focus will be on a new method that is able to learn a dictionary and sparse representations from noise-corrupted nonlinear measurements without signal phase information; experiments in the context of image reconstruction demonstrate significant quality improvements of our method compared to state-of-the-art phase retrieval algorithms that cannot exploit "hidden" sparsity.

B) Meshless methods

Organizers: Alessandra De Rossi and Elisa Francomano

Approximation of scattered data using positive and accurate partition of unity interpolants

Roberto Cavoretto, Alessandra De Rossi, Emma Perracchione
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Dealing with applications, the problem of approximating large and irregular data sets is rather common. In this case, problems as lack of information or ill-conditioning arise. Because of such problems, recent research mainly concentrates on local techniques, such as the Partition of Unity (PU) method.

PU interpolation takes advantage of decomposing the domain into several *subdomains* or *patches* which cover the original one. In literature, except for well-known cases [1], such subdomains consist of hyperspheres of a fixed size. But, in case of irregular data, this might lead to inaccurate approximations.

Thus, considering Radial Basis Functions (RBFs) as local approximants, one can select suitable sizes of the different PU subdomains and safe values for the shape parameter of the local basis functions. For instance, this can be carried out by computing subsequent error estimates [2]. Anyway, always in applications, given a set of samples, we often have additional properties, such as the positivity of the measurements, which we wish to be preserved during the interpolation process.

To this aim, in [3] a global positive RBF approximant is constructed by adding up to the interpolation conditions several positive constraints. However, since a global interpolant is used, adding up other constraints to preserve the positivity implies that the shape of the curve/surface is consequently globally modified and this might lead to a considerable decrease of the quality of the approximating function in comparison with the unconstrained interpolation.

In order to avoid such drawback, focusing on 2D data sets, the PU method is performed by imposing positive constraints on the local RBF interpolants. Such approach enables us to consider constrained interpolation problems only in those PU subdomains that do not preserve the required property. This leads to an accurate method compared with existing techniques.

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On the enhancement of the approximation order of the triangular Shepard method

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The Shepard method is one of the oldest techniques used to interpolate large sets of scattered data [4]. The classical Shepard operator reconstructs an unknown function as a normalized blend of the function values at the scattered points, using the inverse distances to the scattered points as weight functions. Based on the general idea of defining interpolants by convex combinations, Little [2] suggested to extend the bivariate Shepard operator in two ways. On the one hand, he considers a triangulation of the scattered points and substitutes function values with linear polynomials which locally interpolate the given data at the vertices of each triangle. On the other hand, he modifies the classical point-based weight functions and defines, instead, a normalized blend of the locally interpolating polynomials with triangle-based weight functions which depend on the product of inverse distances to the three vertices of the corresponding triangle. The resulting triangular Shepard operator interpolates all data required for its definition, reproduces polynomials up to degree 1, whereas the classical Shepard operator reproduces only constants, and has quadratic approximation order [3]. In this talk we discuss on some improvements of the triangular Shepard operator. In particular, we substitute the linear polynomials with quadratic and cubic polynomials which approximate Bernoulli polynomials on the triangle in a least square sense and locally interpolate at the vertices. The resulting operators reproduce polynomials of degree greater than one and have approximation order greater than 2.

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A stable algorithm for divergence and curl-free radial basis functions in the flat limit

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The direct method used for calculating smooth radial basis function (RBF) interpolants in the flat limit becomes numerically unstable. The RBF-QR algorithm bypasses this ill-conditioning using a clever change of basis technique. We extend this method for computing interpolants involving matrix-valued kernels, specifically divergence-free and curl-free RBFs on the sphere, in the flat limit. Results illustrating the effectiveness of this algorithm are presented as well as applications to computing the Helmholtz-Hodge decomposition of a vector field on the sphere from samples at scattered points. This is joint work with Prof. Grady Wright (Boise State University).

Analysis of the Allee threshold via Moving Least Square approximation.

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Abstract.

Cooperation is a common behavior between the members of predators species, because it can improve theirs skill in hunt, especially in endangered eco-systems. This behavior it is well known to induce the Strong Allee effect, that can induce the extinction when the initial populations' is under a critical density called "Allee threshold ". Here we investigate the impact of the pack hunting in a predator-prey system in which the predator suffers of an infectious disease with frequency and vertical transmission. The result is a three dimensional system with the predators population divided into susceptible and infected individuals. Studying the system dynamics a scenario was identified in which the model presents a bistability. However for a strong hunting cooperation the Allee threshold becomes almost zero, ensuring the survival of the predators.
Thus we present a study to analyze this critical density by considering the basins of attraction of the stable equilibrium points. This paper addresses the question of finding the point lying on the surface which partitions the phase plane. Therefore a Moving Least Square (MLS) method based on compactly supported radial functions has been adopted to reconstruct the separatrix manifold.

Keywords dynamical systems; predator-prey model; basins of attraction; meshless approximation.

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Non-symmetric kernel-based approximation

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Abstract

We analyze kernel-based recovery problems defined by general, and possibly distinct, trial and test spaces.

This setting allows to analyze in a common framework two notable situations, namely point-based interpolation with kernel centers different from the data sites, and non-sym-metric collocation. While the second is frequently observed to give better approximations than symmetric collocation, the first one allows to construct interpolants which are not bounded to be centered on the given data. For some kernel, both of them can be also extended to deal with variable shape parameters.

After discussing error and stability properties of this recovery procedure in an abstract setting, and comparing it with the symmetric methods, we will specialize our analysis to the interpolation case. In particular, we discuss a greedy algorithm to adaptively construct data-dependent test and trial spaces. This algorithm is proven to be an extension of the method introduced in [5], and in particular it is treated by means of bi-orthonormal bases which generalize the Newton basis [4].

Different greedy-selection criteria will be presented, and we will discuss their properties. Some of them will be shown to be extensions of the criteria proposed in [1, 3, 6]. Experimentally, we demonstrate their potential on artificial examples as well as on real world applications.

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CLOUD COARSENING FOR THE MLPG/DMLPG SOLUTION OF DIFFUSION PROBLEMS

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Point clouds are easier to refine/coarse than meshes, this is the main point which makes meshless methods more apt to adaptive strategies than Finite Element Methods. After devising efficient test and trial spaces for Meshless Petrov–Galekin (MLPG) methods [2], and suitable refining strategies [1], in this presentation we introduce a coarsening strategy which is a fundamental key to adaptivity. We analyze by numerical experiments the feasibility of our coarsening strategy. We compare the accuracy of MLPG vs Direct MLPG (DMLPG) when applying our coarsening procedure, in order to identify the most stable and accurate technique.

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Direct Approximation on Spheres Using Generalized Moving Least Squares

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May 31, 2016

Abstract

The *moving least squares* (MLS) approximation has been developed for pure function approximation on spheres by some authors. See for example [2]. The application of MLS for solving partial differential equations (PDEs) on spheres and other manifolds is much involved, because one should evaluate the PDE operators on non-close form and complicated MLS shape functions. This might be the reason why MLS has been rarely used for PDEs on manifolds. In this talk, we avoid this approach and suggest a *direct approximation* using a generalized moving least squares (GMLS). The idea of GMLS was first introduced in [1] in $\mathbb{R}^d$. The new technique eliminates the action of operators on shape functions and replaces them by much cheaper evaluations on spherical harmonics. In fact, GMLS recovers test functionals directly from values at nodes, without any detour via shape functions. The method is meshless, because the unknown quantities are parameterized entirely in terms of scattered nodes on the spheres. The error analysis of the method is given and, as an application, the Laplace–Beltrami equation is solved. In the work's conclusion limitations and suggestions for new researches are presented.

Keywords: Moving least squares, Spherical harmonics, Local polynomial reproduction, Norming sets, Laplace-Beltrami equation.
Mathematics Subject Classification (2010): 41Axx, 65Nxx, 65Dxx.

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Rational Quadrature for Meshless methods

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Abstract

The numerical integration of the Element-Free Galerkin (EFG) forms for Meshless methods is studied and some improvements are provided. Indeed, integrating without taking into account the characteristics of the shape functions reproduced by Meshless methods (rational functions, compact support ...), causes a large integration error that influences the PDE's approximate solution. In this work, the rational quadrature rule [1] and "the bounding box technique" [2] are combined to improve the numerical integration. The performance of the procedure is demonstrated on test problems in 1D .

Keywords: Meshless, Numerical Integration, Element Free Galerkin, Uniform nodes distribution, Rational Quadrature.

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IMPLEMENTATION OF MULTIPLE BOUNDARY CONDITIONS IN RBF COLLOCATION METHOD

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Numerical solutions of the PDEs are routinely computed by researchers in many different areas. Collocation methods based on RBFs have become important when trying to obtain the numerical solution of various ordinary differential equations (ODEs) and partial differential equations (PDEs). Even for the one-dimensional case, how to implement multiple boundary conditions for a time-dependent global collocation problem is not obvious. In this case, we need to enforce two boundary conditions at each end point resulting in a total of four boundary conditions at the two boundary points. Fictitious or ghost point methods have been commonly used as a way to enforce multiple boundary conditions in finite difference methods. The implementation for global collocation methods such as pseudospectral methods is due to Fornberg [1].

The aim of this talk is to show that the Rosenau equation, as an initial-boundary value problem with multiple boundary conditions, can be implemented using RBF approximation methods [2]. For this aim, the fictitious point method and the resampling method are studied in combination with an RBF collocation method. The numerical experiments show that both methods perform well.

Keywords: collocation method, radial basis function, multiple boundary conditions
Classification: MSC 65M70, MSC 35G31

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* Speaker.

C) Multiresolution techniques: recent insights and applications

Organizers: Mariantonia Cotronei and Lucia Romani

Hermite interpolation by incenter subdivision scheme

Chongyang Deng Huining Meng

Abstract

Given a sequence of points and associated tangent vectors, we can get a smooth curve interpolating the initial points by incenter subdivision scheme, in which the new point corresponding to an edge is the incenter of a triangle formed by the edge and the two tangent lines of the two end points. Since the tangents are updated in each subdivision step, in general the limit curves do not interpolate the initial tangent vectors. In this paper, we show that incenter subdivision scheme can also be used to interpolate Hermite data by selected special rules for the first subdivision step.

A multilevel optimization technique with applications to yacht design.

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The performance of a sailing yacht may be improved by optimizing the shape of some of its appendages, like the rudder and the keel. As a first step, we consider the optimization of an appendage section in terms of the drag it generates, while maintaining certain structural features. If the optimization process is based on considering a modification of the original shape by means of a large number of parameters, the procedure to find an 'optimal' shape may be very costly.

We describe a multilevel strategy on the parameter space, based on a multiresolution transform, that involves solving an optimization problem at each resolution level, starting from an initial guess that corresponds to the optimal solution at the previous level. The use of the multilevel technique on several academic examples shows the improvement in performance. This improvement allows us to compute an 'optimal' shape, with respect to the drag it generates, for the sections of the bulb and the keel of a sailing yacht.

Keywords: Optimization, yacht design, sections, multiresolution methods.

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Clifford-Valued B-Splines

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Recently, ideas and methods from the theory of Clifford algebra and analysis have been applied to image and signal analysis. One prominent example is the analysis of four-component seismic data (consisting of a hydrophone (scalar part) and three orthogonally oriented geophones (vector part) by Olhede et al. using quaternionic wavelets.

For the purposes of multichannel signal and image analysis, one naturally needs one direction for each channel. This suggests to consider extensions of complex-valued transforms to higher dimensions, i.e., to quaternion- and Clifford-valued bases and their associated transforms.

In this presentation, we introduce the mathematical background in Clifford algebra and analysis, and define quaternionic B-splines. Several algebraic and analytic properties of quaternionic B-splines are presented. We prove the refinability of these novel B-splines and show that they define a dyadic multiresolution analysis of $L^2(\mathbb{R}, \mathbb{H})$.

Hermite subdivision on manifolds

Caroline Moosmüller*

Hermite subdivision schemes are refinement algorithms that operate on discrete point-vector data and produce a curve and its derivatives in the limit. Most results on Hermite schemes concern data in vector spaces and rules which are linear. We are interested in Hermite schemes that operate on manifold-valued data and are defined by the intrinsic geometry of the underlying manifold (in particular, by geodesics and parallel transport).

We analyse such nonlinear subdivision rules with respect to convergence and C^1 smoothness. Similar to previous work on subdivision in manifolds, we use the method of proximity to conclude convergence and smoothness properties of a manifold-valued scheme from a "close-by" linear one.

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Multigrid methods and Subdivision schemes

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Multigrid is an iterative method for solving linear systems of equations whose coefficient matrices are symmetric and positive definite. Each iteration consists of two steps: the so-called smoother and the coarse grid correction. The convergence rate of multigrid depends on the properties of the smoother and the so-called grid transfer operator appearing at the coarse grid correction step. In this talk, we show the link between the properties of multigrid methods and of subdivision schemes. We construct new grid transfer operators from subdivision symbols. We show that the polynomial generation property and stability of the corresponding basic limit functions are crucial for the fast convergence of the corresponding multigrid method. Our numerical results, both on primal binary and ternary pseudo-splines, illustrate the behaviour of the new grid transfer operators when applied to linear systems arising from elliptic PDE's with different discretizations (finite differences, iso-geometric analysis, etc.).

This work is in collaboration with Maria Charina (University of Vienna), Marco Donatelli (University of Insubria - Como) and Lucia Romani (University of Milano-Bicocca).

Irregular Tight Wavelet Frames: Matrix Approach

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To construct compactly supported tight wavelet frames in the shift-invariant setting there are powerful tools such as the Unitary Extension Principle, or the Oblique Extension Principles for higher vanishing moments. It is well-known that those principles lead to matrix extension problems. The entries of the corresponding matrices are trigonometric polynomials. In the univariate setting, explicit expressions for such extensions are known for a wide class of trigonometric polynomials arising from refinement equations. Our first goal is to reduce such matrix extension problems to the factorization of positive semi-definite matrices with real entries. In the non shift-invariant setting, Chui, He and Stöckler showed how to construct tight frame elements via the factorization of global positive semi-definite matrices derived from B-splines over irregular knot sequences ([1], [2]). Our second goal is to construct such global matrices for general univariate irregular MRA and to simplify their factorizations. Our simplification leads to the factorization of few positive semi-definite matrices of much smaller size.

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On the linear independence of truncated hierarchical generating systems

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Motivated by the necessity to perform adaptive refinement in geometric design and numerical simulation, the construction of hierarchical splines from generating systems spanning nested spaces has been recently studied in several publications [1,2,3,4]. Linear independence can be guaranteed with the help of the local linear independence of the spline basis at each level.

We extends this framework in several ways. Firstly, we consider spline functions that are defined on domain manifolds, while the existing constructions are limited to domains that are open subsets of $\mathbb{R}^d$. Secondly, we generalize the approach to generating systems containing functions which are not necessarily non-negative. Thirdly, we present a more general approach to guarantee linear independence and present a refinement algorithm that maintains this property. The three extensions of the framework are then used in several relevant applications: doubly hierarchical B-splines, hierarchical Zwart-Powell elements, and three different types of hierarchical subdivision splines.

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D) Multivariate polynomial approximation and pluripotential theory

Organizers: Mirosław Baran and Leokadia Białas-Cież

Orthogonal polynomials for equilibrium measures and Chebyshev polynomials

Gökalp Alpan
(Bilkent University, Ankara, Turkey)
The 4th Dolomites Workshop on Constructive Approximation and Applications, Alba di Canazei, Italy, September 2016

In this talk, we consider orthogonal polynomials associated with equilibrium measures on $\mathbb{R}$. We focus on the link between logarithmic capacity and the Hilbert norm of these polynomials. We discuss the implications and possible outcomes of this connection related with Chebyshev polynomials.

Radial functions related to Pleśniak's conditions

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Abstract. It was discovered by W. Pleśniak in 1990 that A. Markov's inequality for polynomials in $\mathbb{C}^N$

$$||D_j P||_E \leqslant M(\deg P)^m ||P||_E,\ j = 1, \ldots, N,\ E \subset \mathbb{C}^N$$

is equivalent to the existence of a positive constant M_2 such that $|P(z)| \leqslant M_1 ||P||_E$ for all $P \in \mathbb{P}_n(\mathbb{C}^N)$ and $\mathrm{dist}\,(z, E) \leqslant 1/n^m$.

If we consider a modified inequality

$$|P(z)| \leqslant M_1 k^m ||P||_E \qquad \text{for } \ \mathrm{dist}\,(z, E) \leqslant (k/n)^m,$$

we get a condition equivalent to V. Markov's estimate

$$||D^\alpha P||_E \leqslant M_2^{|\alpha|} (\deg P)^{m|\alpha|} (1/|\alpha|!)^{m-1} ||P||_E,$$

that is equivalent to the Hölder continuity of the Siciak's extremal function Φ_E. We shall present functions associated to the above Pleśniak's conditions in terms of radial functions $\varphi_n(E, r)$ where

$$\varphi_n(E, r) = \sup_{||z|| \leqslant r} \{||P(x + z)||_E :\ P \in \mathbb{P}_n(\mathbb{C})^N,\ ||P||_E \leqslant 1\}.$$

We shall give two constructions of radial functions and some capacities related to them.

Degrees of Polynomial Appoximation in holomorphic Carleman classes

Moulay Taïb BELGHITI Boutayeb EL AMMARI Laurent P. GENDRE

Ibn Tofaïl University, Morocco and Paul Sabatier University, France

Abstract

In this talk, we extend results of M.S. Baouendi and C. Goulaouic (Ann. Inst. Fourrier, 1971 ; Trans. Amer. Math. Soc, 1974), obtained for compacts of $\mathbb{R}^N$ with analytic boundary. If K is a compact of $\mathbb{C}^N \backsimeq \mathbb{R}^{2N}$, $(N \geq 1)$, $\mathcal{H}_M(K)$ is the space of $\overline{\partial}$-Whitney jets on K which are of class $\{M\}$, where $M(t) = t^t e^{t\mu(t)}$, $t >> 0$ and μ belongs to a Hardy field.
We prove that a jet $F := (F^\alpha)_{\alpha \in \mathbb{N}^{2N}} \in \mathcal{H}_M(K)$ if and only if there exist a constant $C > 0$, such that

$$\lim_{n\to\infty} d_n(F^\alpha, K)\exp(C\overline{\omega}_{K,M}(n)) = 0, \quad \text{for all} \quad \alpha \in \mathbb{N}^{2N}, \tag{1}$$

where $d_n(\cdot, K)$ is the distance, for the uniform norm on K to the complex vectorial space of polynomials of degree at most n, and where $\overline{\omega}_{K,M}$ is a weight depending on the class $\{M\}$ and K.

If K is Whitney-regular

$$\mathcal{H}_M(K) \backsimeq \left\{ f \in \mathcal{E}^\infty(K) \cap \mathcal{O}(\dot{K}) : \exists C > 0,\ \exists \rho > 0,\ \|D^\alpha f\|_K \leqslant C\rho^{|\alpha|}M(|\alpha|),\ (\forall \alpha \in \mathbb{N}^N) \right\}.$$

In this situation, $f \in \mathcal{H}_M(K)$ if and only if $\lim_{n\to\infty} d_n(f, K)e^{C\overline{\omega}(n)} = 0$, where $C > 0$ and $\overline{\omega}$ is a weight depending on {M}. Finally, we annonce similar results in the situation where K is a compact of some Stein manifold. A crucial role is played by a new geometric criteria : the Łojasiewicz-Siciak condition for the Green function of K.

Selected polynomial inequalities

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Polynomial estimates remain an area of intense interest. The investigations are inspired mainly by three classical inequalities:

- *Schur inequality:*
  $$\|p\|_{[-1,1]} \leqslant (n+1) \max_{x\in[-1,1]} |(x-a)\,p(x)|, \qquad a \in [-1,1],$$
- *Bernstein inequality:*
  $$|p'(x)| \leqslant \frac{n}{\sqrt{1-x^2}}\,\|p\|_{[-1,1]}, \qquad x \in (-1,1),$$
- *Markov brother's inequality:*
  $$\|p^{(k)}\|_{[-1,1]} \leqslant \|T_n^{(k)}\|_{[-1,1]}\|p\|_{[-1,1]}, \qquad k \in \mathbb{N}$$
  for every polynomial p of degree at most n, where T_n is the n-th Chebyshev polynomial of the first kind.

In the talk, we focus on selected recent results concerning some generalizations of these inequalities.

FEKETE POINTS AS NORMING SETS

LEN BOS

ABSTRACT. Suppose that $K \subset \mathbb{R}^d$ is a compact set. The Fekete points for K of degree n are those points in K which maximize the Vandermonde determinant associated to the polynomials of degree at most n, restricted to K. A sequence of finite set $\Omega_n \subset K$ is said to be a *norming set* for K if there exists a constant C such that

$$\|p\|_K \leq C\|p\|_{\Omega_n},$$

for all polynomials p of degree at most n. It is said to be of optimal order if $\#(\Omega_n) = O(n^d)$.

We discuss the conjecture that for any constant $A > 1$, the Fekete points of degree An form an optimal order norming set for K and, in particular, give some cases when it holds.

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Equivalence of the global and local Markov inequalities in complex space

Raimondo Eggink

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Building on seminal work by L.P. Bos and P.D. Milman (GAFA, 1995) we will review the current status of open problems related to the (lack of) equivalence of the global and local Markov inequalities in complex space.

Some Asymptotics of Polynomials in Terms of Potential Theory

Alexander Goncharov

(Bilkent University, Ankara, Turkey)

The 4th Dolomites Workshop on Constructive Approximation and Applications, Alba di Canazei, Italy, September 2016

We discuss Chebyshev polynomials and orthogonal polynomials with respect to continuous singular measures. Our concern is the asymptotic behavior of the so-called Widom factors.

Parametric Polynomial Circle Approximation

Gašper Jaklič and Jernej Kozak
University of Ljubljana and University of Primorska, Slovenia

Uniform approximation of a circle arc (or a whole circle) by a parametric polynomial curve is considered. The approximant is obtained in a closed form. It depends on a parameter that should satisfy a particular equation, and it takes only a couple of tangent method steps to compute it. For low degree curves the parameter is provided exactly. The distance between a circle arc and its approximant asymptotically decreases faster than exponentially as a function of polynomial degree. The approximant can be applied for a fast evaluation of trigonometric functions and for a good bivariate parametric polynomial approximation of the sphere.

Generalizations of Markov inequality and the best exponents

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Generalizations of the classical Markov inequality have been the subject of research for 125 years. The development of the theory of the multivariate Markov inequality and the search for the best exponent in this inequality, called the Markov exponent, were very extensive in the second half of the twentieth century. Currently, this inequality is still being generalized in many different ways. Such research was carried on for curves and submanifolds in $\mathbb{R}^N$ (Bos, Brudnyi, Levenberg, Milman, Taylor, Totik, Baran, Pleśniak, Kosek, Coman, Poletski, Gendre); for Julia sets (Kosek); in Banach spaces (Sarantopoulos, Harris, Muñoz, Baran); in L^p norms (Tamarkin, Hille, Szegö, Goetgheluck, Baran, Sroka) in o-minimal structures (Pleśniak, Pierzchała). Searching for the sharp exponent is more difficult if we consider the Markov-type inequality in norms different than the supremum norm. In this case, many more questions remain unanswered.

On multivariate fast decreasing polynomials

András Kroó
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Univariate fast decreasing polynomials p_n of degree n on the interval $[-1, 1]$ attain value 1 at some point $x_0 \in [-1, 1]$, and decrease as we move away from this point according to the relation

$$|p_n(x)| \leq Ce^{-cn\phi(|x-x_0|)},\ x \in [-1, 1],\ n \in \mathbb{N}$$

with $C, c > 0$ independent of n, and ϕ being some positive function tending to zero as $h \to 0$. The characterization of the optimal function $\phi(h)$ providing the fastest possible decrease was found by Ivanov and Totik. The rate of this function depends on x_0 being an inner or boundary point of the interval.

In this talk we will discuss **multivariate** fast decreasing polynomials. Even in the univariate case there is an essential difference between the decrease around inner or boundary points. This phenomena becomes more intricate in the multivariate case. It will be shown that the rate of decrease of the multivariate fast decreasing polynomials is closely related to the *smoothness of the boundary* at the corresponding points. We shall consider both ordinary and homogeneous fast decreasing multivariate polynomials when the underlying domain is locally star like or convex.

Algebra and pluripotential theory on curves and varieties

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Notions of transfinite diameter and Chebyshev constant are important in pluripotential theory. They can be studied in some depth using only polynomial algebra, without reference to plurisubharmonic functions. They naturally extend to algebraic varieties in $\mathbb{C}^n$, with the algebra getting a bit more involved. As in classical pluripotential theory, these notions ought to be related to functions in the Lelong class and the pluricomplex Green function. For an algebraic curve (that has 'generic' good behaviour at infinity) we found an explicit formula linking directional Chebyshev constants to the growth of Sadullaev's Green function along different directions. As an application, Chebyshev constants can be used to deduce some interesting behaviour of the Green function on a sequence of quadratic curves. I will give a possible "potential theoretic" interpretation of this behaviour.

Analysis of the stability and accuracy of discrete least-squares approximation on multivariate polynomial spaces

Giovanni Migliorati*

May 30, 2016

We review the results achieved in the analysis of stability and accuracy for discrete least-squares approximations on multivariate polynomial spaces, with noiseless [2, 3, 1, 4] or noisy evaluations [1, 5] at random points, or with noiseless evaluations at low-discrepancy point sets [6]. Afterwards we present some recent results from [7], where we have proven that a judicious choice of the weights and of the sampling measure provides a stable and accurate weighted least-squares approximation, under the minimal condition that the number of evaluations is linearly proportional to the dimension of the approximation space, up to a logarithmic factor.

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Some recent results on Bernstein type inequalities for rational functions on the complex plane

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Bernstein's inequality states that if P_n is a polynomial with degree n, then for any $t \in (-1,1)$

$$|P_n'(t)| \leq n \, \frac{1}{\sqrt{1-t^2}} \, \|P_n\|_{[-1,1]}$$

where $\|.\|_{[-1,1]}$ is the sup norm over the set $[-1,1]$. Note that on the right there is a factor which is independent of the polynomial (sometimes so-called "geometric factor"). Originally, Bernstein proved this inequality to establish an inverse theorem in approximation theory. In the last decades there were several generalizations and further applications of this inequality.

It is interesting that for arbitrary subsets of the real line $\mathbf{R}$, Bernstein type inequality was established relatively recently by Baran (1994) and Totik (2001) independently. This result uses potential theory (in particular, the density of the equilibrium measure) and it is sharp.

On the complex plane, asymptotically sharp Bernstein type inequality for polynomials was established by Nagy and Totik (2005). In this setting the normal derivative of Green's function naturally enters (geometric factor) and the obtained inequality is asymptotically sharp.

Recently, this Bernstein type inequality was generalized to arcs, first circular arcs (Nagy, Totik 2013), then arbitrary analytic arcs (Kalmykov, Nagy 2015). The latter result was established using Gonchar-Grigorjan type estimates, open-up in Widom's sense, fast decreasing polynomials and direct approximation methods (interpolation). The open-up naturally led from polynomials to rational functions. Some partial results were established by Kalmykov and Nagy (rational functions on analytic Jordan arcs and curves) where open-up is also used to reduce the case of Jordan arcs to the case of Jordan curves.

In this talk we mainly focus on recent results (joint with S. Kalmykov and V. Totik) establishing Bernstein type inequality for rational functions on C^2 smooth Jordan arcs and curves when poles are in a fixed compact set (disjoint from the arc or curve).

PLURIPOTENTIAL NUMERICS

FEDERICO PIAZZON

Abstract. We introduce numerical methods for the approximation of the *extremal plurisubharmonic function* V_E^* of a compact $\mathcal{L}$-regular set $E \subset \mathbb{C}^n$ and its *transfinite diameter* $\delta(E)$.

The methods rely on the computation of a *polynomial mesh* for ∂E and numerical orthonormalization of a suitable basis of polynomials. We prove the convergence of the approximation of $\delta(E)$ and the uniform convergence of our approximation to V_E^* on all $\mathbb{C}^n$ providing an *a priori* estimate on the error. Our algorithms are based on the properties of polynomial meshes and Bernstein Markov measures.

Numerical tests are presented for some simple cases with $E \subset \mathbb{R}^2$ to illustrate the performances of the proposed methods.

This is a joint work with Marco Vianello.

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Key words and phrases. Pluripotential theory, extremal plurisubharmonic function, transfinite diameter, admissible mesh, orthogonal polynomials.

MULTIVARIATE MARKOV-TYPE INEQUALITIES

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Abstract

One of the most important polynomial inequalities is the following Markov's inequality.

Theorem (Markov, 1889) *If P is a polynomial of one variable, then*

$$\|P'\|_{[-1,\,1]} \leq (\deg P)^2 \|P\|_{[-1,\,1]}\,.$$

Moreover, this inequality is optimal, because for the Chebyshev polynomials T_n ($n \in \mathbb{N}_0$), we have $T_n'(1) = n^2$ and $\|T_n\|_{[-1,\,1]} = 1$.

Markov's inequality and its generalizations found many applications in approximation theory, constructive function theory and analysis (for instance, to Whitney-type extension problems), but also in other branches of science (for example, in physics or chemistry). From the point of view of applications, it is important that the constant $(\deg P)^2$ in Markov's inequality grows not too fast (that is, polynomially) with respect to the degree of the polynomial P.

It is natural to ask about similar inequalities if we replace the interval $[-1,\,1]$ by another compact set in $\mathbb{R}^N$ or $\mathbb{C}^N$. In the talk, we will address this issue.

ON A UVAROV TYPE MODIFICATION OF ORTHOGONAL POLYNOMIALS ON THE UNIT BALL

MIGUEL A. PIÑAR

Abstract

In the present work, we study orthogonal polynomials with respect to a Uvarov type modification of the classical inner product on the unit ball. Namely, we consider the inner product

$$\langle f, g\rangle_\mu^\lambda = \frac{1}{\omega_\mu}\int_{\mathbb{B}^d} f(x)g(x)W_\mu(x)dx + \frac{\lambda}{\sigma_d}\int_{\mathbb{S}^{d-1}} f(\xi)g(\xi)d\sigma(\xi),$$

where ω_μ is a normalizing constant, $d\sigma$ denotes the surface measure and σ_{d-1} denotes the surface area. Here $\lambda > 0$ represents a mass uniformly distributed over the sphere.

Using spherical polar coordinates, we shall construct a sequence of mutually orthogonal polynomials with respect to $\langle\cdot,\cdot\rangle_\mu^\lambda$, which depends on a family orthogonal polynomials in one variable. These polynomials are orthogonal with respect to a Uvarov modification of a varying Jacobi measure and therefore they can be expressed in terms of Jacobi polynomials. Explicit representations tor the polynomials, the norms and the kernels will be obtained.

A very interesting problem in the theory of multivariate orthogonal polynomials is that of finding asymptotic estimates for the Christoffel functions. These estimates are useful in the study of the convergence of the Fourier series. Asymptotics for Christoffel functions associated to the classical orthogonal polynomials on the ball were obtained by Y. Xu in 1996 (see [2]). Recently, more general results on the asymptotic behaviour of the Christoffel functions for some kind of regular measures were established by Kroó and Lubinsky [1].

Since our system of orthogonal polynomials does not fit into the above mentioned context, the asymptotic of the Christoffel functions deserves special attention. Not surprisingly, our results show that in any compact subset of the interior of the unit ball Christoffel functions for the Uvarov type modification behave exactly as in the classical case. On the sphere the situation is quite different and we can perceive the influence of the mass λ.

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Bivariate polynomials and structured matrices

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It is shown that inverses of structured matrices, namely Toeplitz, Hankel, and Toeplitz-plus-Hankel matrices, can be suitable introduced as polynomials of two variables, called Bezoutians. How to obtain the coefficients of the involved polynomials and matrix representations of the inverses is discussed. Moreover, recent results of joint papers with Torsten Ehrhardt concerning the reverse problem - the inversion of Bezoutians - are presented.

E) Numerical integration, integral equations and transforms

Organizers: Gradimir V. Milovanović and Donatella Occorsio

On connection coefficients for some perturbed of arbitrary order of the Chebyshev polynomials of second kind

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Keywords: Chebyshev polynomials of second kind; perturbed orthogonal polynomials; connection coefficients; zeros, interception points.

ABSTRACT

Orthogonal polynomials satisfy a recurrence relation of order two, where appear two coefficients. If we modify one of these coefficients at order r, we obtain a perturbed orthogonal sequence. In this work, we consider, in this way, some perturbed of Chebyshev polynomials of second kind and we deal with the problem of finding the connection coefficients [1, 2] that allow to write the perturbed sequence in terms of the original one, and in terms of the canonical basis. From the connection relations obtained, we deduce some results about zeros and interception points of these perturbed polynomials. All the work is valid for any order r of perturbation. Other properties of these polynomials were obtained in [3, 4], for $r \leq 3$. The integral representation of the perturbed forms remain an open problem for $r \geq 2$.

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Approximation of hypersingular integral transforms on the real axis

Maria Carmela De Bonis, Donatella Occorsio*

In this talk we propose some numerical procedures for approximating hypersingular integrals of the type

$$\mathbf{H}_p(fw_{\alpha,\beta},t) \quad = \quad \ =\!\!\!\!\!\!\int_{-\infty}^{+\infty} \frac{f(x)}{(x-t)^{p+1}} w_{\alpha,\beta}(x)dx, \quad t \in \mathbb{R}, \tag{1}$$

where $w_{\alpha,\beta}(x) = |x|^\alpha e^{-|x|^\beta}$ is a generalized Freud weight with $\alpha \geq 0, \beta > 1$ and $0 \leq p \in \mathbb{N}$.

This topic is of interest, for instance, in the numerical solution of hypersingular integral equations, which are often models for physics and engineering problems (see [5, 2, 4]). To our knowledge, most of the papers available in the literature deal with the approximation of Hadamard integrals on bounded intervals (see for instance [4] and the references therein) and the case on the real semiaxis has been considered recently in [1, 3]. We propose here different procedures, which are differently convenient, according that the computation of the integral is required in "many" or "few" values of the parameter t. The convergence and stability of the proposed methods are proved and error estimates are given. Some numerical tests are shown in order to compare their performances.

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A numerical method for a Volterra integral equation related to the initial value problem for the KdV equation

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This talk deals with the following Volterra integral equation

$$k(x, x+y) - \frac{1}{2}\int_0^y \int_{x+\frac{1}{2}(y-x)}^{\infty} q(t)k(t, t+s)dt\, ds = \frac{1}{2}\int_{x+\frac{1}{2}y}^{\infty} q(t)dt, \qquad y \geq 0$$

where $q \in L^1(\mathbb{R}, (1+|x|)dx)$ and k is the bivariate unknown function.

The above equation arises in the solution of the following initial value problem for the Korteweg-de Vries (KdV) equation which governs the propagation of waves in shallow water

$$\begin{cases} \dfrac{\partial q(t,x)}{\partial t} - 6q(t,x)\dfrac{\partial q(t,x)}{\partial x} + \dfrac{\partial^3 q(t,x)}{\partial x^3} = 0, \qquad x \in \mathbb{R}, \qquad t \in \mathbb{R}^+ \\ q(0,x) = q(x). \end{cases}$$

The proposed numerical method takes advantage of an analytical study of the equation according to which it is sufficient to solve it on a specific triangle and not on the entire unbounded domain.

This is a joint work with Cornelis van der Mee and Sebastiano Seatzu.

A NEW CLASS OF CONSTRUCTIVE PRECONDITIONED INTEGRAL EQUATION MODELS FOR SIMULATION OF WAVE PROPAGATION

M. GANESH AND C. MORGENSTERN

ABSTRACT. We consider the Helmholtz acoustic wave propagation model in a bounded media Ω with an inhomogeneous impedance boundary condition on $\partial\Omega$. It is well known that the standard Galerkin variational integral equation formulation of the Helmholtz partial differential equation (PDE) in $H^1(\Omega)$ is indefinite for large wavenumbers, while the Helmholtz PDE is not indefinite. The lack of coercivity (indefiniteness) in the standard integral equation and associated finite element method (FEM) models is one of the major difficulties for simulating wave propagation models using iterative methods.

The concept used in some literature of terming the Helmholtz equation indefinite was questioned in a 2014 SIAM Review article. For the constant coefficient Helmholtz equation case, it was theoretically demonstrated in the article that a non-standard integral equation model can produce a sign-definite variational formulation of the wave propagation model in homogeneous media.

However, the authors of the theoretical article also questioned the practical use of their new sign-definite formulation even for the constant coefficient Helmholtz equation with impedance boundary condition.

Our investigation begins with addressing this key issue of a specific parameter based formulation in the SIAM Review article Through various computer simulations, we provide a concrete answer that the sign-definite formulation analyzed in the 2014 article does not alleviate the key difficulty of reducing the GMRES iterations of the associated FEM model if the choice of parameter that facilitates the proof of sign-definiteness of the formulation used in the article.

We subsequently develop, analyze, and implement a new class of constructive FEM wave propagation models in both homogeneous and heterogeneous media.

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A DISCRETE TOP-DOWN MARKOV PROBLEM IN APPROXIMATION THEORY

Walter Gautschi

Abstract

The Markov brothers' inequalities in approximation theory concern polynomials p of degree n and assert bounds for the kth derivatives $|p^{(k)}|$, $1 \leq k \leq n$, on $[-1, 1]$, given that $|p| \leq 1$ on $[-1, 1]$. Here we go the other direction, seeking bounds for $|p|$, given a bound for $|p^{(k)}|$. For the problem to be meaningful, additional restrictions on p must be imposed, for example, $p(-1) = p'(-1) = \cdots = p^{(k-1)}(-1) = 0$. The problem then has an easy solution in the continuous case, where the polynomial and their derivatives are considered on the whole interval $[-1, 1]$, but is more challenging, and also of more interest, in the discrete case, where one focusses on the values of p and $p^{(k)}$ on a given set of $n - k + 1$ distinct points in $[-1, 1]$. Analytic solutions are presented and their fine structure analyzed by computation.

Collocation-quadrature for the notched half plane problem

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A Cauchy singular integral equation describing the notched half plane problem of two-dimensional elasticity theory is considered. This equation contains an additional fixed singularity represented by a Mellin convolution operator. We study a polynomial collocation-quadrature method for its numerical solution which takes into account the "natural" asymptotic of the solution at the endpoints of the integration interval and for which until now no criterion for stability is known. We present a new technique for proving that the operator sequence of the respective collocation-quadrature method belongs to a certain C^*-algebra, in which we can study the stability of these sequences. One of the main ingredients of this technique is to show that the part of the operator sequence, associated with the Mellin part of the original equation, is "very close" to the finite section of particular operators belonging to a C^*-algebra of Toeplitz operators. Moreover, basing on these stability results numerical results are presented obtained by an implementation of the proposed method.

The talk is based on joint work with Robert Kaiser, Chemnitz, Germany.

On the numerical solution of integral equations of Mellin type in weighted L^p spaces

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We are interested in the numerical solution of second kind integral equations with fixed singularities of Mellin convolution type given by

$$f(y) + \int_0^1 k(x,y)f(x)dx + \int_0^1 h(x,y)f(x)dx = g(y), \quad y \in (0,1], \tag{1}$$

where f is the unknown, h and g are smooth functions and

$$k(x,y) = \pm\frac{1}{x}\bar{k}\left(\frac{y}{x}\right) \tag{2}$$

is a Mellin kernel, defined by means of a function $\bar{k} : [0, +\infty) \to [0, +\infty)$ satisfying suitable assumptions.
Since the kernel $k(x,y)$ has a fixed-point singularity at $x = y = 0$, the corresponding integral operator

$$(Kf)(y) = \int_0^1 k(x,y)f(x)dx$$

is non-compact. Consequently, the standard stability proofs for numerical methods do not apply and a modification of the classical methods in a neighbourhood of the endpoint $y = 0$ is needed.
Generalizing the results in [1], we propose to approximate the solutions of (1) in weighted L^p spaces by applying a "modified" Nyström method which uses a Gauss-Jacobi quadrature formula. The modification of the classical method essentially consists in a new suitable approximation of the integral transform $(Kf)(y)$ in points very close to 0, where the convergence of the Gaussian rule is not assured.
The stability and the convergence are proved in weighted L^p spaces and error estimates are also given. Moreover, some numerical results show the effectiveness of the method.

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Computing integrals of highly oscillatory special functions using quadrature processes

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Abstract

The standard methods for numerical integration are not applicable to integration of rapidly oscillating functions, which appear in the theory of special functions, as well as in many applications in theoretical physics, quantum chemistry, the theory of transport processes, acoustic scattering, problems in electromagnetics, fluid mechanics, etc. Conventional techniques for computing values of special functions are power series, asymptotic expansions, continued fractions, differential and difference equations, and so on. Using suitable integral representation of special functions, in this lecture, we show how existing or specially developed quadrature formulas can be successfully applied to effectively calculation values of some special functions, such as highly oscillatory integrals of Fourier type with Hankel kernel, oscillatory Bessel transformation, Bessel-Hilbert transformation, etc. Theoretical results and numerical examples illustrate the efficiency and accuracy of the proposed methods.

On the Numerical Approximation of Some Non-standard Volterra Integral Equations

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American option pricing can be formulated as a free boundary problem for the Black-Scholes Equation (BSE) and as soon as the free boundary is evaluated, the option price will be known. Different approaches of investigation of the BSE, such as Fourier and Laplace transforms and Green function method give us different integral equations, different at least in their form [2, 3]. Some of them can be classified as follows

$$x(t) = \varphi(x(t), t) + \int_0^t K(t, s, x(t), x(s))ds, \quad t \in [a, b], \tag{1}$$

where the kernel $k(t, s, x, y)$ is smooth or weakly singular function. The aim this paper is to introduce these classes of integral equations and other ones arising in this field. The existence issue of the Eq. (1) by the Schauder fixed point theorem is discussed.

A computational method based on barycentric rational interpolatory quadrature [1, 4] is introduced for approximating Eq. (1) and for simplicity of the presentation is analyzed for the special case

$$x(t) = y(t) + \int_0^t K(t, s, x(t), x(s))ds, \quad t \in [a, b]. \tag{2}$$

Finally, the results will be compared with other methods in the financial math literature, especially an implicit Runge-Kutta discretization and methods based on fixed point iteration [5].

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ORTHOGONAL POLYNOMIALS FOR POLLACZECK–LAGUERRE WEIGHTS ON THE REAL SEMIAXIS

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The weighted polynomial approximation for functions defined on $(0, +\infty)$ which can grow exponentially at 0^+ and/or $+\infty$ has been considered only recently in the literature [3, 4, 5, 6], dealing in particular with estimates for the best weighted approximation, polynomial inequalities and Gaussian rules. Also, some properties of the orthonormal system related to the weight $\sigma(x) = \mathrm{e}^{-x^{-\alpha}-x^{\beta}}$, with $\alpha > 0$, $\beta > 1$ and $x \in (0, +\infty)$, have been studied.

Here, in order to construct approximation processes such as Lagrange interpolation and Fourier sums, we consider the orthonormal system $\{p_m(w)\}_m$ associated to weight

$$w(x) = x^{\gamma}\mathrm{e}^{-x^{-\alpha}-x^{\beta}} \qquad x \in (0, +\infty)\,,$$

where $\alpha > 0$, $\beta > 1$ and $\gamma \geq 0$. We observe that the weight w can be seen as a combination of a Pollaczeck-type weight $\mathrm{e}^{-x^{-\alpha}}$ and a Laguerre-type weight $x^{\gamma}\mathrm{e}^{-x^{\beta}}$. Nevertheless the properties of $\{p_m(w)\}_m$ cannot be deduced from previous results concerning these two weights.

We show that the weight w can be reduced to a weight belonging to the Levin–Lubinsky class $\mathcal{F}(C^2+)$ [2] and then we obtain estimates for the polynomials $p_m(w)$, their zeros and the associated Christoffel function. Moreover, some applications to Gaussian rules and Lagrange interpolation will be discussed.

We remark that the weight w is nonclassical and the coefficients of the three terms recurrence relation for $\{p_m(w)\}_m$ are not explicitly known. So, for the computation of the zeros and the Christoffel number we use a procedure given in [6] and the Mathematica package `OrthogonalPolynomials` [1].

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ANTI-GAUSSIAN QUADRATURE FORMULAE BASED ON THE ZEROS OF STIELTJES POLYNOMIALS

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Abstract. It is well known that a practical error estimator for the Gauss quadrature formula is by means of the corresponding Gauss-Kronrod quadrature formula developed by Kronrod in 1964. However, recent advances show that Gauss-Kronrod formulae fail to exist, with real and distinct nodes in the interval of integration and positive weights, for several of the classical measures. An alternative to the Gauss-Kronrod formula, as error estimator for the Gauss formula, is the anti-Gaussian and the averaged Gaussian quadrature formulae presented by Laurie in 1996. These formulae always exist and enjoy the nice properties that, in several cases, Gauss-Kronrod formulae fail to satisfy. Now, it is remarkable that, for a certain, quite broad, class of measures, for which the Gauss-Kronrod formulae exist, the anti-Gaussian and averaged Gaussian formulae, based on the zeros of the corresponding Stieltjes polynomials, have elevated degree of exactness, and the estimates provided for the error term of the Gauss formula by either the Gauss-Kronrod or the averaged Gaussian formulae are exactly the same.

EXTENDED LAGRANGE INTERPOLATION ON THE REAL SEMIAXIS AND APPLICATIONS TO THE QUADRATURE

DONATELLA OCCORSIO, MARIA GRAZIA RUSSO

Abstract

Let $w(x) = e^{-x^\beta} x^\alpha,$, $\beta > \frac{1}{2}$, be a given Generalized Laguerre weight. Setting $\bar{w}(x) = xw(x)$ and denoting by $\{p_m(w)\}_m, \{p_n(\bar{w})\}_n$ the corresponding sequences of orthonormal polynomials, then the zeros of $Q_{2m+1} = p_{m+1}(w)p_m(\bar{w})$ are simple [2]. The Lagrange interpolation of a given function f, continuous in $(0, +\infty)$, at the zeros of Q_{2m+1} is called *Extended Lagrange Interpolation.*

Here we will give some results on the behaviour of this interpolation process in suitable subspaces of $L^p_u((0, +\infty))$, $1 \le p < +\infty$ with a special attention to the case $p = 1$.

Moreover as an application we show how to use the above mentioned extended interpolation process for approximating integrals of the type

$$\int_0^{+\infty} f(x)k(x,y)w(x)dx, \qquad y > 0,$$

where $k(x, y)$ should also be a weakly singular function for $x = y$.

The idea is to construct a "mixed" sequence of *product integration rules* obtained by approximating alternatively f by means of the ordinary Lagrange polynomial based on the zeros of $p_{m+1}(w)$ and the introduced extended Lagrange polynomial.

In this way we can double the number of nodes of the quadrature formula, but reusing $m+1$ already computed samples of the function f. This approach is really relevant when m is "large" and the procedure for computing the zeros and the weights of the quadrature rule can fail or is too much expensive [1].

We will prove the stability and the convergence of the proposed quadrature procedure, showing also some numerical tests which confirm the theoretical estimates.

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PRODUCT INTEGRATION RULES ON THE SQUARE

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This talk deals with the approximation of integrals of the type

$$I(f;\mathbf{y}) = \int_{\mathbf{D}} K(\mathbf{x},\mathbf{y}) f(\mathbf{x}) \omega(\mathbf{x}) d\mathbf{x}, \quad \mathbf{x}=(x_1,x_2), \quad \mathbf{y}=(y_1,y_2),$$

$$\mathbf{D} = [-1,1]\times[-1,1],$$

where the function f is sufficiently smooth inside the square, with possible algebraic singularities on the border, ω is the product of two Jacobi weights and the kernel K is a "nearly"singular kernel. Kernels of this type, appear, for instance, in the numerical solution of integral for BEM 3D (see [1], [4]) and in the numerical solution of bivariate integral equations (see [2], [3]). The stability and the convergence of the cubature rule is proved, and some numerical tests, which confirm the theoretical estimates, are proposed.

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High-dimensional integration of kinks and jumps – smoothing by preintegration

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In many applications, including option pricing, integrals of d-variate functions with "kinks" or "jumps" are encountered. (Kinks describe simple discontinuities in the derivative, jumps describe simple discontinuities in function values.) The standard analyses of sparse grid or Quasi-Monte Carlo methods fail completely in the presence of kinks or jumps not aligned to the axes, yet the observed performance of these methods can remain reasonable.

In recent joint papers with Michael Griebel and Frances Kuo we sought an explanation by showing that many terms of the ANOVA expansion of a function with kinks can be smooth, because of the smoothing effect of integration. The problem settings have included both the unit cube and d-dimensional Euclidean space. The underlying idea is that integration of a non-smooth function with respect to a well chosen variable, say x_j, can result in a smooth function of $d-1$ variables.

In still more recent joint work with Andreas Griewank, Hernan Leovey and Frances Kuo we have extended the theoretical results from kinks to jumps, and have turned "preintegration" into a practical method for evaluating integrals of non-smooth functions over d-dimensional Euclidean space. In this talk I will explain both the method and the ideas behind "smoothing by preintegration".

New classes of the index transforms and their applications to solutions of higher order PDE's

Semyon Yakubovich

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Abstract

We discuss new index transforms, which are associated with the modified Bessel functions as the kernel. The boundedness and invertibility are examined for these operators in the Lebesgue weighted spaces. Inversion theorems are proved. Important particular cases are exhibited. The results are applied to solve initial value problems for higher order PDE, involving the Laplacian.

F) Sparse approximation

Organizers: Annie Cuyt and Wen-shin Lee

Sublinear-Time Fourier Algorithms

Sina Bittens*

In 2010, M.A. Iwen (in Found. Comput. Math., 10(3):303-338, 2010) introduced a deterministic combinatorial sublinear-time Fourier algorithm for estimating the best k term Fourier representation for a given frequency sparse signal, relying heavily on the Chinese Remainder Theorem and combinatorial concepts. In 2016 a different deterministic sublinear Fourier algorithm for input signals with small support length was proposed, which employs periodizations of the signal and requires that the signal length is a power of 2 (Plonka and Wannenwetsch in Numerical Algorithms, 71(4):889-905, 2016).

In this talk we will develop Iwen's algorithm from examples for the case of an input function with small support length, combining the Chinese Remainder Theorem approach for arbitrary signal lengths with the structure given by the small support. This reduces the runtime of the algorithm as the effortful combinatorial part can be omitted.

A hybrid Fourier-Prony method

Matteo Briani[1], Annie Cuyt[1], and Wen-shin Lee[1]

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The FFT algorithm that implements the Discrete Fourier Transform is considered one of the top ten algorithms of the 20th century. Its main strengths are the low computational cost of $\mathcal{O}(n \log n)$ and its stability. It is one of the most common algorithms that is used to analyze signals with a dense frequency representation. In recent years there has been an increasing interest in sparse signal representation and a need for new algorithms that exploit such structure. We propose a new technique that combines the properties of the Discrete Fourier Transform with the sparsity of the signal. This is achieved by integrating ideas of Prony's method into Fourier's method. The resulting technique has the same frequency resolution as the original FFT algorithm but uses fewer samples and can achieve a lower computational cost.

A Region Based Easy Path Wavelet Transform for Sparse Image Representation

Renato Budinich, joint work with Gerlind Plonka
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In [1] G. Plonka proposed an innovative method for image compression: successively finding a suitable path in the image (i.e. reducing it to a one-dimensional signal) and applying one level of a one dimensional wavelet transform. This yields a sparse representation which behaves better than the typical tensor product wavelet transform. However there are adaptivity costs: for each level one has to store the path, i.e. a permutation of the pixel points.

We propose a variation on this method, which consists in first segmenting the image into regions, then successively for every level find in each region a path (in some canonical manner, not depending on the pixel values) and apply a one-dimensional wavelet transform to it. We will discuss various details of this approach and present some numerical examples.

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Identification problems in sparse sampling

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Abstract

We consider the interpolation of an n-variate exponential sum

$$F(x_1,\ldots,x_n)=\sum_{j=1}^{t}c_je^{f_{j,1}x_1+f_{j,2}x_2+\cdots+f_{j,n}x_n}.$$

In the univariate case, $n=1$, there is an entire branch of algorithms, which can be traced back to Prony's method dated in the 18th century, devoted to the recovery of the $2t$ unknowns, $c_1,\ldots,c_t,f_1,\ldots,f_t$, in

$$F(x)=\sum_{j=1}^{t}c_je^{f_jx}.$$

In the multivariate case, $n>1$, it remains an active research topic to identify and separate distinct multivariate parameters from results computed by a Prony-like method from samples along projections.

On top of the above, if the $f_{j,k}$ are allowed to be complex, the evaluations of the imaginary parts of distinct $f_{j,k}$ can also collide. This aliasing phenomenon can occur in either the univariate or the multivariate case.

Our method interpolates $F(x_1,\ldots,x_n)$ from $(n+1)\cdot t$ evaluations. Since the total number of parameters c_j and $f_{j,k}$ is exactly $(n+1)\cdot t$, we interpolate $F(x_1,\ldots,x_n)$ from the minimum possible number of evaluations. The method can also be used to recover the correct frequencies from aliased results.

Essentially, we offer a scheme that can be embedded in any Prony-like algorithm, such as the least squares Prony version, ESPRIT, the matrix pencil approach, etc., thus can be viewed as a new tool offering additional possibilities in exponential analysis.

A low-rank matrix completion approach to data-driven signal processing

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Abstract

In filtering, control, and other mathematical engineering areas it is common to use a model-based approach, which splits the problem into two steps:

1. model identification and
2. model-based design.

Despite its success, the model-based approach has the shortcoming that the design objective is not taken into account at the identification step, i.e., the model is not optimized for its intended use.

In this talk, we show a data-driven approach, which combines the identification and the model-based design into one joint problem. The signal of interest is modeled as a missing part of a trajectory of the data generating system. Subsequently, the missing data estimation problem is reformulated as a mosaic-Hankel structured matrix low-rank approximation/completion problem. A local optimization method, based on the variable projections principle, is then used for its numerical solution.

The missing data estimation approach for data-driven signal processing and the local optimization method for its implementation in practice are illustrated on examples of control, state estimation, filtering/smoothing, and prediction. Currently, we are missing fast algorithms with provable properties in the presence of measurement noise and disturbances. Development of such methods will make the matrix completion approach for data-driven signal processing a practically feasible alternative to the model-based methods.

Reference

I. Markovsky. A low-rank matrix completion approach to data-driven signal processing. Technical report, Vrije Univ. Brussel, 2015. http://homepages.vub.ac.be/~imarkovs/publications/ddsp.pdf

On rational functions without Froissart doublets

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Abstract

In this talk we consider the problem of working with rational functions in a numeric environment. A particular problem when modeling with such functions is the existence of Froissart doublets, where a zero is close to a pole. We discuss three different parameters which allow one to monitor the absence of Froissart doublets for a given general rational function. These include the euclidean condition number of an underlying Sylvester-type matrix, a parameter for determing coprimeness of two numerical polynomials and bounds on the spherical derivative. We show that our parameters sharpen those found in previous papers [1], [2].

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THE SPARSE GAUSS-NEWTON ALGORITHM FOR UNDERDETERMINED SYSTEMS OF EQUATIONS

ANNA OLEYNIK AND MÅRTEN GULLIKSSON

We develop the algorithm to find sparse solutions to a nonlinear underdetermined system of equations

$$\begin{aligned} f_1(x_1,\ldots,x_N) &= 0 \\ &\vdots \\ f_m(x_1,\ldots,x_N) &= 0 \end{aligned}$$

or simply

$$f(x) = 0. \tag{1}$$

Here $x \in \mathbb{R}^N$, $f : D \subset \mathbb{R}^N \to \mathbb{R}^m, m < N$, is twice continuously differentiable on the open convex set D and $0 \in f(D)$.

Given

$$\|x\|_0 = \sharp\{i : x_i \neq 0\}$$

the problem of finding the most sparse solution to (1) reads

$$\begin{array}{l} \min_x \|x\|_0 \\ \text{s.t. } f(x) = 0. \end{array} \tag{2}$$

Due to the combinatorial complexity the problem (2) is considered to be intractable and the current algorithms do not guarantee that a sparse solution attained is a solution to (2).

However, there is a numerical algorithm to find sparse (but not necessarily the most sparse) solutions of (1) based on a convex relaxation of the problem, a so called ℓ_1 - update method. We suggest an alternative approach that we refer to as the sparse Gauss-Newton method. This is a line search method where we calculate $x_{k+1} = x_k + \alpha_k p_k$ to update the approximation x_k, with p_k being a search direction and α_k the step length. The search direction is chosen to provide a descent of the merit function and is based on linear optimization greedy algorithms, and α_k satisfies some standard criteria to ensure global convergence.

We show that starting from some $k \geq K \in \mathbb{N}$ the method is equivalent to the Gauss-Newton method for underdetermined system of equations and converges globally with a quadratic rate of local convergence.

We test the algorithm versus the ℓ_1 - update method to illustrate its advantages and convergence properties.

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Approximate inversion of the Black-Scholes formula using a parametric Barycentric representation

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Abstract

In this presentation, we tackle an important practical problem: the real-time evaluation of implied volatility. It is well-known that the Black–Scholes implied volatility (IV) is one of the most useful and important concepts for options traders. Since a closed–form solution for the involved inverse problem does not exist, numerical root–finding methods are typically employed. In practice, such numerical methods are the computational bottleneck when millions of options need to be inverted in real–time situations. An attractive alternative are analytical approximations which can deliver IVs instantaneously.

Based on S&P 500 index option data, we first illustrate that existing bivariate approximations may not be sufficiently accurate. Then we introduce our bivariate rational approximation to the IV. We give an overview of its construction and highlight some of the important decisions taken to reach the final result.

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Sparse high-dimensional FFT using rank-1 (Chebyshev) lattices

Toni Volkmer, TU Chemnitz

We consider the (approximate) reconstruction of a high-dimensional (e.g. $d = 10$) periodic signal from samples using a trigonometric polynomial $p_I \colon \mathbb{T}^d \simeq [0,1)^d \to \mathbb{C}$,

$$p_I(\boldsymbol{x}) := \sum_{\boldsymbol{k}\in I} \hat{p}_{\boldsymbol{k}} \, \mathrm{e}^{2\pi\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}, \quad \hat{p}_{\boldsymbol{k}} \in \mathbb{C},$$

where $I \subset \mathbb{Z}^d$ is a suitable and unknown frequency index set. For this setting, we present a method which adaptively constructs the index set I of frequencies belonging to the non-zero or (approximately) largest Fourier coefficients in a dimension incremental way. This method computes projected Fourier coefficients from samples along suitable rank-1 lattices $\Lambda(\boldsymbol{z}, M) := \{\frac{j}{M}\boldsymbol{z} \mod \boldsymbol{1} \colon j = 0, \ldots, M-1\}$, $\boldsymbol{z} \in \mathbb{Z}^d$, and then determines the frequency locations. For the computation, only one-dimensional fast Fourier transforms (FFTs) and simple index transforms are used. When we assume that the signal has sparsity $s \in \mathbb{N}$ in frequency domain and the frequencies $\boldsymbol{k}$ are contained in the cube $[-N, N]^d \cap \mathbb{Z}^d$, $N \in \mathbb{N}$, our method requires $\mathcal{O}(d\, s^2 N)$ samples and $\mathcal{O}(d\, s^3 + d\, s^2 N \log(s\, N))$ arithmetic operations in the case $\sqrt{N} \lesssim s \lesssim N^d$.

This method can be transfered to the non-periodic case, where we consider the (approximate) reconstruction of a multi-dimensional signal restricted to the domain $[-1,1]^d$ using an algebraic polynomial $a_{\tilde{I}} \colon [-1,1]^d \to \mathbb{R}$ in Chebyshev basis,

$$a_{\tilde{I}}(\boldsymbol{x}) := \sum_{\boldsymbol{k}\in\tilde{I}} \hat{a}_{\boldsymbol{k}} \, T_{\boldsymbol{k}}(\boldsymbol{x}) = \sum_{\boldsymbol{k}\in\tilde{I}} \hat{a}_{\boldsymbol{k}} \prod_{t=1}^{d} T_{k_t}(x_t), \quad \hat{a}_{\boldsymbol{k}} \in \mathbb{R},$$

where $\tilde{I} \subset \mathbb{N}_0^d$, $\boldsymbol{x} := (x_1, \ldots, x_d)^\top \in [-1,1]^d$, $\boldsymbol{k} := (k_1, \ldots, k_d)^\top \in \mathbb{N}_0^d$ and $T_k(x) := \cos(k \arccos x)$ for $k \in \mathbb{N}_0$. Here, we sample along suitable rank-1 Chebyshev lattices $\mathrm{CL}(\boldsymbol{z}, M) := \{\cos(\frac{j}{M}\pi\boldsymbol{z}) \colon j = 0, \ldots, M\}$, $\boldsymbol{z} \in \mathbb{N}_0^d$, and again only use one-dimensional FFTs / discrete cosine transforms as well as simple index transforms.

This is joint work with Daniel Potts.

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A multivariate generalization of Prony's method

Ulrich von der Ohe

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Motivated by a problem from physics, in 1795 de Prony proposed a method to reconstruct the parameters of an exponential sum, *i. e.* a linear combination of exponential functions, given a finite set of samples of sufficient cardinality [2]. By his approach the original interpolation problem is translated into the problem of solving a single univariate polynomial equation. Several variants and generalizations of Prony's method have been studied recently, *cf.* Plonka-Tasche [4] for a survey. In particular the multivariate case has been studied [1, 5, 3]. This talk is about a generalization of Prony's method to the case of multivariate exponential sums that is based on solving systems of multivariate polynomial equations. In particular, we consider some of its algebraic properties.

This talk is based on joint work with Stefan Kunis, H. Michael Möller, Thomas Peter, and Tim Römer. Support by DFG-GRK1916 is gratefully acknowledged.

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P) Poster session

Organizers: Roberto Cavoretto and Marco Vianello

Determining system poles using rows of sequences of orthogonal Hermite-Padé approximants

Nattapong Bosuwan and Guillermo López Lagomasino

Abstract

We introduce a new definition of orthogonal Hermite-Padé approximants. We study a relation of the convergence of poles of row sequences of orthogonal Hermite-Padé approximants and the system poles of the vector of approximated functions. We give necessary and sufficient conditions for the convergence with geometric rate of the common denominators of orthogonal Hermite-Padé approximants. Thereby, we obtain analogues of the theorems of Montessus de Ballore [3] and Gonchar [2] which extend results in [1].

Keywords: Padé approximants of orthogonal expansions, Padé-orthogonal approximation, Orthogonal polynomials, Fourier-Padé approximation, Inverse problems, Montessus de Ballore's theorem, Orthogonal Hermite-Padé approximants.
2010 Mathematics Subject Classification: Primary 30E10, 41A27; Secondary 41A21.

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Bivariate orthogonal polynomials, 2D Toda lattices and Lax–type representation

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Abstract

We explore the connection between an infinite system of points in $\mathbb{R}^2$ described by a bi–dimensional version of the Toda equations with the standard theory of orthogonal polynomials in two variables. We consider a Toda lattice in one–time variable t and two dimensional space variables describing a mesh of interacting points over the plain. We prove that this Toda lattice is related with the matrix coefficients of the three term relations for bivariate orthogonal polynomials associated with an exponential modification of the measure. Moreover, Lax–type pairs for block matrices is deduced.

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OpenCL based parallel algorithm for RBF-PUM interpolation

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We present a parallel algorithm for multivariate Radial Basis Function Partition of Unity Method (RBF-PUM) interpolation [1]. The concurrent nature of the PUM allows us to deal with a large number of scattered data-points in high space dimensions. To efficiently exploit this concurrency, our algorithm makes use of shared-memory parallel processors through the OpenCL standard. This efficiency is achieved by a parallel space partitioning strategy with linear computational-time complexity with respect to the input and evaluation points.

The speed of our algorithm allows for computationally more intensive construction of the interpolant. In fact, the RBF-PUM can be coupled with a cross validation technique that searches for optimal values of the shape parameters associated with each local RBF interpolant [2], thus reducing the global interpolation error. The numerical experiments support our claims by illustrating the interpolation errors and the running times of our algorithm.

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Meshless methods for pulmonary image registration

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Lung is one of the most important organs in the human respiratory system but is also one of the most prone to be lesioned. Hence, analysis of pulmonary images for the diagnosis and treatments of lung diseases deserves to be considered. Moreover, during the implementation of image-guided and computer-aided operations, compensating the deformation of pulmonary images is crucial. Based on these reasons, image registration of lungs has become a key technology. In this poster, meshless methods and state of the art for non-rigid pulmonary image registration are reviewed. We also discuss the research trend and the role of pulmonary image registration in clinical implications.

Spectral filtering for the resolution of the Gibbs phenomenon in MPI applications

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Polynomial interpolation on the node points of Lissajous curves using Chebyshev series is an effective way for a fast image reconstruction in Magnetic Particle Imaging. Due to the nature of spectral methods, a Gibbs phenomenon occurs in the reconstructed image if the underlying function has discontinuities. A possible solution for this problem are spectral filtering methods acting on the coefficients of the interpolating polynomial.

In this work, after a description of the Gibbs phenomenon in two dimensions, we present an adaptive spectral filtering process for the resolution of this phenomenon and for an improved approximation of the underlying function or image. In this adaptive filtering technique, the spectral filter depends on the distance of a spatial point to the nearest discontinuity. We show the effectiveness of this filtering approach in theory, in numerical simulations as well as in the application in Magnetic Particle Imaging.

A rescaled method for RBF approximation

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In the recent paper [1], a new method to compute stable kernel-based interpolants has been presented. This *rescaled interpolation* method combines the standard kernel interpolation with a properly defined rescaling operation, which smooths the oscillations of the interpolant. Although promising, this procedure lacks a systematic theoretical investigation.

Through our analysis, this novel method can be understood as standard kernel interpolation by means of a properly rescaled kernel. This point of view allow us to consider its error and stability properties.

First, we prove that the method is an instance of the Shepard's method, when certain weight functions are used. In particular, the method can reproduce constant functions.

Second, it is possible to define a modified set of cardinal functions strictly related to the ones of the not-rescaled kernel. Through these functions, we define a Lebesgue function for the rescaled interpolation process, and study its maximum - the Lebesgue constant - in different settings.

Also, a preliminary theoretical result on the estimation of the interpolation error is presented.

As an application, we couple our method with a partition of unity algorithm. This setting seems to be the most promising, and we illustrate its behavior with some experiments.

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Integration on manifolds by mapped low-discrepancy points and greedy minimal k_s-energy points

Stefano De Marchi[1] and Giacomo Elefante[2]

Abstract

We know that low-discrepancy points are the best choice in order to integrate function through quasi-Monte Carlo method (qMCM) in the unit cube $[0,1]^d$. When we map them to a manifold, the map should preserve their *uniformity* in the unit cube, with respect to the Lebesgue measure, so that they will remain a set of good points for the qMCM on the manifold.

Thanks to the Poppy-seed Bagel Theorem (cf. [1]) we know that the minimal Riesz s-energy are asymptotically uniformly distributed with respect to the normalized Hausdorff measure, and so they are an appropiate choice of points to integrate functions on manifolds via the qMCM.

A method for extracting minimal Riesz s-energy points from a discretization of the manifold is the so-called *greedy algorithm*. In principle we do not know if these extracted points remain a good choice to integrate functions by using the qMC approach. Through numerical experiments we attempt to answer to this question.

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Abstract

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The Dynamical Functional Particle Method

Let $\mathcal{F}$ be an operator and $v = v(x), v : X \to \mathbb{R}^k, k \in \mathbb{N}$, where X is a Banach space that will be defined by the actual problem setting. Consider the abstract equation

$$\mathcal{F}(v) = 0 \tag{1}$$

that could be, e.g., a system of linear equations or a differential equation. Our main idea is to solve the differential equation

$$\mu u_{tt} + \eta u_t = \mathcal{F}(u). \tag{2}$$

for $u = u(x,t), u : X \times T \to \mathbb{R}^k$ such that $u_t, u_{tt} \to 0$ when $t \to t_1, t_1 \leq \infty$, i.e., $\lim_{t\to t_1} u(x,t) = v(x)$.

The parameters $\mu = \mu(x,u(x,t),t), \eta = \eta(x,u(x,t),t)$ are the mass and damping parameters that can be used to improve the convergence. In addition, the two initial conditions $u(t_0)$ and $u_t(t_0)$ can be chosen for a good inital starting approximation.

For a differential equation (1) and (2) need to be discretized to attain a numerical solution. For simplicity, we exemplify by applying finite differences but it is possible to use, e.g., finite elements, basis sets or any other method of discretization. We define a grid $x_1,x_2,\ldots$ and approximate $v(x_i)$ by v_i and assume that the discretized version of (1) can be written as

$$F_i(v_1 \ldots, v_n) = 0, i = 1, \ldots, n \tag{3}$$

where $F_i : \mathbb{R}^n \to \mathbb{R}$.

Turning to the dynamical system (2) it is discretized such that $u_i(t)$ approximates $u(x_i,t)$ and $\mu_i(t) = \mu(x_i,u_i(t),t), \eta_i(t) = \eta(x_i,u_i(t),t)$ for $i = 1,\ldots,n$. Further, $\mathcal{F}(u)$ is discretized as $\mathcal{F}(v)$ in (3) and we approximate (2) with the system of ordinary differential equations

$$\mu_i \ddot{u}_i + \eta_i \dot{u}_i = F_i(u_1, \ldots, u_n), i = 1, \ldots, n \tag{4}$$

with initial conditions $u_i(t_0), \dot{u}_i(t_0)$. Our idea in the discrete setting is to solve (3) by solving (4) such that $\dot{u}_i(t), \ddot{u}_i(t) \to 0$ when $t \to t_1, t_1 \leq \infty$, i.e., $\lim_{t\to t_1} u_i(t) = v_i$. The overall approach for solving (1) using (4) is named the *Dynamical Functional Particle Method*, DFPM.

We will show how to apply DFPM to solve linear equations, linear eigenvalue problems, nonlinear equations, optimization problems with nonlinear constraints, the stationary nonlinear Schrödinger equation (NLSE), and (NLSE) with additional constraints (in the setting of mean-field description of bosonic atoms).

Approximation with Faber–Walsh polynomials on disconnected compact sets in the complex plane

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We present our recent results from [1] on Faber–Walsh polynomials, which allow the series expansion of analytic functions on disconnected compact sets.

More precisely, let $E \subseteq \mathbb{C}$ be a compact set with connected complement. E itself may consists of several disjoint components. Then the Faber–Walsh polynomials $b_k(z)$ for E have degree k. Any analytic function f on E has an expansion in a Faber–Walsh series

$$f(z) = \sum_{k=0}^{\infty} a_k b_k(z), \quad z \in E,$$

which is uniformly and maximally convergent on E. The Faber–Walsh series generalizes several classical series expansions, in particular the Taylor series on a disk, the Chebyshev series on $[-1, 1]$ and the Faber series on a simply connected compact set.

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Around operators not increasing the degree of polynomials

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Abstract

We present a generic operator J simply defined as a linear map not increasing the degree from the vectorial space of polynomial functions into itself and we address the problem of finding the polynomial sequences that coincide with the (normalized) J-image of themselves. The technique developed assembles different types of operators and initiates with a transposition of the problem to the dual space.

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Quadrature of Quadratures: Compressed Sampling by Tchakaloff Points

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We show that a discrete version of Tchakaloff theorem on the existence of positive algebraic cubature formulas, entails that the information required for multivariate polynomial approximation can be suitably compressed; cf. [4, 5, 6]. Extracting such "Tchakaloff points" from the support of discrete measures by NonNegative Least Squares (NNLS) applied to the moment system, we are able to compress algebraic quadrature, least square approximation and polynomial meshes on multivariate compact sets; cf. [1, 2, 3, 5].

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Variational Bézier or B-spline curves and surfaces

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Abstract

Given a control polygon, we use some least-square criterion, to derive "B-curves" (i.e. Bernstein, B-spline, hyperbolic or circular spline curves...) which are closer the control polygon, still being in the same vectorial space as the original one. We usually loose the convex hull property, and better preserve the general shape of the control polygon. The idea is simple : we minimize some L^2-distance between the curve and the control polygon.

Furthermore, by increasing (resp. decreasing) the degree of the parametric polynomial curve, in the same way we derive a curve still closer (resp. further) the control polygon. Similarly we obtain the same type of results by increasing (resp. decreasing) the number of knots of the spline curve

Actually the so-obtained curves (or surfaces, or any multi-dimensional object) are in the space generated by the original B-functions and some "new control points" which are easily derived. The obtained curve (or surface) is the quasi-interpolation, by using the original B-functions, of the so-obtained "new control polygon". We so keep all the known properties of the original quasi-interpolation (including convex hull property), express in this "new control polygon".

We give ways to derive new B-functions which are linear combinations of the original B-functions (i.e. Bernstein polynomials, B-splines, hyperbolic or circular B-spline curves...), such that the associated Bézier curve is closer the control polygon than the usual one (still being in the same functional space, but possibly not in the convex hull of the control polygon), and better preserving the general shape of the control polygon. We also give ways to derive associated basis functions such that the so-obtained curve is further form the control polygon (more cutting the angles), while being in the same functional space.

Finally we propose to mix this least-square criterion together with a least-square distance between some points of the curve and the control points, and with a variational criterion aiming to cut down the oscillations of the curve. Various curves are presented to show the interest of these new curves.

The same strategy is possible for surfaces by using corresponding B-surfaces, such as tensorial product of Bernstein polynomials, of B-splines, or by using polyharmonic B-splines.

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