1 Metodi matematici - G. Bellin

Soluzioni al compito del 7 dicembre 2012

Esercizio 1. Ricorda che $\sim A =_{df} A \supset \bot$.
Si costruisca una derivazione nel sistema di deduzione naturale intuizionistico **NJ** delle seguenti formule dalle assunzioni indicate.

(a) $A \supset (B \supset C) \vdash (A \cap B) \supset C$

$$\dfrac{\dfrac{f : A \supset (B \supset C) \qquad \dfrac{\dfrac{(1)}{x : A \cap B}}{\pi_0 x : A} \cap_0 \mathrm{E}}{(f)\pi_0 x : B \supset C} \supset \mathrm{E} \qquad \dfrac{\dfrac{(1)}{x : A \cap B}}{\pi_1 x : B} \cap_1 \mathrm{E}}{(1)\ \dfrac{((f)\pi_0 x)\pi_1 x : C}{\lambda x.((f)\pi_0 x)\pi_1 x : (A \cap B) \supset C} \supset \mathrm{I}} \supset \mathrm{E}$$

(b) $(A \cap B) \supset C \vdash A \supset (B \supset C)$

$$\dfrac{f : (A \cap B) \supset C \qquad \dfrac{\dfrac{(1)}{x : A} \qquad \dfrac{(2)}{y : B}}{\langle x, y\rangle : A \cap B} \cap \mathrm{I}}{(f)\langle x, y\rangle : C} \supset \mathrm{E}$$
$$(2)\ \dfrac{(f)\langle x, y\rangle : C}{\lambda y.(f)\langle x, y\rangle : B \supset C} \supset \mathrm{I}$$
$$(1)\ \dfrac{\lambda y.(f)\langle x, y\rangle : B \supset C}{\lambda x.\lambda y.(f)\langle x, y\rangle : A \supset (B \supset C)} \supset \mathrm{I}$$

(c) $A \vdash (B \supset A)$

$$\dfrac{x : A}{\lambda y.x : B \supset A} \supset \mathrm{I}$$

(d) $(A \supset\sim B) \vdash (B \supset\sim A)$

$$\dfrac{\dfrac{f : A \supset\sim B \qquad \dfrac{(2)}{x : A}}{(f)x : B \supset \bot} \qquad \dfrac{(1)}{y : B}}{((f)x)y : \bot} \supset \mathrm{E}$$
$$(2)\ \dfrac{((f)x)y : \bot}{\lambda x.((f)x)y : A \supset \bot} \supset \mathrm{I}$$
$$(1)\ \dfrac{\lambda x.((f)x)y : A \supset \bot}{\lambda y.\lambda x.((f)x)y : B \supset\sim A} \supset \mathrm{I}$$

(e) $(A \cap \sim B) \vdash \sim (A \supset B)$

$$\dfrac{\dfrac{\dfrac{x : A \cap \sim B}{\pi_1 x : \sim B}\cap_1 \text{E} \quad \dfrac{\overset{(1)}{f : A \supset B} \qquad \dfrac{x : A \cap \sim B}{\pi_0 x : A}\cap_0 \text{E}}{(f)\pi_0 x : B}\supset \text{E}}{(\pi_1 x)(f)\pi_0 x : \bot}\supset \text{E}}{(1)\ \dfrac{(\pi_1 x)(f)\pi_0 x : \bot}{\lambda f.(\pi_1 x)(f)\pi_0 x :\sim (A \supset B)}\supset \text{I}}$$

(f) $\sim (A \supset B) \vdash (\sim\sim A) \cap \sim B$

$$\dfrac{\dfrac{\sim (A \supset B) \qquad (1)\ \dfrac{\dfrac{\overset{(2)}{\sim A} \qquad \overset{(1)}{A}}{\dfrac{\bot}{B}\bot_{int}}}{A \supset B}\supset \text{I}}{(2)\ \dfrac{\bot}{\sim\sim A}\supset \text{I}} \quad \dfrac{\sim (A \supset B) \qquad \dfrac{\overset{(3)}{B}}{A \supset B}\supset \text{I}}{(3)\ \dfrac{\bot}{B \supset \bot}\supset \text{I}}}{(\sim\sim A) \cap \sim B}$$

(g) $\sim (A \cup B) \vdash \sim A \cap \sim B$

$$\dfrac{\dfrac{\sim (A \cup B) \qquad \dfrac{\overset{(1)}{A}}{A \cup B}\cup_0 \text{I}}{(1)\ \dfrac{\bot}{A \supset \bot}\supset \text{I}} \quad \dfrac{\sim (A \cup B) \qquad \dfrac{\overset{(2)}{B}}{A \cup B}\cup_1 \text{I}}{(2)\ \dfrac{\bot}{B \supset \bot}\supset \text{I}}}{(\sim A) \cap (\sim B)}$$

(h) $((A \cup B) \supset C) \vdash (A \supset C) \cap (B \supset C)$

$$\dfrac{\dfrac{(A \cup B) \supset C \qquad \dfrac{\overset{(1)}{A}}{A \cup B}\cup_0}{(1)\ \dfrac{C}{A \supset C}\supset \text{I}} \quad \dfrac{(A \cup B) \supset C \qquad \dfrac{\overset{(2)}{B}}{A \cup B}\cup_1}{(2)\ \dfrac{C}{B \supset C}\supset \text{I}}}{(A \supset C) \cap (B \supset C)}$$

(i) $(A \supset C) \cap (B \supset C) \vdash (A \cup B) \supset C$.

$$\dfrac{\dfrac{A \cup B \quad (1) \qquad \dfrac{\dfrac{(A \supset C) \cap (B \supset C)}{A \supset C}\cap_0 E \qquad (2) \quad A}{C} \qquad \dfrac{\dfrac{(A \supset C) \cap (B \supset C)}{B \supset C}\cap_1 E \qquad (3) \quad B}{C}}{C}(2)(3)}{(A \cup B) \supset C}(1) \supset \mathrm{I}$$

(l) $((A \supset B) \supset A) \vdash \sim\sim A$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(1) \quad \sim A \qquad A \quad (2)}{\bot}\supset \mathrm{E}}{B}\bot_{int}}{A \supset B}(2) \supset \mathrm{I} \qquad (A \supset B) \supset A}{A} \qquad (1) \quad \sim A}{\bot}}{(\sim A) \supset \bot}(1)$$

Esercizio 2. Per le derivazioni (a) - (e) si decorino le deduzioni con termini del lambda calcolo con prodotti.