Logica Computazionale 2009
Exercise 2

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Exercise 1. Show that it is possible to falsify the following statement:

"If everybody loves someone, then someone is loved by everybody"

Hint: Construct a model $\mathcal{M} = (M, L)$, i.e., a set of individuals M and a two places relation L, such that $L(x, y)$ is an interpretation of *"x loves y"* and also satisfies the first condition (*"everybody likes someone"*), but for a given individual $m \in M$ it is not true that for all $i \in M$, $L(i, m)$.

3 marks

Exercise 2. Apply the semantic tableaux procedure, using the *breadth-first* search strategy, to show the validity of the following statement:

"If someone is loved by everybody, then everybody loves someone"

3 marks

Hint: We may use the *antecedent* Γ and the *succedent* Δ of a sequent $\Gamma \Rightarrow \Delta$ to represent a *queue*: we invert the rule which applies to the leftmost formula in the antecedent and in the succedent (to the left and to the right of the $\Rightarrow$ sign).[1] To implement the *breadth-first* [or the *depth-first*] strategy it suffices to add the new formulas *to the end of* [*in front of*] the queue in the antecedent and in the succedent. Here are the rules for the quantifiers in the *breadth-first* strategy and then the rules for the universal quantifier in the *depth-first* strategy:

[1] Here we assume that in a sequent $\Gamma \Rightarrow \Delta$, the antecedent Γ and the succedent Δ are lists of formulas. The letters $a_0, \ldots, a_i$ to the left of each sequent are names of generic individuals declared so far, with the aim of making all formulas in Γ true and all formulas in Δ false.

Breadth-First search-strategy

$$\frac{a_0,\dots,a_i,a_{i+1};\Gamma \Rightarrow \Delta, A(a_{i+1})}{a_0,\dots,a_i;\Gamma \Rightarrow \forall x.A,\Delta}\ \forall\text{-R} \qquad \frac{a_0,\dots,a_i;\Gamma, A(a_0),\dots,A(a_i),\forall x.A \Rightarrow \Delta}{a_0,\dots,a_i;\forall x.A,\Gamma \Rightarrow \Delta}\ \forall\text{-L}$$

$$\frac{a_0,\dots,a_i;\Gamma \Rightarrow \Delta, A(a_0),\dots,A(a_i),\exists x.A}{a_0,\dots,a_i;\Gamma \Rightarrow \exists x.A,\Delta}\ \exists\text{-R} \qquad \frac{a_0,\dots,a_i,a_{i+1};\Gamma, A(a_{i+1}) \Rightarrow \Delta}{a_0,\dots,a_i;\exists x.A,\Gamma \Rightarrow \Delta}\ \exists\text{-L}$$

Depth-First search-strategy

$$\frac{a_0,\dots,a_i,a_{i+1};\Gamma \Rightarrow A(a_{i+1}),\Delta}{a_0,\dots,a_i;\Gamma \Rightarrow \forall x.A,\Delta}\ \forall\text{-R} \qquad \frac{a_0,\dots,a_i;A(a_0),\dots,A(a_i),\forall x.A,\Gamma \Rightarrow \Delta}{a_0,\dots,a_i;\forall x.A,\Gamma \Rightarrow \Delta}\ \forall\text{-L}$$

Exercise 3. (i) Apply the semantic tableaux procedure to the sequent

$$\Rightarrow (\exists z \forall v.B(v,z)) \vee (A \to A)$$

Use a *breadth-first* search procedure, starting from the leftmost formula.

2 marks

(ii) What happens if you use a *depth-first* search procedure, starting from the leftmost formula?

2 marks

TOTAL 10 MARKS