Ring epimorphisms and tilting theory

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Plan:

1. Reminder on adjoint functors
2. Ring epimorphisms
3. Universal localization
4. Homological epimorphisms
5. Tilting modules and recollements
6. Tilting modules arising from ring epimorphisms
7. Classification of tilting modules

1. Reminder on adjoint functors

Two functors $q : \mathcal{C} \to \mathcal{D}$, $i : \mathcal{D} \to \mathcal{C}$ between preadditive categories $\mathcal{C}, D$ form an *adjoint pair* (q, i) if for any $C \in \mathcal{C}$, $D \in \mathcal{D}$ there is a natural iso

$$\mathrm{Hom}_{\mathcal{D}}(q(C), D) \cong \mathrm{Hom}_{\mathcal{C}}(C, i(D))$$

Then there are natural morphisms

$\eta : Id_{\mathcal{C}} \to iq$ *unit* (choose $D = q(C)$ and set $1_{q(C)} \mapsto \eta_C$)

$\theta : qi \to Id_{\mathcal{D}}$ *counit* (choose $C = i(D)$ and set $\theta_D \leftarrow 1_{i(D)}$)

such that

$q(C) \xrightarrow{q(\eta_C)} qiq(C) \xrightarrow{\theta_{q(C)}} q(C)$ coincides with $1_{q(C)}$, and

$i(D) \xrightarrow{\eta_{i(D)}} iqi(D) \xrightarrow{i(\theta_D)} i(D)$ coincides with $1_{i(D)}$.

1. Reminder on adjoint functors

Example. Let R, S be two rings and ${}_RQ_S$ a bimodule. Then

$$q = - \otimes_R Q : \mathrm{Mod}\text{-}R \to \mathrm{Mod}\text{-}S$$

$$i = \mathrm{Hom}_S(Q, -) : \mathrm{Mod}\text{-}S \to \mathrm{Mod}\text{-}R$$

form an adjoint pair (q, i) via the natural isomorphisms for M_R, N_S:

$$\mathrm{Hom}_S(M \otimes_R Q, N) \cong \mathrm{Hom}_R(M, \mathrm{Hom}_S(Q, N))$$

$$m \otimes x \mapsto f(m)(x) \leftarrow \quad f$$

References

General facts about adjoint functors and ring epimorphisms:

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