

Verona: Minicourse on
**Model Theoretic and Functor Theoretic Methods in Representation
Theory**

The use of functor categories in the representation theory of finite-dimensional algebras is long-established and has been very successful. More recently, the use of concepts, results and techniques from the model theory of modules has become more common and has been used to prove a number of results. In fact the two sets of techniques, functor-theoretic and model-theoretic, are very closely linked. This course of lectures will be an introduction to these sets of techniques and will explain some of the links between them.

1. Pp formulas and finitely presented modules

Pp formulas are projected systems of linear equations, their solution sets are the key definable sets in the model theory of modules. The lattice of pp formulas is essentially the lattice of pointed finitely presented modules.

2. The functor category

Functor categories generalise, but are not essentially different from, module categories. Every pp formula defines a finitely presented functor on finitely presented modules, indeed, these are the finitely generated subfunctors of the forgetful functor. More generally, there is an equivalence between the category of pp-pairs and the category of finitely presented functors.

3. The tensor embedding and purity

There is a full embedding of modules into a functor category. This induces the pure-exact structure on the category of modules and, at the same time, brings modules and the duals of the functors on them together in a single category.

4. The Ziegler spectrum and definable categories

The pure-injective modules are those which become injective in the functor category; they are also those which are algebraically compact. The indecomposable pure-injectives play a key role, algebraically and model-theoretically; they form the points of a topological space - the Ziegler spectrum - whose closed sets generate the definable subcategories of modules.

5. Duality

There is a duality which runs through this whole picture, manifesting itself at every level, from pp formulas, through modules and functors, to definable subcategories. It is a key technical and conceptual tool.

6. Localisation

Restriction to a definable subcategory is equivalent to localisation in the functor category. This is methodologically useful and also gives rise to various measures of complexity of module (and, more generally, definable) categories.

7. Perspectives

A broad look at what is known and at current open questions and areas of exploration.