Some recent results on pretorsion theories in lextensive categories

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Pretorsion theories are defined as “non-pointed torsion theories”, where the zero object and the zero morphisms are replaced by a class of “trivial” objects and a suitable ideal of morphisms respectively. Thus, the notion of pretorsion theory can be defined in any arbitrary category $\mathbb{C}$, starting from a pair $(\mathcal{T}, \mathcal{F})$ of full replete subcategories of $\mathbb{C}$ where $\mathcal{T}$ and $\mathcal{F}$ consist of the classes of “torsion” and “torsion-free” objects, and whose intersection defines the class of “trivial objects” [5, 6]. Under some natural mild assumptions, this new setting allows one to obtain many of the basic results that are well known for classical torsion theories in the abelian and homological frameworks.

In this talk we shall first recall the basic properties of pretorsion theories with a focus on the context of lextensive categories [3]. We discuss some examples in the category $\mathsf{PreOrd}(\mathbb{C})$ of (internal) preorders in pretopos $\mathbb{C}$ [1, 2, 5] and in the category Cat of small categories [4]. Then, we present a way to associate a pointed “stable” category $\mathsf{Stab}(\mathbb{L})$ to any given pretorsion theory in a lextensive category $\mathbb{L}$, showing that the canonical functor $\Sigma\colon \mathbb{L} \to \mathsf{Stab}(\mathbb{L})$ from $\mathbb{L}$ into the stable category satisfies a suitable universal property and sends the pretorsion theory into a “genuine” torsion theory in the stable category [2, 3]. Thus, this construction gives us the “universal torsion theory” associated with the given pretorsion theory.

References

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